AI HW 2

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Q1

a.

Let S be the sentence [(Food⇒Party)∨(Drinks⇒Party)]⇒[(Food∧Drinks)⇒Party]

Food	Drinks	Party	(Food⇒Party)∨(Drinks⇒Party)	(Food∧Drinks)⇒Party	S
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	T	T
F	T	T	T	T	T
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	F	F	T
T	T	T	T	T	T

The sentence S is valid since it is true in all models.

b.

Left Hand Side: (Food⇒Party)∨(Drinks⇒Party)

1. Eliminate ⇒, replace α⇒β with ¬α∨β

   (¬Food∨Party)∨(¬Drinks∨Party)

   2. Simplify

   ¬Food∨Party∨¬Drinks∨Party

   ¬Food∨¬Drinks∨Party

Right Hand Side: [(Food∧Drinks)⇒Party]

1. Eliminate ⇒, replace α⇒β with ¬α∨β

   ¬(Food∧Drinks)∨Party

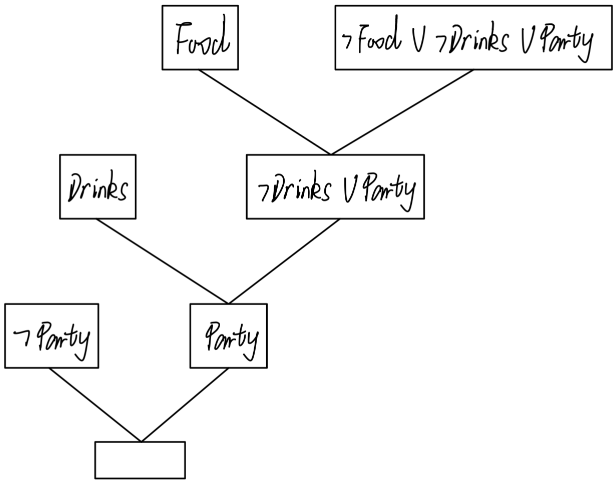
2. Move ¬ using de Morgan's rules

   ¬Food∨¬Drinks∨Party

Since we finally get (¬Food∨¬Drinks∨Party) on both sides, we know that the values on both sides are always the same. For both conditions: (True⇒True) and (False⇒False), the value is True, we get that the sentence is valid.

c.

KB: ¬Food∨¬Drinks∨Party, α : ¬Food∨¬Drinks∨Party, ¬α: Food∧Drinks∧¬Party



Since it finally get Nil, KB ∧ ¬α is unsatisfiable, so the sentence is valid.

Q2

a. Paris and Marseilles are both in France.

(i). 2: Conjunction cannot be used in a term.

(ii). 1

(iii). 3: Disjunction means "or", not "... and ... are both ..."

b. There is a country that borders both Iraq and Pakistan

(i). 1

(ii). 3: This means every country borders to Iraq and Pakistan.

(iii). 2: c should be used in the scope of its quantifier.

(iv). 2: Conjunction cannot be used in a term.

c. All countries that border Ecuador are in South America

(i). 1

(ii). 1

(iii). 3: The sentence means that every country borders Ecuador and is in South America. And particularly, for every non-country c, c is in South America.

(iv). 3: The sentence with universal quantifier should not uses conjunction as main connective.

d. No region in South America borders any region in Europe

(i). 1

(ii). 1

(iii). 3: This means there is a country in South America that borders every country in Europe.

(iv). 1

e. No two adjacent countries have the same map color

(i). 1

(ii). 1

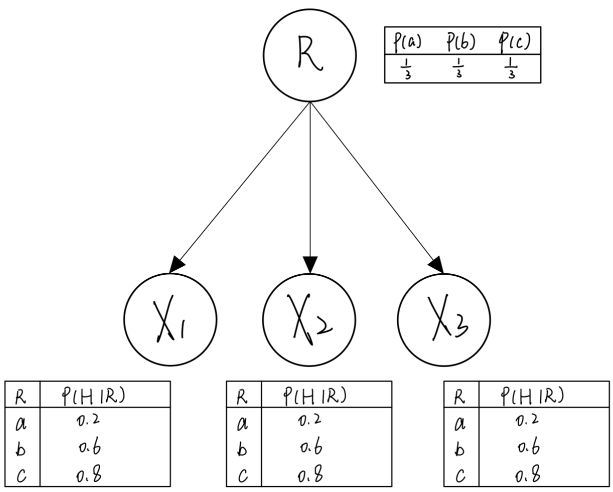
(iii). 3: The sentence with universal quantifier should not uses conjunction as main connective.

(iv). 2: Inequality cannot be used in a term.

Q3.

a.

The Bayesian network:



With R denoting which coin we drew, the network has R at the root and X_1, X_2 and X_3 as the children.

The CPT for R is:

R	P(R)
a	$\frac{1}{3}$
b	$\frac{1}{3}$
c	$\frac{1}{3}$

The CPT for X_i (i = 1, 2, 3) given R is the same which is shown below:

R	X_i	$P(X_i \mid R)$
a	Head	0.2
b	Head	0.6
c	Head	0.8

b.

According to the question, let α be the observed flips come out heads twice and tail one, we can know that we need to find the coin R that maximizes $P(R|\alpha)$.

$$P(R|\alpha) = \frac{P(R \cdot \alpha)}{P(\alpha)} = \frac{P(\alpha|R) * P(R)}{P(\alpha)}$$

Since $P(a) = P(b) = P(c) = \frac{1}{3}$, $P(\alpha)$ is constant, maximizing $P(R|\alpha)$ equals to maximizing $P(\alpha|R)$.

$$P(\alpha|R)$$
$$= P(H|\alpha) * P(H|\alpha) * P(T|\alpha) + P(H|\alpha) * P(T|\alpha) * P(H|\alpha) + P(T|\alpha) * P(H|\alpha) * P(H|\alpha)$$
$$= 3 * P(H|\alpha) * P(H|\alpha) * P(T|\alpha)$$

Therefore:

$$P(\alpha|a) = 3 * 0.2 * 0.2 * 0.8 = 0.096$$
$$P(\alpha|b) = 3 * 0.6 * 0.6 * 0.4 = 0.432$$
$$P(\alpha|c) = 3 * 0.8 * 0.8 * 0.2 = 0.384$$

Since $P(\alpha|b)$ is the max value, b is the most likely one to have been drawn.