1 Introduction

Here I will discuss an attempt to use a Hidden Markov Model to predict and make simulated investments in the stock market. The problem of predicting the stock market is extremely complex - its rise and fall is dictated by too many factors to be able to predict accurately. However, my idea is we may not need to predict the stock market completely accurately in order to be able to profit from it - we just need to be able to predict it better than random. If we can consistently predict the market better than random, then repeated investments may allow an investor to beat the market.

The Markov Model approach to predicting the market is motivated by this philosophy. The MM is a relatively simple case of Artificial Intelligence wherein we try to predict the future using only the events of the recent past. Every event in a Markov chain has a set of possible states that have a certain probability of being the true state for the duration of that particular event. We can make an educated guess about the state of the current event by tabulating the sequential co-occurrence of states in the training data. For example, assume that we let the relative rise or fall of a stock price be the "state" we are measuring in the stock market. If we see a big jump in the stock market preceded by two moderate jumps at several occasions in the training data, our model will be likely to invest big after it sees two moderate jumps in the test data.

One could make the argument that this would be a weak predictor of the rise in the stock market; however, a strong predictor may not be needed. Over time, a repeated string of weak successes could compound on top of each other to create a large increase in income, and a large increase in performance of the stock market.

2 The Markov Model Framework

[MORE RESEARCH ON THE MATH OF HMMS NEEDED]
The Markov Model framework assumes a system with a finite number of possible states exists at every moment time, and it assumes that the state of the system at one moment depends entirely on the state of the system at the previous moment. This is in contrast to models that assume a completely independent relationship between neighboring states (such as predicting the outcome of a coin flip), or models in which subsequent states depend on the entire sequence of past states [INSERT EXAMPLE].

In this specific case, we create a model which repeatedly makes a series of short-term investments in the stock market. Each short-term investment is based on a very small number of previous states - this is done because it allows us to have much more accurate information on all the possible combinations of prior states we are examining. For example, assuming there are s states and

our visibility (i.e. the number of prior states we use to make our prediction) is v, the total number of distinct prior combinations of states we would need to understand from the training data is v^s - so the amount of training data we need to make an accurate decision increases exponentially in the visibility and the number of states.
We could also differentiate between combinations of prior states that occur in a different order - for example, should we consider a sequence of states $S_1 = \{st_1, st_2\}$ different from the sequence of states $S_2 = \{st_2, st_1\}$?
[NOTE: THIS IS SOMETHING I SHOULD TRY OUT - COULD BE VERY IMPORTANT!]

In our model, we have initially set the visibility to 2 and the number of states to 7.

2.1 Meaning of "States"

Every state in our HMM model represents a different level of change in the stock we are trading on. More specifically, we split the distribution of the magnitude of daily change into s equal parts, and represent each sector with one of s states. Assume we are attempting to make investments in Apple's stock. We then select a set of historical daily stock closing prices for Apple ($A = \{a_0, a_1, ..., a_n\}$) and use this as our training data. To create states for our HMM, we want to analyze the differences in stock price from day to day - let's denote this as: $A' = \{a'_0, a'_1, ..., a'_{n-1}\}$, where $a'_k = a_{k+1} - a_k$.
Then, each state we define will be of the form $s_t = s_j$ iff $[a'_{\frac{n*j}{s'}}, a'_{\frac{n*(j+1)}{s'}}]$, where s_t is the state assigned to the model at time t. Thus, the first state will represent the 0th through $\frac{100}{7} = 14$th percentile of the daily stock market changes, the second will represent the 14th through the 28th, etc. Let $\{S_j\}$ represent this set of 7 states, and $\{S_T\}$ represent the series of states we encounter over time.

2.2 Predicting Future States

As mentioned previously, the model predicts the state s_{t+1} based on s_t and s_{t-1}. More specifically, it uses the training data to compute the probability distribution $\{P(s_{t+1} = s_j | s_t, s_{t-1}) \forall s_j \in \{S_j\}\}$. This prediction is based on the results that were seen in the training data. For every combination of states $\{s_t, s_{t-1}\}$ that occur in the training data, the state s_{t+1} is recorded. After all the training data is recorded, the set of outcomes s_{t+1} that were seen in the data is coverted into a probability distribution - note that this relies on the assumption that the test data will have the probability distribution of future states as the training data. See the pseudocode algorithm below for more details.

```
statemap = dictionary([s_a, s_b] ↦ list(len(S_j)))
for s_t in training_data:
...stateinput=sort(s_{t-1}, s_t)
...statelist ← statemap[stateinput]
```

```
...statelist[s_{t+1}] += 1
...statemap[stateinput] ← statelist
end for

stateprobs ← statemap / sum(statemap)
```

Note that we sort the inputs $\{s_{t-1}, s_t\}$ to the dictionary because we want to compute these probabilities in a manner independent of the order in which the past states occur (otherwise there might be too many possibilities to predict accurately).

2.3 Computing the Utility of an Investment

We have now discussed the way in which our model tries to predicts the stock market. One final component we need to discuss is the decision-making algorithm, where the model decides, based on the probability distribution of potential future states (which we will call P_{S_j}), how much of its money to invest in the stock market.

The model is given the ability to choose the most efficient of nine potential amounts of money to invest: $\{0, \frac{1}{8}M, \frac{2}{8}M, ..., 1 * M\}$, where M is our total wealth. For each amount, we compute the `moneychangeratio`, which is the fraction of our current wealth that will be present after we invest. In order to calculate this, we will need to know the expected gain ratio corresponding to each state in $\{S_j\}$ - we compute this by finding the average gain of qualifying examples from the training data, and we denote it $E_g[s_j]$.

Below, we compute the expected monetary value of our investment (I_{t+1,s_j}) after the next close of the stock market, assuming that the change is denoted by state s_j:

$$I_{t+1,s_j} = I_t * E_g[s_j]$$

Next, we compute the expected gain ratio for a given investment amount I_t and a given state s_j denoted $E_g[I_t, s_j]$ - note that after the investment, the wealth *not* invested ($M - I_t$) is unchanged and the wealth invested grows from I_t to I_{t+1,s_j}, so the future wealth is $M - I_t + I_{t+1,s_j}$:

$$E_g[I_t, s_j] = \frac{1}{M}[(M - I_t) + I_{t+1,s_j}]$$

Once we know the expected fraction of change in wealth $E_g[I_t, s_j]$, we apply a special utility function $util0$ which maps the moneychangeratio to a user-specified utility value u. The function then adds up the value u computed for each potential state s_j, weighted by the probability of encountering the state s_j. The $util0$ function allows the user to control the investment behavior of the model with a function such as $util0 = 2^{E_g[I_t,s_j]} - 1$, a potential monetary gain is given a utility that is very large in the positive direction, while a loss ($E_g[I_t, s_j] < 1$) is given a utility that is very small in the negative direction.

This creates a model that is that is very willing to take risks, in spite of the high costs it may encounter.

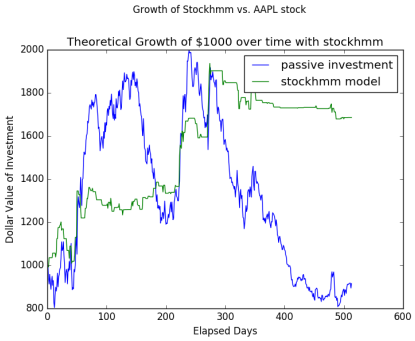
$$\text{Util}(I_t) = \Sigma_{s_j \in S_j} P(s_{t+1} = s_j | s_t, s_{t-1}) * util0(E_g[I_t, s_j])$$

Above, we computed the final utility for the investment I_t. We will compute such a utility for all possible investment amounts, and then choose to invest the amount with the highest utility.

Note that in our current model, we use the utitlity function: $f(E_g) = \{E_g$ iff $E_g \geq 1$, $ln(E_g) + 1$ iff $E_g < 1\}$. Thus, the model is very risk-averse.

3 Results

The results for the current model ($f(E_g) = \{E_g$ iff $E_g \geq 1$, $ln(E_g) + 1$ iff $E_g < 1\}$) are shown below for the stock AAPL.



[INSERT MORE RESULTS FOR OTHER TESTS.]