Formulario di Trigonometria

Indice degli argomenti

Formule fondamentali

$$\sin^2\alpha + \cos^2\alpha = 1$$
$$\tan x = \frac{\sin x}{\cos x}$$
$$\cot x = \frac{\cos x}{\sin x}$$
$$\sec x = \frac{1}{\cos x}$$
$$\csc x = \frac{1}{\sin x}$$

Valori noti delle funzioni trigonometriche

	Angolo															
Op.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\sin$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\tan$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\nexists$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\nexists$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$
$\cot$	$\nexists$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\nexists$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$

Simmetrie delle funzioni trigonometriche

$$\begin{aligned}
\sin(-x) &= -\sin x \\
\cos(-x) &= \cos x \\
\tan(-x) &= -\tan x \\
\cot(-x) &= -\cot x
\end{aligned}$$

Relazioni tra funzioni goniometriche elementari

$\sin\alpha = \pm\sqrt{1-\cos^2\alpha}$	$\sin\alpha = \pm\dfrac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$	$\sin\alpha = \pm\dfrac{1}{\sqrt{1+\cot^2\alpha}}$
$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$	$\cos\alpha = \pm\dfrac{1}{\sqrt{1+\tan^2\alpha}}$	$\cos\alpha = \pm\dfrac{\cot\alpha}{\sqrt{1+\cot^2\alpha}}$
$\tan\alpha = \pm\dfrac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$	$\tan\alpha = \pm\dfrac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$\tan\alpha = \dfrac{1}{\cot\alpha}$
$\cot\alpha = \pm\dfrac{\sqrt{1-\sin^2\alpha}}{\sin\alpha}$	$\cot\alpha = \pm\dfrac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$	$\cot\alpha = \dfrac{1}{\tan\alpha}$

Formule sugli angoli complementari e supplementari

	Angolo						
Operatore	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$
$\sin$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$
$\cos$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$
$\tan$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$	$\tan\alpha$	$\cot\alpha$	$-\cot\alpha$	$-\tan\alpha$
$\cot$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$	$\cot\alpha$	$\tan\alpha$	$-\tan\alpha$	$-\cot\alpha$

Formule di addizione e sottrazione

Formule di addizione

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha}$$

Formule di sottrazione

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$
$$\cot(\alpha-\beta) = \frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$$

Formule di duplicazione e bisezione

Formule di duplicazione

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$
$$\cot 2\alpha = \frac{\cot^2\alpha-1}{2\cot\alpha}$$

Formule di bisezione

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$
$$\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$$

Formule di prostaferesi

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$	$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$	$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
$\tan\alpha + \tan\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$	$\tan\alpha - \tan\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$
$\cot\alpha + \cot\beta = \frac{\sin(\beta+\alpha)}{\sin\alpha\sin\beta}$	$\cot\alpha - \cot\beta = \frac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

Formule parametriche $\left(t = \tan\frac{\alpha}{2}\right)$

$$\sin\alpha = \frac{2t}{1+t^2}$$
$$\cos\alpha = \frac{1-t^2}{1+t^2}$$
$$\tan\alpha = \frac{2t}{1-t^2}$$
$$\cot\alpha = \frac{1-t^2}{2t}$$

Formule di Werner

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$
$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

Funzioni goniometriche inverse

Le funzioni goniometriche inverse sono: arcoseno, arcocoseno, arcotangente e arcocotangente.

$$\arcsin x = \sin^{-1} x$$
$$\arccos x = \cos^{-1} x$$
$$\arctan x = \tan^{-1} x$$
$$arccotx = \cot^{-1} x$$

Funzioni iperboliche

Le funzioni iperboliche sono: seno iperbolico, coseno iperbolico, tangente iperbolica, cotangente iperbolica.

Formule fondamentali

$$\cosh^2\alpha - \sinh^2\alpha = 1$$
$$\tanh x = \frac{\sinh x}{\cosh x}$$
$$\coth x = \frac{\cosh x}{\sinh x}$$
$$sech\ x = \frac{1}{\cosh x}$$
$$csch\ x = \frac{1}{\sinh x}$$

Forma esponenziale

Per calcolare il valore delle funzioni iperboliche dobbiamo considerare la loro espressione in forma esponenziale.

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Simmetrie delle funzioni iperboliche

$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$
$$\tanh(-x) = -\tanh x$$
$$\coth(-x) = -\coth x$$

Formule di addizione e sottrazione

$$\sinh(\alpha+\beta) = \sinh\alpha\cosh\beta + \cosh\alpha\sinh\beta$$
$$\cosh(\alpha+\beta) = \cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta$$
$$\tanh(\alpha+\beta) = \frac{\tanh\alpha + \tanh\beta}{1+\tanh\alpha\tanh\beta}$$
$$\sinh(\alpha-\beta) = \sinh\alpha\cosh\beta - \cosh\alpha\sinh\beta$$
$$\cosh(\alpha-\beta) = \cosh\alpha\cosh\beta - \sinh\alpha\sinh\beta$$
$$\tanh(\alpha-\beta) = \frac{\tanh\alpha - \tanh\beta}{1-\tanh\alpha\tanh\beta}$$

Formule di duplicazione e bisezione

$$\sinh 2\alpha = 2\sinh\alpha\cosh\alpha$$
$$\cosh 2\alpha = \cosh^2\alpha + \sinh^2\alpha$$
$$\tanh 2\alpha = \frac{2\tanh\alpha}{1+\tanh^2\alpha}$$
$$\sinh\frac{\alpha}{2} = \pm\sqrt{\frac{\cosh\alpha-1}{2}}$$
$$\cosh\frac{\alpha}{2} = \sqrt{\frac{\cosh\alpha+1}{2}}$$
$$\tanh\frac{\alpha}{2} = \sqrt{\frac{\cosh\alpha-1}{\sinh\alpha}}$$

Formule di prostaferesi

$\sinh\alpha+\sinh\beta = 2\sinh\frac{\alpha+\beta}{2}\cosh\frac{\alpha-\beta}{2}$	$\sinh\alpha-\sinh\beta = 2\cosh\frac{\alpha+\beta}{2}\sinh\frac{\alpha-\beta}{2}$
$\cosh\alpha+\cosh\beta = 2\cosh\frac{\alpha+\beta}{2}\cosh\frac{\alpha-\beta}{2}$	$\cosh\alpha-\cosh\beta = -2\sinh\frac{\alpha+\beta}{2}\sinh\frac{\alpha-\beta}{2}$

Formule parametriche $\left(t = \tanh\frac{\alpha}{2}\right)$

$$\sinh\alpha = \frac{2t}{1-t^2}$$
$$\cosh\alpha = \frac{1+t^2}{1-t^2}$$
$$\tanh\alpha = \frac{2t}{1+t^2}$$

Formule di Werner

$$\sinh\alpha\cosh\beta = \frac{1}{2}[\sinh(\alpha+\beta)+\sinh(\alpha-\beta)]$$
$$\cosh\alpha\cosh\beta = \frac{1}{2}[\cosh(\alpha+\beta)+\cosh(\alpha-\beta)]$$
$$\sinh\alpha\sinh\beta = \frac{1}{2}[\cosh(\alpha-\beta)-\cosh(\alpha+\beta)]$$

Funzioni iperboliche inverse

Le funzioni iperboliche inverse sono: settore seno iperbolico, settore coseno iperbolico, settore tangente iperbolica e settore cotangente iperbolica.

$$\mathit{settsinh}\ x = x\log(x+\sqrt{x^2+1}) \qquad \forall x \in \mathbb{R}$$
$$\mathit{settcosh}\ x = x\log(x+\sqrt{x^2-1}) \qquad \text{con } x \geq 1$$
$$\mathit{setttanh}\ x = \frac{1}{2}\log\frac{1+x}{1-x} \qquad \text{con } -1 < x < 1$$
$$\mathit{settcoth}\ x = \frac{1}{2}\log\frac{x+1}{x-1} \qquad \text{con } x < -1 \ e\ x > 1$$