Thesis Notes

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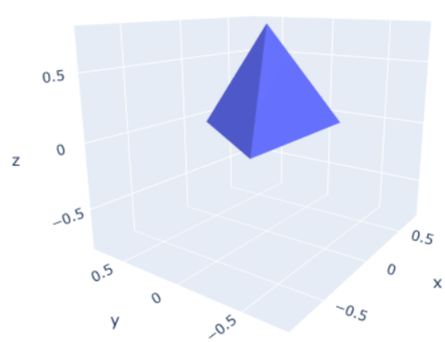
January 4, 2024

Part I
ASCII Stereolithography (.ast) Files

```
solid Mesh
  facet normal 0.000000 0.000000 -1.000000
    outer loop
      vertex -0.500000 -0.288675 0.000000
      vertex 0.000000 0.577350 0.000000
      vertex 0.500000 -0.288675 0.000000
    endloop
  endfacet
  facet normal -0.816497 0.471405 0.333333
    outer loop
      vertex 0.000000 0.000000 0.816497
      vertex 0.000000 0.577350 0.000000
      vertex -0.500000 -0.288675 0.000000
    endloop
  endfacet
  facet normal 0.000000 -0.942809 0.333333
    outer loop
      vertex 0.000000 0.000000 0.816497
      vertex -0.500000 -0.288675 0.000000
      vertex 0.500000 -0.288675 0.000000
    endloop
  endfacet
  facet normal 0.816497 0.471405 0.333333
    outer loop
      vertex 0.000000 0.000000 0.816497
      vertex 0.500000 -0.288675 0.000000
      vertex 0.000000 0.577350 0.000000
    endloop
  endfacet
endsolid Mesh
```



3.

Conclusion

Figure 1: ASCII STL code to create a tetrahedron.

1 Converting the .ast File to a Data Structure

The .ast file was parsed for four strings which are used to denote the beginning and end of the description of faces and vertices: ”facet normal”, ”end facet”, ”outer loop”, and ”end loop”, respectively. The data was then converted into tuple with python:

[['Face 0', [normal vector xyz], [[point 1 xyz], [point 2 xyz], [point 3 xyz]]], ['Face 1', ...]]

Part II
Python Code Libraries

2 Meshpy

3 Gudhi

Part III
Homology

4 Complexes

5 Homologies

6 Betti Numbers

Part IV
Python Custom Defined Functions

7

Part V
Methods to build Simplicial Complexes

https://www.youtube.com/watch?v=fnkvPy4ZCNY

- X = Finite d-dimensional sub-metric euclidean space.
- depend on non-negative real parameter, r

8 Cech

$Cech_r(X) = \{\sigma \subseteq X \mid \cap_{x \in \sigma} B_r(X) \neq \emptyset\}$

- Balls grow around points of metric space, every time k+1 balls intersect, add a k-dimensional simplex to complex.

9 Vietoris-Rips

$VR_r(X) = \{\sigma \subseteq X \mid diam\sigma \leq 2r\}$

- if subsets of metric space have diameter less than or equal to 2*r, add simplex

10 Delaunay

$Del(X) = \{\sigma \subseteq X \mid \cap_{x \in \sigma} V_x \neq \emptyset\}$
$V_x = \{y \in \mathbb{R}^2 \mid ||1x|| \leq ||y - z||, z \in X\}$

- Do not depend on a parameter or intersecting balls (no "time")
- Intersecting voronoi cells determine simplices in complex

10.1 Voronoi cells

11 Alpha

$Alpha_r(X) = \{\sigma \subseteq X \mid \cap_{x \in \sigma} (B_r(X) \cap V_x) \neq \emptyset\}$

- in-between cech and delauney complexes: to construct, take into account both voronoi cell-associated points in metric spce and growing balls around these points

11.1 Formula