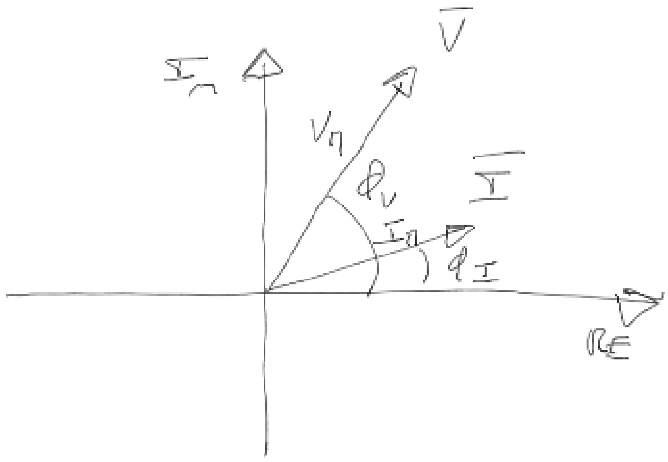
Lezione 25

Metodo dei fasori (Dominio della frequenza)

$$v(t) = V_M \operatorname{sen}(\omega t + \varphi_V) \longrightarrow \boxed{\bar{V} = V_M e^{j\varphi_V}}$$

FASORE

$$i(t) = I_M \operatorname{sen}(\omega t + \varphi_I) \longrightarrow \bar{I} = I_M e^{j\varphi_I}$$



$$v_1(t) + v_2(t) = v_3(t)$$

$$\bar{V}_1 + \bar{V}_2 = \bar{V}_3$$

$$\bar{V}_1 = V_{M_1} e^{j\varphi_{V_1}} \qquad \bar{V}_2 = V_{M_2} e^{j\varphi_{V_2}}$$

$$\bar{V}_1 = V_{M_1}(\cos\varphi_{V_1} + j \operatorname{sen}\varphi_{V_1}) = a_1 + jb_1$$

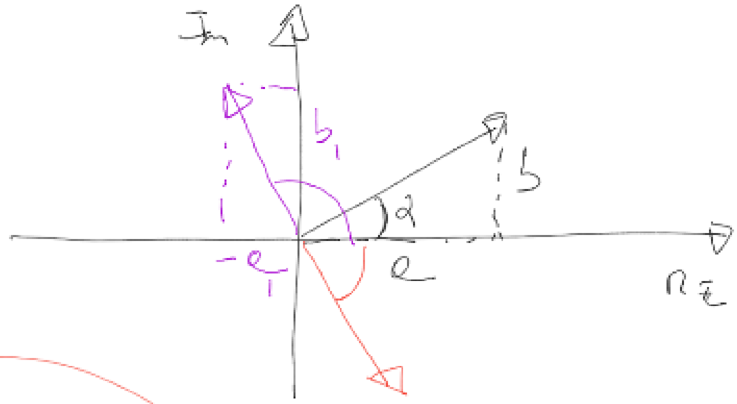
$$\bar{V}_2 = V_{M_2}(\cos\varphi_{V_2} + j \operatorname{sen}\varphi_{V_2}) = a_2 + jb_2$$

$$\bar{V}_3 = (a_1 + a_2) + j(b_1 + b_2) \longrightarrow v_3(t)$$

$$\bar{V}_3 = V_{M_3} e^{j\varphi_{V_3}} \longrightarrow v_3(t) = V_{M_3} \operatorname{sen}(\omega t + \varphi_{V_3})$$

$$|\bar{V}_1| = |a_1 + jb_1| = \sqrt{a_1^2 + b_1^2}$$

$$\arg(\bar{V}_1) = \operatorname{arctg}\frac{b_1}{a_1} \quad \text{o} \quad \operatorname{arctg}\frac{b_1}{a_1} + \pi \quad \text{se } a_1 < 0$$



$$\alpha = \operatorname{arctg}\left(\frac{b}{a}\right)$$

$$\operatorname{arctg}\left(\frac{b_1}{-a_1}\right) = \operatorname{arctg}\left(-\frac{b_1}{a_1}\right) + \pi$$

ESEMPIO

$$I_g(t) = 2 \operatorname{sen}(\omega t + 0{,}1) \longrightarrow \bar{I}_g = 2e^{j0{,}1}$$

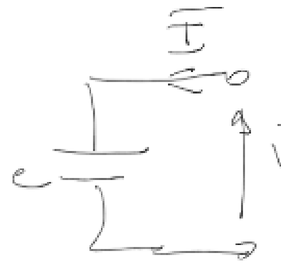
RESISTORE

$$v(t) = \underbrace{R I_M}_{V} \operatorname{sen}(\omega t + \underbrace{\varphi_I}_{\varphi})$$

$$\bar V = R I_M e^{j\varphi_I}$$

$$\boxed{\bar V = R \cdot \bar I}$$

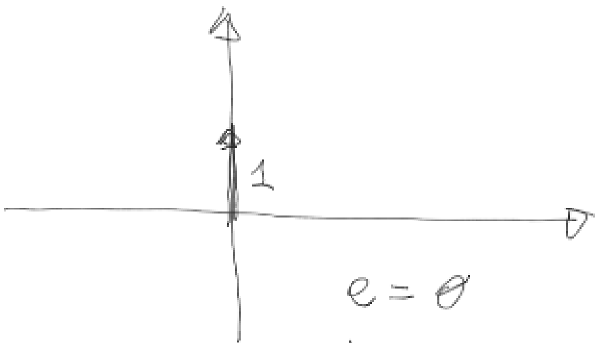
Condensatore



$$i(t) = \underbrace{\omega C V_M}_{I_M} \sin(\omega t + \underbrace{\varphi_V + \tfrac{\pi}{2}}_{\varphi_I})$$

$$\bar I = I_M e^{j\varphi_I} = \omega C V_M e^{j(\varphi_V + \frac{\pi}{2})} =$$

$$= \omega C \underbrace{V_M e^{j\varphi_V}}_{\bar V} e^{j\frac{\pi}{2}}$$



$$e^{j\frac{\pi}{2}} = j$$

$a = 0$

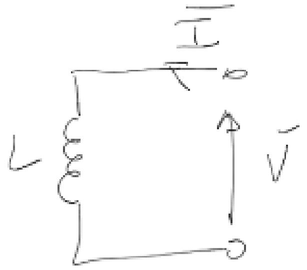
$b = 1$

$$\bar I = j\omega C \bar V$$

$$\boxed{\bar V = \frac{1}{\ } \bar I}$$

$$\boxed{\overline{V} = \frac{1}{j\omega C}\,\overline{I}}$$

INDUTTORE



$$v(t) = \underbrace{\omega L I_M}_{V_M} \sin(\omega t + \underbrace{\varphi_I + \tfrac{\pi}{2}}_{\varphi_V})$$

$$\overline{V} = V_M e^{j\varphi_V} = \omega L I_M e^{j(\varphi_I + \frac{\pi}{2})} =$$

$$= \omega L I_M e^{j\varphi_I} \cdot \underbrace{e^{j\frac{\pi}{2}}}_{j}$$

$$= j\omega L \underbrace{I_M e^{j\varphi_I}}_{\overline{I}}$$

$$\boxed{\overline{V} = j\omega L\,\overline{I}}$$

Con il metodo dei fasori otteniamo tutte relazioni costitutive algebriche del tipo:

$$\boxed{\overline{V} = \dot{Z}\,\overline{I}}$$

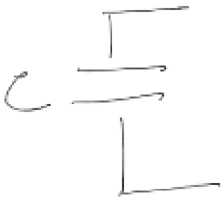
$\dot{Z}$ si chiama <u>IMPEDENZA</u>.

RESISTORE $\longrightarrow$ $\dot{Z} = R$

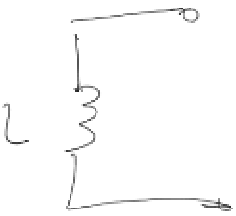
CONDENSATORE $\longrightarrow$ $\dot{Z} = \frac{1}{JWC} = -\frac{J}{WC}$

INDUTTORE $\longrightarrow$ $\dot{Z} = JWL$

$$\boxed{\dot{Y} = \frac{1}{\dot{Z}} \longrightarrow \text{AMMETTENZA}}$$

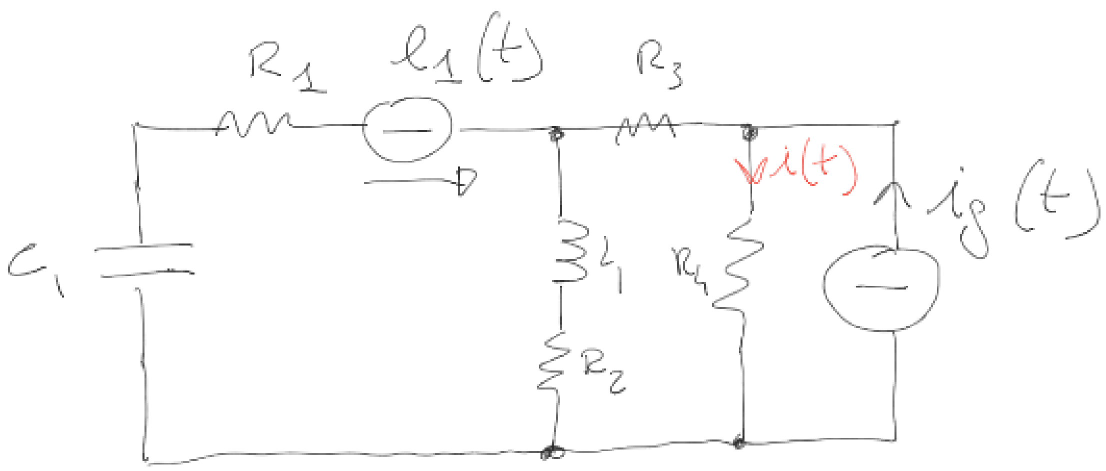


$C = 10\,\mu F \longrightarrow \frac{1}{J(2\pi f)10 \cdot 10^{-6}} = \dot{Z} \quad \left(\frac{1}{JWC}\right)$



$L = 2H \longrightarrow \dot{Z} = J(2\pi f)2 \quad (JWL)$

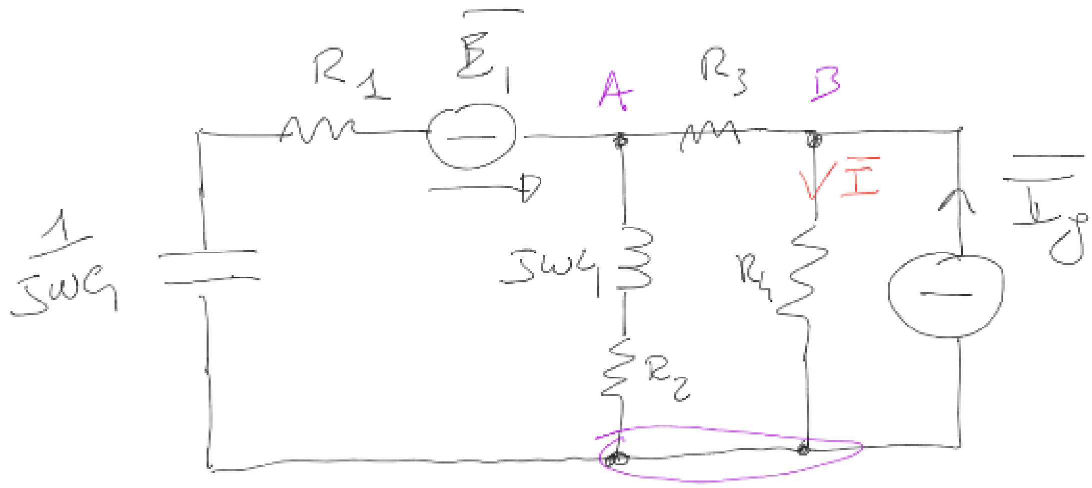
ESEMPIO



REGIME PERMANENTE SINUSOIDALE CON $f = 50\,Hz$

$$e_1(t) = 2\sin(\omega t + 0{,}1)$$

$$i_g(t) = 3\sin(\omega t - 0{,}2)$$



$$\bar{E}_1 = 2e^{j0{,}1} \qquad \bar{I}_g = 3e^{-j0{,}2}$$

$$\begin{array}{c|cc} & A & B \\ A & \frac{1}{R_1 + \frac{1}{j\omega C_1}} + \frac{1}{R_3} + \frac{1}{R_2 + j\omega L_1} & -\frac{1}{R_3} \\ B & -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} \end{array} \begin{bmatrix} \bar{V}_A \\ \bar{V}_B \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\bar{E}_1}{R_1 + \frac{1}{j\omega C_1}} \\ + \bar{I}_g \end{bmatrix}$$

$$\bar{I} = \frac{\bar{V}_B}{\dot{Z}_4} = \frac{\bar{V}_B}{R_4} = 4 - j5$$

$$\bar{I} = I_M e^{j\varphi_I} \longrightarrow i(t) = I_M \operatorname{sen}(2\pi f + \varphi_I)$$

$$I_M = |\bar{I}| = |4 - j5| = \sqrt{4^2 + 5^2}$$

$$\varphi_I = \operatorname{arctg}\left(\frac{-5}{4}\right)$$