## Data Science Methods for Clean Energy Research

Week 4, Lecture 1: Hypothesis testing and p-values

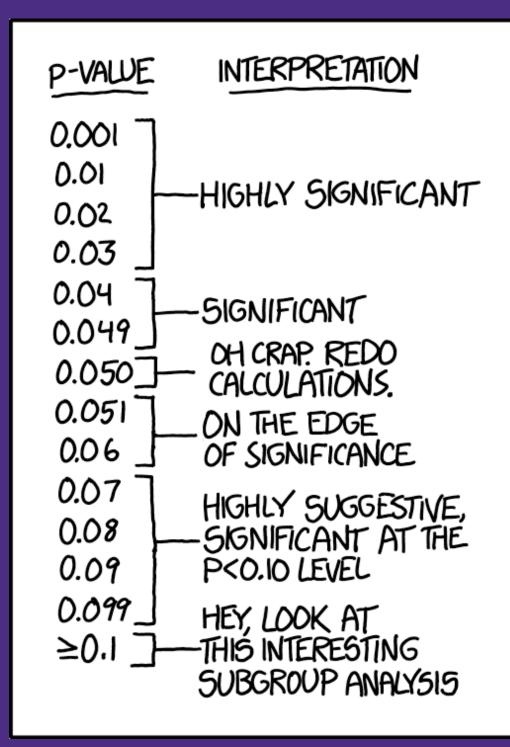
January 23, 2017

Please start a new Jupyter notebook and load the same software stack we have been using.

We will use later in the lecture

No file download





#### P values via xkcd



#### **Outline**

- > Reading assignment for W4
  - Please do this
- > Quick review
- > Warmup
- > One sample t-test
  - Theory
  - Practice
- > Nuzzo, the ASA, and the onslaught against p-values



### Key concepts from last time

- > Histogram
- > PDFs
- > Central limit theorem



### Warm up – 5 minutes discussion

- > Choose someone at the table who is not wearing jeans - they are the facilitator.
  - Tie break: whoever has birthday closest to today
- > Each person
  - 1. Give an example of data you have collected (either in research or class)
  - 2. What did you do with the data? (e.g., error analysis, graphing, p-value calculation)
  - 3. Did you make any scientific conclusions about the data?
  - 4. What statistical methods (if any) were the conclusions based on?

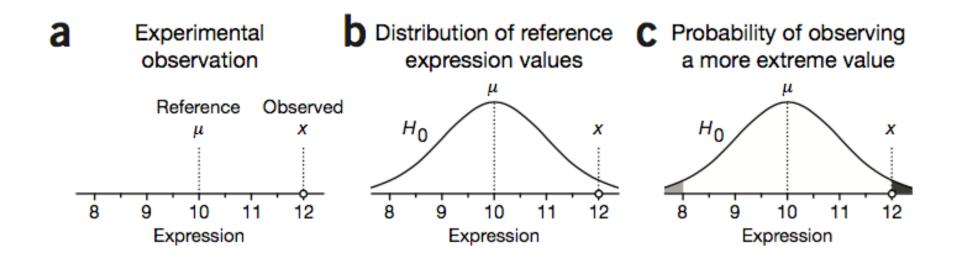


# The concept of a hypothesis test in statistics

- > A scientific hypothesis: Adding additional hydrophobic groups to conjugated polymers will accelerate their crystallization rate in polar solvents
- > A scientific experiment to test the hypothesis: We will synthesize P3AT of various –R length, and measure crystallinity as a function of time in different solvents and compare to consensus values from literature
- A statistical hypothesis: There is a 95% probability that the measured crystallization rates (with varying -R) are drawn from different distributions than the consensus literature values

# The concept of a hypothesis test in statistics

- > Panel a illustrates the situation: we want to know if a reference value is difference from the observation
- > Panel b proposes there is some underlying distribution around the reference (assume normal for now)
- > **Panel c illustrates the hypothesis test:** what is the probability the data at least as extreme as measured?
  - The p-value is the shaded area under the curve, e.g., p=.05



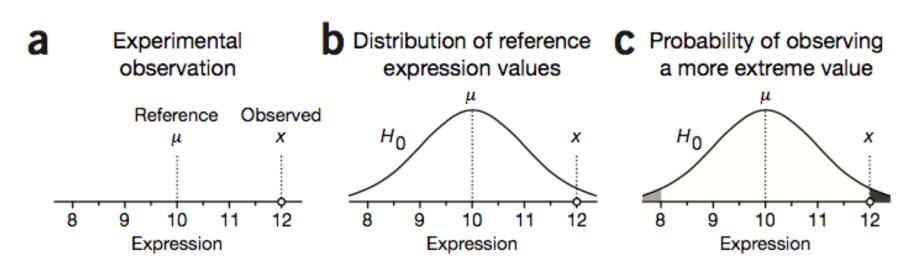
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"Informally, a *p*-value is the probability under a specified statistical model that a statistical summary of the data (e.g., the sample mean difference between two compared groups) would be equal to or more extreme than its observed value." – From the American Statistical Association statement on p-values

### The null hypothesis

- > H<sub>0</sub> is the null hypothesis, it states that the observations (measurements) are drawn from the same distribution as the the reference. It is characterized by a null distribution (also H<sub>0</sub>)
- You will find many statistics texts that contrast the null with the so-called alternate hypothesis (H<sub>A</sub>) which may state something about the likelihood that the data are "non random". These are falling out of favor.
- > Presumably because of the way the this shapes your thinking about what the data are showing you (more on this soon)



#### **Short interlude – the CLT and SEM**

> Before we explain how to characterize H<sub>0</sub> (the distribution) and test its significance (the hypothesis) we have add one more piece of information about what happens when we sample means and how to characterize the error of the means.



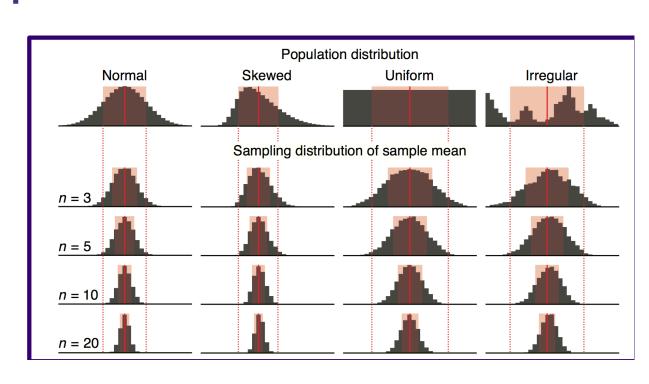
### Sampling from distributions and the central limit theorem

- > Recall from the CLT that the sampling distribution of the sample mean will always drift towards a normal distribution
- > We can define the standard error of the mean (SEM) as a descriptor of this distribution:

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_{i}$$

$$S_{x} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \overline{x})^{2}}$$

$$SEM = \frac{S_x}{\sqrt{n}}$$



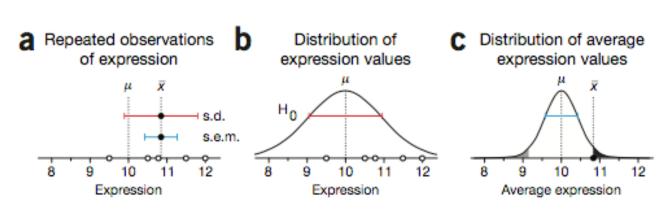
# Sampling from distributions and the central limit theorem

- > Panel a shows the calculation of the sample mean, standard deviation, and standard error
- > **Panel b highlights a major assumption:** the variance of your sample is the same as the variance of the null distribution H<sub>0</sub>
- > Panel c shows how the p-value is actually estimated: Assuming the s.e.m. of your sample is distributed about your reference value

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_{i}$$

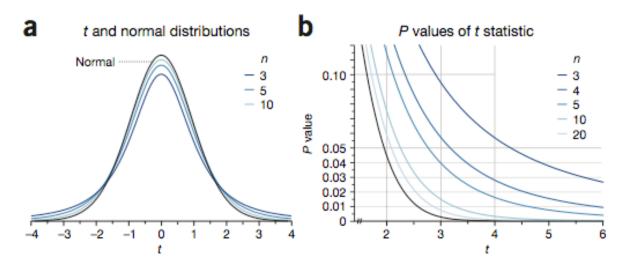
$$s.d. = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \overline{x})^{2}}$$

$$s.e.m = \frac{S_{x}}{\sqrt{n}}$$



## Just one more detail: The sampling distribution

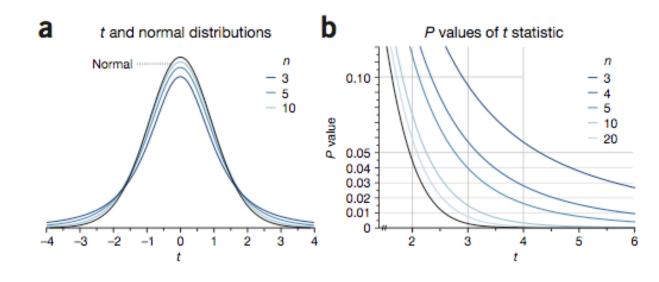
- > In general, it is known that the sample variance is an underestimate of the true distribution variance.
- > There is a distribution closely related to the normal called "the t-distribution" or "student's t-distribution"
- > The t-distribution gives you an easy recipe for calculating p-values with small sample sizes



### The recipe for a 1 sample t-test

- > State your hypothesis (H<sub>0</sub>) and collect data
- > Determine your desired significance level (usually called  $\alpha$ ). We usually use P=0.05
- > Calculate so-called t-statistic
- > Use a lookup table (or function) to determine P
- > If  $P < P_{\alpha}$ , you may reject  $H_0$

$$t = \frac{\overline{x} - \mu}{s.e.m} = \frac{\overline{x} - \mu}{s_x / \sqrt{n}}$$



#### **Practice**

- > Please open a python notebook lets "capacity build!"
- > Our goal will be to reproduce experiments drawn from this distribution and test their significance level

Distribution of expression values

