Lecture 04 - Inference with Two Qualitative Variables

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Inference with Two Qualitative Variables

We are going to begin our discussion of statistical inference with a simple case of two qualitative variables.

As a refresher, a qualitative variable is one whose values indicate either 1) nominal categories or, 2) ordinal categories.

In either case, comparison of means is inappropriate without an adjustment to the standard t-tests you've discussed in prior statistics classes.

Inference with Two Qualitative Variables - Outline

Contingency Tables

- Contingency table (a.k.a., a cross-tab).
  - Illustrates the joint distribution of two variables
  - Defined by the # of rows and the # of columns (R x C)
- 2 x 2 contingency table

	$\sim X$	X	Total
$\sim Y$	f_{11}	f_{12}	$f_{1\cdot}$
Y	f_{21}	f_{22}	$f_{2\cdot}$
Total	$f_{\cdot 1}$	$f_{\cdot 2}$	n

Contingency Tables

- An alternative representation
  - Relative frequencies instead of frequencies
  - Joint probabilities in the middle, unconditional probabilities on the margins

	$\sim X$	X	Total
$\sim Y$	p_{11}	p_{12}	$p_{1\cdot}$
Y	p_{21}	p_{22}	$p_{2\cdot}$
Total	$p_{\cdot 1}$	$p_{\cdot 2}$	n

- $p_{22} = p(X \cap Y)$
- $p_{2\cdot} = p_{21} + p_{22} = p(\sim X \cap Y) + p(X \cap Y) = p(Y)$

Relationship Between Two Categorical Variables

- Example: juvenile and adult arrests

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	40	20	60
Yes (A)	10	30	40
Total	50	50	100

- Are juvenile and adult arrests independent?
  - Does knowing that someone was arrested as a juvenile help us predict whether or not they are arrested as an adult?
  - Not necessary to be a perfect prediction - only *better than chance* or *more likely than not.*

Conditional Probabilities

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	40	20	60
Yes (A)	10	30	40
Total	50	50	100

- Are juvenile & adult arrest independent?
  - Recall the rule of independence: If $p(A \mid B) = p(A)$ or $p(B \mid A) = p(B)$ then A and B are independent.
  - Are J and A independent in this cross-tab?
    - $p(A) = \frac{40}{100} = 0.4$
    - $p(A \mid J) = \frac{30}{50} = 0.6$
  - Not independent - worth investigating more.

Conditional Probabilities Continued

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	40	20	60
Yes (A)	10	30	40
Total	50	50	100

What is the nature of the relationship between juvenile and adult arrest?

- Arrested juveniles are 200% **more likely** to be arrested as an adult than non-arrested juveniles.
  - $p(A \mid J) = \frac{30}{50} = 0.6$
  - $p(A \mid \sim J) = \frac{10}{50} = 0.2$
- % difference $= \frac{0.60 - 0.20}{0.20} * 100 = 200.0\%$

Setting Up a Hypothesis Test

Now it's time to introduce you to the formal way to set up a hypothesis test.

A formal hypothesis test includes five steps:

1) Formally state your hypotheses
2) Choose a probability distribution
3) Make decision rules
4) Compute test statistic
5) Make a decision about the null hypothesis

Step 1. Formally State Hypotheses

Two hypotheses are stated here:

1) The Alternative Hypothesis (H_A or H_1)
2) The Null Hypothesis (H_0)

The former is your hypothesis for the test and, depending upon the test being conducted, can be directional or non-directional.

The latter is the antithesis of the alternative hypothesis, and reflects either a) no effect or, b) an effect in the opposite direction as expected in H_A.

Step 2. Choose Probability Distribution

There are numerous probability distributions we will use throughout this class. This is the step where you identify that distribution.

These distributions are often dependent upon the degrees of freedom of your test.

Lower degrees of freedom = wider distributions, generally. This make it *more difficult* to reject the null hypothesis.

Step 3. Make Decision Rules

This is where you select your alpha level and identify its associated critical score for your test statistic.

The alpha level is the degree to which you will tolerate falsely rejecting a true null hypothesis. Standard levels include .05 (5 times in 100), .01 (1 time in 100), and .001 (1 time in 1000).

A **critical score** is merely the value of the test statistic at that alpha level. Any value exceeding that occurs in the **critical region** where you will reject the null hypothesis.

Step 4. Compute Test Statistic

Here is where you apply the formula specific to that probability distribution.

Your result can then be plotted on this probability distribution and you can determine the probability that you would expect a result like that is the null hypothesis were true.

Step 5. Make a Decision About H_0

If the test statistic falls within the **critical region** this indicates that the probability of finding a test statistic of that size or larger (given the null hypothesis is true) is smaller than your alpha level.

The language in this step is always in reference to the null hypothesis - we either accept it or we reject it.

The conclusion must be tailored to your hypothesis. Directional and non-directional hypotheses will result in differently worded conclusions.

Hypothesis Testing Errors

In hypothesis testing, there are two types of error that we must be aware of. A type I error (**false positive**) is when we reject the null hypothesis, but in fact it is true. A type II error (**false negative**) is when we accept the null hypothesis, but it in fact is false.

	Decision	
Reality	Accept H_0	Reject H_0
H_0 is true	Correct decision	Type I Error (α)
H_0 is false	Type II Error (β)	Correct decision

We can reduce the probability of a type I error by decreasing level of significance. However, if we decrease the probability of falsely rejecting the null hypothesis (say 0.001), we necessarily increase the probability of falsely accepting the null hypothesis.

Chi-Square (χ^2) Test of Independence

- Formal test of whether two categorical (N,O) variables are **statistically independent**.
  - Compare *observed* frequencies with *expected*
  - χ^2 relies on the multiplication rule for independent events
    - $p(A \cap B) = p(A) * p(B)$
- How much does our contingency table depart from what we would **expect** if the two variables were truly independent?

Chi-Square (χ^2) Test of Independence

Formula for (χ^2):

$$\chi^2 = \sum_{k=1}^{k} \frac{(f_{obs} - f_{exp})^2}{f_{exp}}$$

Where: 1) f represents *frequency* 2) f_{obs} is *observed frequency* 3) f_{exp} is *expected frequency* 4) and k is the total number of cells in the table

χ^2 is associated with its own probability distribution (refer to the appendix in your textbook).

The table is organized by **degrees of freedom** which is calculated as follows:

$$df = (\#\ \text{rows} - 1) * (\#\ \text{columns} - 1)$$

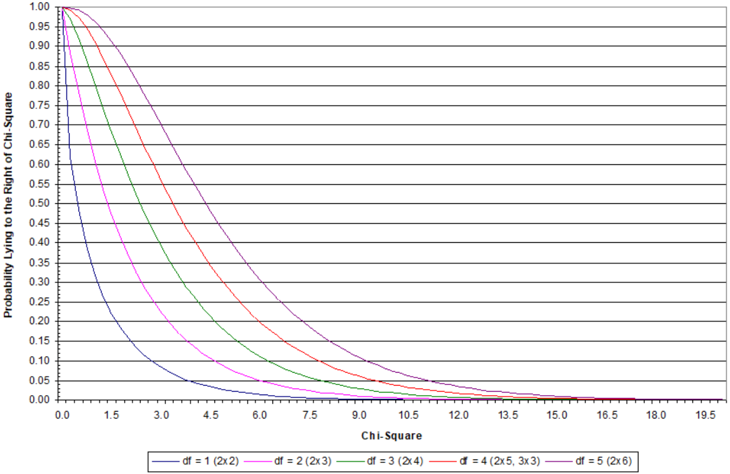
Chi-Square Probability Distribution

- Probability function for χ^2 (don't freak out - you don't have to compute this):

$$p(x) = \frac{(1/2)^{(k/2)}}{\Gamma(k/2)} x^{k/(2-1)} e^{-x/2}$$

- x represents a given χ^2 value that you want to know the probability of
- k represents the **degrees of freedom**
- $\Gamma(\cdot)$ represents the **gamma** function
  - An extension of the factorial function (e.g., $z!$) to non-integer and complex numbers

Chi-Square Probability Distribution



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Chi-Square Probability Distribution - Critical Values

A **critical value** is the value of some probability distribution statistic where we are comfortable calling the result *unlikely* enough to be statistically **significant**.

You can find the full version of the table below in your text's appendix.

	Significance Level (α)			
Degrees of Freedom	.10	.05	.01	.001
1	2.706	3.841	6.635	10.827
2	4.605	5.991	9.210	13.815
3	6.251	7.815	11.341	16.268
4	7.779	9.448	13.277	18.465
5	9.236	11.070	15.086	20.517

Alternative Method to Find the Chi-Square Critical Value

An easy way to identify the χ^2 critical value is to use the qchisq() function in R.

The qchisq() function takes the following general form:

qchisq(p, df, lower.tail=FALSE)

Where *p* is your alpha level (.05, .01, or .001), *df* is your degrees of freedom, and lower.tail=FALSE tells R to return the χ^2 value necessary to reject the null hypothesis (i.e., the value that separates out .05 probability points at or above it).

Juvenile & Adult Arrest Hypothesis Test

- Research Question
  - Is juvenile arrest a predictor of adult arrest?
- Step 1) Formally state hypotheses
  - Hypotheses stated in terms of χ^2
    - Under H_0, $f_{obs} = f_{exp}$, so χ^2 will be zero
    - Under H_1, $f_{obs} \neq f_{exp}$, so χ^2 will be positive
    - Note - you cannot do a directional (one-tailed) significance test with χ^2

$$H_1 : \chi^2 > 0$$

$$H_0 : \chi^2 = 0$$

Juvenile & Adult Arrest Hypothesis Test

- Step 2. Choose a probability distribution
  - χ^2 distribution
  - $df = (\#\text{rows} - 1) * (\#\text{columns} - 1) = (2 - 1) * (2 - 1) = 1$

Juvenile & Adult Arrest Hypothesis Test

- Step 3. Make decision rules (refer to table above or in the text appendix)
  - $\alpha = .05$ (I will always provide this for you)
  - $\chi^s_{crit} = 3.841$
  - Reject H_0 if the test statistic (TS) > 3.841

I took this critical value from a table, but could also find it using the following code in inline code (within grave accents):

r qchisq(.05, 1, lower.tail=FALSE)

Juvenile & Adult Arrest Hypothesis Test

- Compute the test statistic

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	40	20	60
Yes (A)	10	30	40
Total	50	50	100

- Get expected frequencies (should sum to n): $f_{exp} = \frac{RM * CM}{n}$

Cell	f_{obs}	f_{exp}
11	40	$(60 * 50)/100 = 30$
12	20	$(60 * 50)/100 = 30$
21	10	$(40 * 50)/100 = 20$
22	30	$(40 * 50)/100 = 20$
	100	100

Juvenile & Adult Arrest Hypothesis Test

- Compute test statistic (continued)
  - We now have two different contingency tables:
    - One showing the **observed** relationship between A & J
    - One showing the **expected** relationship between A & J where the are independent by construction.

Observed Cross-Tab

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	40	20	60
Yes (A)	10	30	40
Total	50	50	100

$p(A) = .40$
$p(A \mid J) = .60$

Expected Cross-Tab

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	30	30	60
Yes (A)	20	20	40
Total	50	50	100

$p(A) = .40$
$p(A \mid J) = .40$

Juvenile & Adult Arrest Hypothesis Test

- Compute the test statistic (continued)
  - Compute the χ^2 statistic

$$\chi^2 = \sum \frac{(f_{obs} - f_{exp})^2}{f_{exp}}$$

Cell	f_{obs}	f_{exp}	$f_{obs} - f_{exp}$	$(f_{obs} - f_{exp})^2$	$(f_{obs} - f_{exp})^2 / f_{exp}$
11	40	30	10	100	3.33
12	20	30	-10	100	3.33
21	10	20	-10	100	5.00
22	30	20	10	100	5.00
	100	100			16.66

Step 4: R Method - via creating data in R

```
cell<-c("11","12","21","22") # create character variable to indicate cell positions
obs<-c(40,20,10,30) # input observed frequencies
exp<-c((60*50)/100,(60*50)/100,(40*50)/100,(40*50)/100) # calculate expected frequencies
dev<-obs-exp # compute differences
dev_sq<-dev^2 # square the differences
chi<-dev_sq/exp # divide differences by expected frequencies
chi2<-sum(chi) # sum chi2 values for individual cells
chi2 # print out chi square value
```

```
## [1] 16.66667
```

Step 4: Automatic Method

```
juv_adult_arr<-matrix(c(40,10,20,30),ncol=2) # column values in pairs
# (top to bottom, left to right)
chisq.test(juv_adult_arr, correct=FALSE)# run chi-square test
```

```
##
##  Pearson's Chi-squared test
##
## data:  juv_adult_arr
## X-squared = 16.667, df = 1, p-value = 0.00004456
```

The *correct=FALSE* part is required to suppress a test correction that makes it harder to reject the null hypothesis. Normally, it is best to leave that as correct=TRUE in practice but for this example I wanted the χ^2 to be the same across all examples.

You cannot use this method for submitting your work, but can use it to check your math!

Juvenile & Adult Arrest Hypothesis Test

- Step 4. Compute the test statistic (continued):
  - $TS = 16.66$
- Step 5. Make a decision about the null hypothesis
  - Reject H_0
- Conclusion?
  - There is a significant relationship between juvenile arrest & adult arrest.
  - Or, juvenile arrest & adult arrest are not independently distributed.

A Comment on Interpretation of Step 5.

- What does it mean to reject the null hypothesis of no relationship between juvenile & adult arrest?
  - **Does not** mean that all (or even most) people arrested as juveniles will be arrested as adults.
  - Only means that people arrested as juveniles are **more likely** to be arrested as adults than if they had not been arrested as juveniles.
- Does not imply causality, either!
  - This result only confirms a statistical association exists between the two variables.

Military Service and Drug Use

	Military Service?		
Use Drugs?	No ($\sim M$)	Yes (M)	Total
No ($\sim D$)	3426	407	3833
Yes (D)	629	108	737
Total	4055	515	4570

- Are drug use & military service independent?
  - $p(D) = \frac{737}{4570} = 0.161$
  - $p(D \mid M) = \frac{108}{515} = 0.21$
- Research Question
  - Is there a relationship between military service & drug use?

Military Service and Drug Use - Hypothesis Test

- Step 1: Formally state hypothesis
  - $H_1 : \chi^2 > 0$
  - $H_0 : \chi^2 = 0$
- Step 2: Choose a probability distribution
  - χ^2 distribution
  - $df = (2-1) * (2-1) = 1$
- Step 3: Make decision rules
  - $\alpha = .01$
  - $\chi^2_{crit} = 6.635$
  - Reject H_0 if TS > 6.635

Military Service and Drug Use - Hypothesis Test

- Step 4: Compute test statistic

Cell	f_{obs}	f_{exp}	$(f_{obs} - f_{exp})^2/f_{exp}$
11	3426	$(3833 * 4055)/4570 = 3401.05$	0.18
12	407	$(3833 * 515)/4570 = 431.95$	1.44
21	629	$(737 * 4055)/4570 = 653.95$	0.95
22	108	$(737 * 515)/4570 = 83.05$	7.50
	4570	4570.00	10.07

- Step 5: Make a decision about the null hypothesis
  - TS = 10.07
  - Conclusion? Reject H_0, conclude that military service and drug use are **not** independent of one another.

Computational Formula for χ^2

- Computational formula:
$$\chi^2 = \left(\sum_{k=1}^{k} \frac{f_{obs}^2}{f_{exp}} \right) - n$$
- An advantage is that this formula does not require you to compute squared deviations
- **Caution**: Do not forget to subtract off n

Employment & Delinquency - Hypothesis Test

	Employment Status			
Delinquent Acts	0 Hours	1-20 Hours	21+ Hours	Total
0	3642	1605	1441	6688
1 - 4	637	374	427	1438
5+	318	201	289	808
Total	4597	2180	2157	8934

- Research question: Is employment associated with delinquency?
- **Step 1**: Formally state hypothesis
  - $H_1 : \chi^2 > 0$
  - $H_0 : \chi^2 = 0$

Employment & Delinquency - Hypothesis Test

	Employment Status			
Delinquent Acts	0 Hours	1-20 Hours	21+ Hours	Total
0	3642	1605	1441	6688
1 - 4	637	374	427	1438
5+	318	201	289	808
Total	4597	2180	2157	8934

- **Step 2**: Choose a probability distribution
  - χ^2 distribution
  - $df = (3 - 1) * (3 - 1) = 4$
- **Step 3**: Make decision rules
  - $\alpha = .001$
  - $\chi^2_{crit} = 18.465$
  - Reject H_0 if TS > 18.465

Employment & Delinquency - Hypothesis Test

- **Step 4**: Calculate the test statistic

Cell	f_{obs}	f_{exp}	$f^2_{\text{obs}}/f_{\text{exp}}$
1	3642	3441.32	3854.38
2	1605	1631.95	1578.50
3	1714	1614.73	1285.96
4	1524	739.92	548.40
5	1179	350.89	398.63
6	671	347.19	525.16
7	221	415.76	243.23
8	109	197.16	204.91
9	8604	195.08	428.14
	8934	8934.00	9067.31

Employment & Delinquency - Hypothesis Test

	Employment Status			
Delinquent Acts	0 Hours	1-20 Hours	21+ Hours	Total
0	3642	1605	1441	6688
1 - 4	637	374	427	1438
5+	318	201	289	808
Total	4597	2180	2157	8934

- **Step 5**: TS = 9067.31-8934 = 133.31
- Conclusion? Reject H_0, conclude that employment and delinquency are **not** independent.

Measures of Association

- A chi-square test will provide an answer as to whether the relationship between two variables is **statistically significant**
  - But...The test says nothing about whether the relationship is **substantively significant**
- Measures of association inform us about the strength of the relationship
  - Ex., How strong is the relationship between juvenile & adult arrest, or between military service & drug use?

Measures of Association

MOA #1: Phi (ϕ) - only applicable to 2x2 tables

$$\phi = \sqrt{\frac{\chi^2}{\mathrm{n}}}$$

If you square the value of ϕ you can interpret this statistic as the percentage of the variance in one variable that may be explained by the other.

Measures of Association

MOA #2: Contingency (C) - works for any size table.

$$C = \sqrt{\frac{\chi^2}{\mathrm{n} + \chi^2}}$$

MOA #3: Cramer's V (V) - works for any size table

$$V = \sqrt{\frac{\chi^2}{n * \min(r-1, c-1)}}$$

Where r is rows, c is columns, and $min(r-1, c-1)$ returns the smaller number of the two.

Measures of Association

MOA #4: Lambda (λ)

$$\lambda = \sqrt{\frac{(\sum f_{IV}) - f_{DV}}{\mathrm{n} - f_{DV}}}$$

Where f_{IV} are the largest f's in each category of the independent variable, and f_{DV} is the largest marginal of the dependent variable.

Measures of Association

- Useful rule for interpreting measures of association (except Lambda)
  - 0.00 to 0.29 = Weak relationship
  - 0.30 to 0.59 = Moderate relationship
  - 0.60 to 1.00 = Strong relationship
- M.O.A.'s for ordinal variables can also be positive or negative
  - Closer to 0.0 $\Rightarrow$ Weaker relationship
  - Closer to +1.0 $\Rightarrow$ Stronger positive relationship
  - Closer to –1.0 $\Rightarrow$ Stronger negative relationship

Measures of Association - Juvenile & Adult Arrest

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	40	20	60
Yes (A)	10	30	40
Total	50	50	100

$$\phi = \sqrt{\frac{\chi^2}{n}} = \sqrt{\frac{16.667}{100}} = 0.408$$

$$V = \sqrt{\frac{\chi^2}{n * \min(r-1, c-1)}} = \sqrt{\frac{16.667}{100 * (2-1)}} = 0.408$$

ϕ and Cramer's V will always produce the same answer in a 2x2 table.

Measures of Association - Juvenile & Adult Arrest

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	40	20	60
Yes (A)	10	30	40
Total	50	50	100

$$C = \sqrt{\frac{\chi^2}{\text{n} + \chi^2}} = \sqrt{\frac{16.667}{100 + 16.667}} = 0.378$$

$$\lambda = \sqrt{\frac{(\sum f_{IV}) - f_{DV}}{\text{n} - f_{DV}}} = \sqrt{\frac{(40+30) - 60}{100 - 60}} = 0.5$$

λ is a measure of proportionate reduction in error (PRE), telling us how better we can predict the DV using information about the IV.

Measures of Association - Military Service & Drug Use

	Military Service?		
Use Drugs?	No ($\sim M$)	Yes (M)	Total
No ($\sim D$)	3426	407	3833
Yes (D)	629	108	737
Total	4055	515	4570

$$V = \sqrt{\frac{\chi^2}{n * \min(r-1, c-1)}} = \sqrt{\frac{10.068}{4570 * (2-1)}} = 0.047$$

Measures of Association - Military Service & Drug Use

	Military Service?		
Use Drugs?	No ($\sim M$)	Yes (M)	Total
No ($\sim D$)	3426	407	3833
Yes (D)	629	108	737
Total	4055	515	4570

$$C = \sqrt{\frac{\chi^2}{\text{n} + \chi^2}} = \sqrt{\frac{10.068}{4570 + 10.068}} = 0.047$$

$$\lambda = \sqrt{\frac{(\sum f_{IV}) - f_{DV}}{\text{n} - f_{DV}}} = \sqrt{\frac{(3426 + 407) - 3833}{4570 - 3833}} = 0$$

Employment and Delinquency Example

	Employment Status			
Delinquent Acts	0 Hours	1-20 Hours	21+ Hours	Total
0	3642	1605	1441	6688
1 - 4	637	374	427	1438
5+	318	201	289	808
Total	4597	2180	2157	8934

$$V = \sqrt{\frac{\chi^2}{n * \min(r-1, c-1)}} = \sqrt{\frac{133.31}{8934 * (3-1)}} = 0.086$$

$$C = \sqrt{\frac{\chi^2}{\mathrm{n} + \chi^2}} = \sqrt{\frac{133.31}{8934 + 133.31}} = 0.121$$

Employment and Delinquency Example

	Employment Status			
Delinquent Acts	0 Hours	1-20 Hours	21+ Hours	Total
0	3642	1605	1441	6688
1 - 4	637	374	427	1438
5+	318	201	289	808
Total	4597	2180	2157	8934

$$\lambda = \sqrt{\frac{(\sum f_{IV}) - f_{DV}}{\mathrm{n} - f_{DV}}} = \sqrt{\frac{(3642 + 1605 + 1441) - 6688}{8934 - 6688}} = 0$$

An Alternative Test of Independence in 2x2 Tables

- A z-test for two sample proportions

$$z = \frac{p_1 - p_2}{\sqrt{\pi * (1 - \pi)} * \sqrt{\frac{n_1 + n_2}{n_1 * n_2}}}$$

  - π is the **unconditional** probability of an event for the entire sample
  - p_1 and p_2 are the **conditional** probabilities for the two subsamples of interest
  - n_1 and n_2 are the number of cases in the two subsamples (i.e., column totals)

An Alternative Test of Independence in 2x2 Tables

Probability distribution for z is shown in the appendix of your text, but use this one instead for now.

	Significance Level (α)			
	.10	.05	.01	.001
One-tailed test	1.282	1.645	2.326	3.090
Two-tailed test	1.645	1.960	2.576	3.291

The advantage of using z over χ^2 is that we can test **directional hypotheses**.

We will give more formal treatment to the z distribution in an upcoming lecture.

A z-test for Two Proportions - Juvenile & Adult Arrest

	Juvenile Arrest?		
Adult Arrest?	No ($\sim J$)	Yes (J)	Total
No ($\sim A$)	40	20	60
Yes (A)	10	30	40
Total	50	50	100

- Let's try the z-test:
  - **Unconditional probability** of arrest as an adult
    - $\pi = p(A) = 40/100 = 0.40$
  - **Conditional probabilities** of arrest as an adult
    - $p_J = p(A \mid J) = 30/50 = 0.60$
    - $p_{\sim J} = p(A \mid \sim J) = 10/50 = 0.20$

A z-test for Two Proportions - Juvenile & Adult Arrest

- **Step 1**: Formally state hypotheses
  - Hypotheses stated in terms of π, the population probability of arrest as an adult
  - Under H_1, adult arrest is more likely when a juvenile arrest (J) has occurred than when it has not $(\sim J)$.

$$H_1 : \pi_J > \pi_{\sim J}$$

$$H_0 : \pi_J \leq \pi_{\sim J}$$

  - Notice that this is a directional test. Specifically, a *right-tailed* test. How would we have written it if we wanted a *left-tailed* test instead?

A z-test for Two Proportions - Juvenile & Adult Arrest

- **Step 2**: Choose a probability distribution
  - z distribution
- **Step 3**: Make decision rules (refer to table)
  - $\alpha = 0.05$ (one-tailed)
  - $z_{crit} = 1.645$
  - Reject H_0 if TS > 1.645

A z-test for Two Proportions - Juvenile & Adult Arrest

- **Step 4**: Compute the test statistic

$$z = \frac{p_1 - p_2}{\sqrt{\pi * (1-\pi)} * \sqrt{\frac{n_1+n_2}{n_1 * n_2}}} = \frac{0.6 - 0.2}{\sqrt{.4 * .6} * \sqrt{\frac{50+50}{2500}}} = \frac{0.4}{0.0980} = 4.08$$

- **Step 5**: Make a decision about the null hypothesis
  - Reject H_0, conclude that the probability of adult arrest is significantly *higher* if you are arrested as a juvenile.

A z-test for Two Proportions - Juvenile & Adult Arrest

- The fact that we obtained the test statistic we did is not a coincidence
  - $z = 4.08$ (from the z test)
  - $\chi^2 = 16.66$ (from χ^2 test)
  - Notice - $4.08 = \sqrt{16.66}$, or $16.66 = 4.08^2$
- χ^2 and z are very closely related
  - Your conclusion from a χ^2 test will be equivalent to conducting a two-tailed z test.
  - A z test has the added flexibility of allowing us to do one-tailed tests.

Two Questions

Time for your two questions!