

## White Paper

### Group 1:

### Project: Risk model: Generalized Autoregressive Conditional Heteroscedastic (GARCH)-GJR Model

Applied Quantitative Risk Management – ECON 6295

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## Investment Case

Our client wants to forecast the portfolio risks involving shorting 100 stocks of Spy (S & P 500) and long hedging 100 stocks of Apple. Our client has a cash reserve of \$10,000 and our respective individual stock prices are \$186.79 for Apple and \$281.12 for Spy (S & P 500) .

To analyze the portfolio risks for these long and short hedging strategies, we also analyzed the historic, adjusted returns of each trading day over a period of one year. This distribution of adjusted returns provides context, and an estimated baseline of the distribution of each security's volatility for the given year. However, each year's volatility changes due to circumstances, so we wanted to simulate an expected daily range of volatility over the same period of time. Simulating risks generalizes the expected volatility any given year, which allows for a robust estimate of risk.

We used the GARCH-GJR risk model to simulate the distribution of daily volatility over a one year window. In brief, GARCH modeling involves calculating the current time period volatility based on the previous time period's volatility. Unlike GARCH (1,1), we used GARCH-GJR because we wanted more conservative estimates on daily returns, and more liberal estimates on daily risks. When GARCH-GJR calculates the current period's volatility it has higher weights on the previous day's risks, than on previous day's returns. This essentially preserves the multi-day effects of risks than expected multi-day returns in a time period, which emphasizes a risk-averse model when hedging the client's portfolio. We then simulated a 10-day forecasted volatility based on our simulated distributions.

Once we obtained simulated distributions of volatility, value at risk analysis was performed using the hedging specifications and portfolio holdings of the client to estimate loss (in dollars) for this hedging operation.

## Exploratory Data Analysis

Our dataset comes from Yahoo finance's API for Apple and Spy (S & P 500 index). We used one year of historical adjusted closing price from the year 2022-01-01 to 2023-11-01. We then calculated the daily historical returns for each security using the adjusted closing price. Overall, Spy is appraised at a higher price than Apple, which is not a surprise since it is a benchmark security of the S & P 500 index.

Figure 1: Box-plot of Apple and SPY

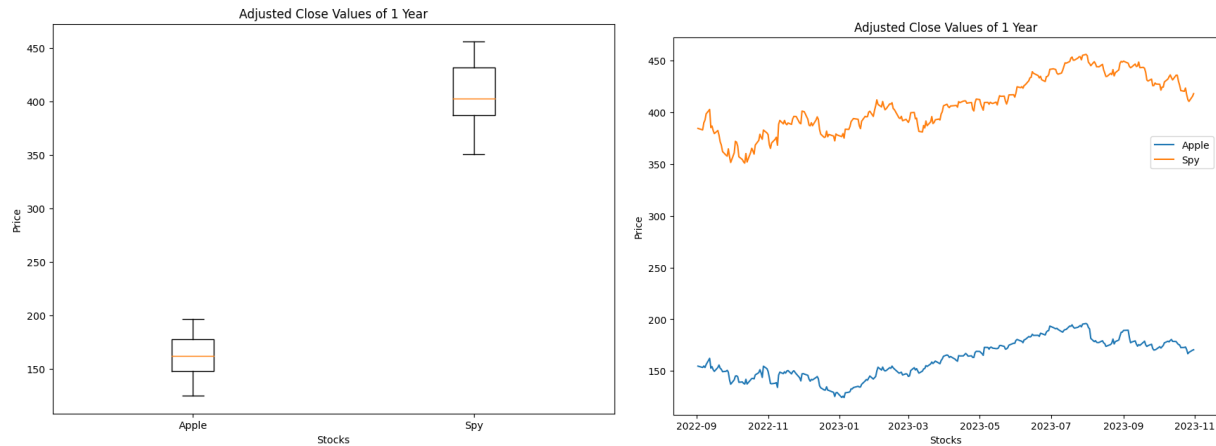


Figure 2: Historical adjusted close prices for Apple and Spy from September, 2022 to November, 2023.

## Model Description

In this analysis, we chose the asymmetric Generalized Autoregressive Conditional Heteroscedastic (GARCH) model, Glosten, Jagannathan and Runkle (GARCH GJR). The main differences between GARCH (1,1) and GARCH GJR, is that the latter reflects the asymmetric impact of past volatility shocks on current volatility. This allows the model to account for the fact that negative and positive shocks are weighted differently on volatility. In short, the parameter (gamma) weights daily losses more than daily returns, which makes the model more conservative by inflating volatility. For reference, GJR-GARCH is calculated when  $p = q = 1$ , as follows:

$$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1})\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

In our model, we chose to model daily volatility, so we can expect our model to simulate daily volatility higher than the historic daily volatility. Using GARCH GJR offers more generalizable daily volatility for any given year for our securities. This means that our results are robust for any given year, compared to relying on the intricacies of historical daily volatility.

Based on the equation, daily volatility is calculated from the prior day's volatility plus any returns and loss..... [add more here]

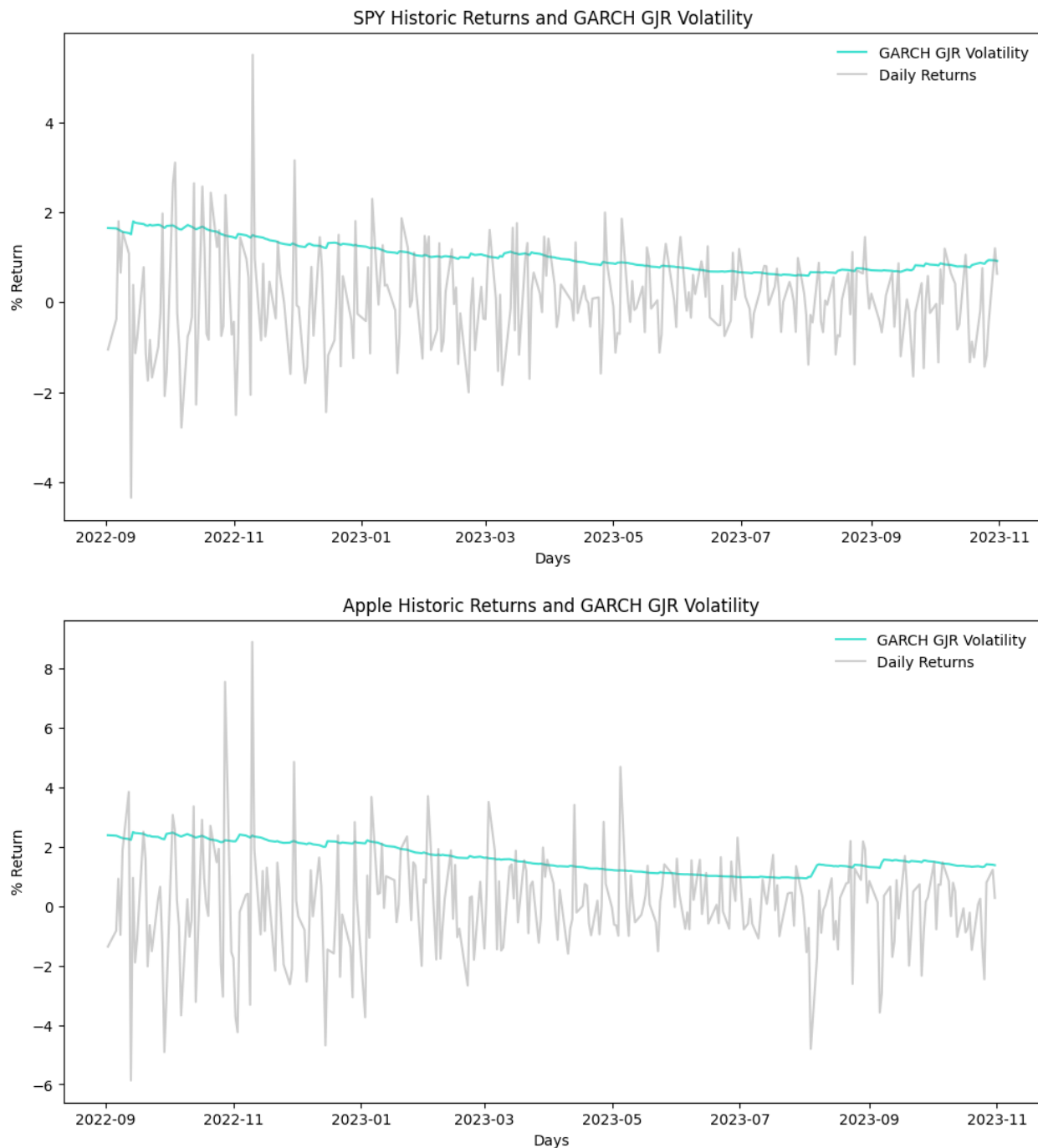


Figure 3: GARCH GRJ (in blue) results on daily volatility (in grey) for Apple and Spy from September, 2022 to November, 2023.

We then forecast 10 trading day volatilities, where the 10th day volatility is used in calculating the portfolio risk in our shorting of Spy and longing of Apple. We compared this portfolio value to the daily simulated volatility from our GARCH GJR models to construct loss distributions. We calculated the probability of our portfolio loosing

in Value-at-Risk (VaR) to examine and evaluate the market risk of holdings. Due to the lack of daily profit and loss data of holdings, we take one year of sample to backward testing. And further, used to forecast the 10 days value at risk. Aftermath, we conclude the GJR- GARCH works well in Value at Risk (VaR) forecasting.

The generalized autoregressive conditional heteroskedasticity (GARCH) process is an econometric term developed in 1982 by Robert F. Engle, who later won the Nobel Memorial Prize for Economics in 2003. GARCH describes an approach to estimate volatility in financial markets. Later Glosten, Jagannathan and Runkle advanced it and named it GARCH-GJR. The GJR-GARCH model extends the basic GARCH(1,1) by weighting the previous time period's volatility more than the previous time period's returns for each calculation of the current time period. In the GARCH, (1,1) in parentheses notation refers to the first number as how many ARCH or autoregressive lags appear in the equation, while the second one means how many moving average lags are specified in number of GARCH terms. In some cases, more than one lag is needed to have good variance forecasts.

## Value at Risk (VaR)

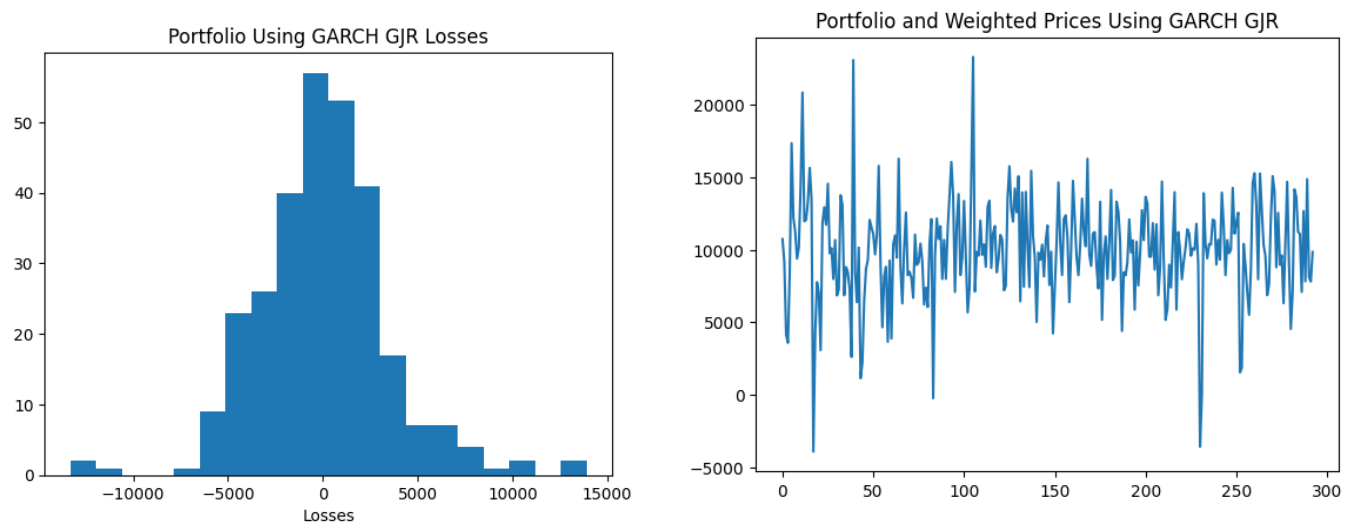


Figure 4: GARCH GRJ loss distribution (left) and GARCH GJR weighted prices (right) from the portfolio.

Value at risk is the percentile of the loss distribution or vice versa. Specific confidence levels of the loss distribution correspond to dollar amounts of loss or gain and their probabilities. It was found that the expected loss at the 5% risk at the next 10 days is \$5793.16 or greater.

## Backtesting, Assumptions and Limitations:

Backtesting is used to measure the performance of VaR that has been estimated. As provided in Table 1, the MAPE criteria is most indicative of a successful model if the weight is above .30, so for both Apple and SPY it seems our MAPE value meets this criteria so the model is sound enough for use. Additionally, compared to historical daily volatility  $\mu$ , both models reflect a similar  $\mu$  with the expected difference that GJR simulates 'hot' or slightly overshoots the model when penalizing risks more than returns.

Index	MAE	MSE	MAPE
Apple	0.935	0.878	0.409
SPY	0.385	0.152	0.782

Though VaR can offer volatility probabilities, it fails to account how volatility clusters during the year, and from internal and external shocks. For example, VaR may not accurately predict the losses during periods of heightened market volatility. Essentially, it is during this period that the distributions of volatility are changing in some way, and VaR's current loss distribution may not reflect those changes. As a result, extreme and or frequent losses break the expected predicted value from the VaR loss distributions. However, VaR with GARCH capture the conditional volatility into the account and make the prediction more accurate than before.

## References

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