Lecture 1 §1.1–1.2

A linear equation in the variables $x_1 \dots x_n$ is of the form

$$a_1 x_1 + a_2 x_2 + \dots + a_n x_n = b$$

where a_i & b are known coefficients (real or complex).

A system of m linear equations is of the form

$$\begin{aligned} a_{11}x_1 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

Most basic case: $m=n$

ex
$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 2 \\ 2x_1 + 5x_2 - 2x_3 &= 4 \\ 3x_1 + 4x_2 - 3x_3 &= -1 \end{aligned}$$

$r_2 - 2r_1$, $r_3 - 3r_1$

$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 2 \\ x_2 + 2x_3 &= 0 \\ -2x_2 + 3x_3 &= -7 \end{aligned}$$

$r_3 + 2r_2$

$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 2 \\ x_2 + 2x_3 &= 0 \\ 7x_3 &= -7 \end{aligned}$$

Back substitution

row 3 $\boxed{x_3 = -1}$

row 2 $x_2 + 2\cdot -1 = 0$ $\boxed{x_2 = 2}$

row 1 $x_1 + 2\cdot 2 - 2\cdot -1 = 2$

$x_1 + 6 = 2$

$\boxed{x_1 = -4}$

Key technique $\boxed{\text{Linear system operation 1}}$ (LSO1)

Add a multiple of one row to another.

Defn Two systems of linear equations are equivalent if they have the same set of solutions.

So doing LSO1 repeatedly yields an equivalent system of equations.

Matrices & Vectors

A matrix is a rectangular array of numbers, e.g.

$$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}, \quad \begin{pmatrix} 5 & 6 & 4 \\ 2 & 3 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ -5 \end{pmatrix}, \quad \begin{pmatrix} 4 & 5 & 7 \end{pmatrix}.$$

We use the notation

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} = (a_{ij}).$$

So a_{ij} is in the ith row & jth column.

The size of a matrix is written (# rows) × (# columns), e.g. $m \times n$.

So the matrices above have sizes 2×2, 2×3, 5×1, 1×3 respectively

If A & B have the same size then

$$A + B = (a_{ij} + b_{ij}), \text{ e.g., } \begin{pmatrix} 4 & 1 \\ 0 & 2 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 9 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 1 & 11 \\ 1 & 0 \end{pmatrix}.$$

If c is a number, then

$$cA = (ca_{ij}), \text{ e.g., } -4\begin{pmatrix} 4 & 1 \\ 0 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} -16 & -4 \\ 0 & -8 \\ -12 & 4 \end{pmatrix}.$$

The product of A & B is defined when the # columns of A ~~equal~~ equals the # rows of B, e.g., A is 3×4 & B is 4×2.

~~A~~ By definition, $AB = \left(\sum_{k=1}^{n} a_{ik} b_{kj}\right)$ when A is m×n & B is n×p.

ex

$$\underset{2\times 2}{\begin{pmatrix} 3 & 4 \\ -1 & 0 \end{pmatrix}} \underset{2\times 3}{\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}} = \begin{pmatrix} 3\cdot 1 + 2\cdot 4 & 3\cdot 0 + 4\cdot 1 & 3\cdot -1 + 4\cdot 0 \\ -1\cdot 1 + 0\cdot 2 & -1\cdot 0 + 0\cdot 1 & -1\cdot -1 + 0\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 4 & -3 \\ -1 & 0 & 1 \end{pmatrix}.$$

See the text for more examples.

A vector (or column vector) is an n×1 matrix, usually denoted by a lower case letter like x or b. We will write

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

A row vector is a 1×n matrix, e.g.

$$(1 \quad 0 \quad -2 \quad 3)$$

Row vectors will be much less common.