

## **Article Draft**

# **A double-sampling extension of the German National Forest Inventory for design-based small area regression estimation of timber volume resources on forest district levels**

**Large scale application to the federal state of Rhineland-Palatinate**

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## Abstract

The German National Forest Inventory consists of a systematic grid of permanent sample plots and provides a reliable evidence-based assessment of the state and the development of Germany's forests on national and federal state level in a 10 year interval. However, the data have yet been scarcely used for estimation on smaller management levels such as forest districts due to insufficient sample sizes within the area of interests and the implied large estimation errors. In this study, we present a double-sampling extension to the existing German National Forest Inventory (NFI) that allows for the application of recently developed design-based small area regression estimators. We illustrate the implementation of the estimation procedure and evaluate its potential by the example of timber volume estimation on two small scale management levels (45 and 405 forest district units respectively) in the federal German state of Rhineland-Palatinate. An airborne laserscanning (ALS) derived canopy height model and a tree species classification map based on satellite data were used as auxiliary data in an ordinary least square regression model to produce the timber volume predictions on the plot level. The results support that the suggested double-sampling procedure can substantially increase estimation precision on both management levels: the two-phase estimators were able to reduce the variance of the SRS estimator by 43% and 25% on average for the two management levels respectively.

**Keywords.** National forest inventory, small area estimation, double sampling for regression within strata, cluster sampling, LiDAR canopy height model, tree species classification

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# 1 Introduction

The German National Forest Inventory (NFI) provides reliable evidence-based and accurate information of the current state and the development of Germanys forest over time. The NFI thereby has to satisfy various information needs and amongst others reports to public and state forestry administrations, wood-based industries and the public on the national level, as well as to the Food and Agriculture Organization of the United Nations (FAO) and United Nations Framework Convention on Climate Change (UNFCCC) on the international level ([Polley et al, 2010](#)). At the current time, the inventory design of the German NFI solely rests upon a terrestrial cluster inventory that is carried out at sample locations systematically distributed over the entire forest state area of Germany. As this implies covering a large area of 114'191 ha ([Thünen-Institut, 2014](#)), the sample size has been chosen according to satisfy high estimation accuracies for forest attributes on the national and federal state level. This however leads to very low sampling intensities and consequently, sample sizes often drop dramatically when entering spatial units below the federal state level. This is particularly true for forest management levels such as forest districts for which the estimation uncertainties turn out be unacceptably large due to the very limited number of sample plots within these units. For this reason, the German NFI data have not yet been extensively incorporated in operational forest planning on forest district management levels. In most German federal states, management strategies are thus still based on expert judgements from time-consuming standwise inventories (SFI), which are prone to systematic deviations [Kuliešis et al \(2016\)](#) and do not provide any measure of uncertainty.

Some German federal states, such as Lower Saxony, have approached this problem by establishing a regional Forest District Inventory (FDI) with a much higher sampling density than used by the NFI in order to base their regional management strategies on quantitative and accurate information ([Böckmann et al, 1998](#)). However, such FDIs are cost-intensive and, facing increasing restrictions in budget and staff resources, there has been a need for more cost-efficient inventory methods ([von Lüpke, 2013](#)). One method which has proven to be efficient is double-or two-phase sampling ([Särndal et al, 2003; Gregoire and Valentine, 2007; Köhl et al, 2006; Mandallaz, 2008](#)). Double sampling incorporates inexpensive auxiliary information and can be used to either increase estimation precision under given terrestrial sample size, or maintain estimation precision under reduced terrestrial sample size. A double sampling for stratification procedure has e.g. been used in the FDI of Lower Saxony ([Saborowski et al, 2010](#)), and [Grafström et al \(2017\)](#) lately illustrated how to use the auxiliary information to determine optimised balanced terrestrial sample designs. Recent studies have lately extended double-sampling to triple-sampling estimation methods using auxiliary information in two different sampling intensities. An example can be found in [von Lüpke et al \(2012\)](#) who illustrated an extension of the existing two-phase FDI Lower Saxony to a three-phase design that uses updates of past inventory data as additional auxiliary information and allows for a significant reduction of the terrestrial sample size in intermediate inventories. An other example is [Massey et al \(2014\)](#) who developed a triple-sampling extension based on the ideas of [Mandallaz \(2013b\)](#) for the Swiss NFI that can significantly reduce the increase in estimation uncertainty caused by the new annual inventory design.

## 1. INTRODUCTION

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Two-phase and three-phase samplings techniques have also been used in the service of small area estimation (SAE). SAE techniques particular address the situation where the number of samples within a subunit, so-called small area (SA), of the entire sampling frame is too small to provide reliable estimates for that unit. A broad range of SA estimators used in forest inventories ([Köhl et al, 2006](#)) originally comes from official statistics. A commonly applied SAE method is thereby known as indirect estimation ([Rao, 2015](#)), where statistical models are used to convert auxiliary information into predictions of the target variable that is rarely or not available in the small area. The statistical models are thereby often developed by "borrowing strength" from data outside the small area. There are numerous applications of SAE in forestry ([Breidenbach and Astrup, 2012](#); [Goerndt et al, 2011](#); [Steinmann et al, 2013](#); [Mandallaz et al, 2013](#)), and most of the studies use unit-level models, i.e. the statistical models are fitted using data from inventory plots. Especially unit-level models for timber volume estimation under the use of various remote sensing data have been intensively investigated with respect to timber volume prediction ([Koch, 2010](#); [Naesset, 2014](#)). There are also few studies that have investigated area level-models, where the auxiliary information is only provided on the SA-level ([Magnussen et al, 2017](#)). Some studies have illustrated that even NFI data of low sampling densities can be used in small area estimation procedures to provide estimations of acceptable accuracy on much smaller management levels. One example is [Breidenbach and Astrup \(2012\)](#) who used data from the Norwegian NFI for SAE estimations of standing timber volume for 14 municipalities where the number of NFI samples within these areas were between 1 and 35. Estimation errors under the applied model-based and design-based SAE estimators turned to be markedly smaller than achieved under simple random sampling (SRS). Another example is [Magnussen et al \(2014\)](#) who recently used the Swiss NFI data for SAE estimation of timber volume within 108 Swiss forest districts with sample sizes between 9 and 206. Despite these promising results, a similar study in Germany using the German NFI data for SAE estimation has not yet been conducted.

The aim of this study was to investigate whether the German NFI data can provide acceptable estimation precision on two forest district levels when incorporated in small area estimation procedures. We therefore conducted a study in the German federal state Rhineland-Palatinate where we extended the German NFI to a double-sampling design and applied three types of design-based small area regression estimators in order to derive point and variance estimates of mean standing timber volume for 45 and 405 forest districts respectively. The SA-estimators we considered were the *pseudo-small*, *extended pseudo-synthetic* and the *pseudo-synthetic* design-based small area estimator suggested by [Mandallaz \(2013a\)](#); [Mandallaz et al \(2013\)](#). Auxiliary data were obtained from a countrywide airborne Laser scanning (LiDAR) canopy height model (CHM) and a tree species classification map and used for regression within tree species strata. The estimation accuracies were compared to those achieved under SRS sampling. The chosen double-sampling estimators were favoured for several reasons: **(i)** the design-based frame considerably relaxes requirements on the regression model which seemed appropriate facing severe quality restrictions in the LiDAR data; **(ii)** the estimators can consider *non-exhaustive*, i.e. non wall-to-wall, auxiliary information; **(iii)** all estimators are explicitly formulated for cluster sampling which has not yet been the case for frequently used model-dependent estimators; and **(iv)** the asymptotically unbiased g-weight variance accounts for the design-dependency of the regression coefficients on the sample (*internal model*

approach) and is also robust to heteroscedasticity of the model residuals.

The results from this study were considered to provide valuable information whether the suggested procedure a) might be a cost-saving alternative to a regional FDI and b) can be used as a reliable validation for SDI data. A secondary objective was to address the potential effects of auxiliary data quality on estimation precision, and to identify challenges when transferring the existing German NFI design into the suggested double sampling estimation procedure.

## **2 Terrestrial sampling design of the German NFI**

The German National Forest Inventory (German NFI) is a periodic inventory that is carried out every 10 years over the entire forest area of Germany. The most recent inventory (BWI3) was conducted in the years 2011 and 2012. While information was originally gathered at a systematic 4x4 km grid, some federal states such as Rhineland-Palatinate have switched to a densified 2x2 km grid. The German NFI uses a cluster sampling design, which means that a sample unit consists of maximal four sample locations (also referred to as *sample plots*) that are arranged in a square (so called *cluster*) with a side length of 150 metres (Picture). The number of plots per cluster can however vary between 1 and 4 depending on forest/non-forest decisions by the field crews on the individual plot level ([Bundesministerium für Ernährung, 2011](#)). In the field survey of the BWI3, sample trees for timber volume estimation are selected according to the angle count sampling technique ([Bitterlich, 1984](#)), using a basal area factor (BAF) of 4 that is respectively adjusted for sample trees at the forest boundary by a geometric intersection of the boundary transect with the tree-individual inclusion circle ([Bundesministerium für Ernährung, 2011](#)). A further inventory threshold for a tree to be recorded is a diameter at breast height (dbh) of at least 7 cm. For each sample tree that is selected by this procedure, the dbh, the absolute tree height, the tree diameter at 7 m (D7) and the tree species is measured and used to calculate a volume estimation on the tree level. These volume estimations are based on the application of tree species specific taper curves that are adjusted to the set of diameters and corresponding height measurements taken from the respective sample tree ([Kublin et al, 2013](#)).

## **3 Double sampling in the infinite population approach**

The estimators used in this study have been proposed by ([Mandallaz, 2013a; Mandallaz et al, 2013](#)) and build upon the so called infinite population approach (IPA) in order to bridge the inventory procedure and the derived information to the mathematics behind the estimators. Therefore, we shall first provide a short introduction into this general estimation frame. We start by assuming that the population  $P$  of trees  $i \in 1, 2, \dots, N$  within a forest of interest  $F$  is exactly defined, and each tree  $i$  has a directly or indirectly observable response variable  $Y_i$  (e.g., its timber volume) that allows to specify the population mean  $Y$  (e.g., the average timber volume per unit area) over  $F$ . If a full census of all tree population individuals is not possible,  $Y$  has to

be estimated by the conduction of an inventory. The infinite population approach assumes that the spatial distribution of the local density  $Y(x)$  (e.g., the timber volume per unit area) at each point or location  $x$  in the forest  $F$  is given by a fixed (i.e. non stochastic) piecewise constant function. The population mean  $Y$  is thus mathematically equivalent to the integral of the density function surface divided by the forest area  $\lambda(F)$ , i.e.  $Y = \frac{1}{\lambda(F)} \sum_{i=1}^N Y_i = \frac{1}{\lambda(F)} \int_F Y(x) dx$ , and thus the population mean  $Y$  corresponds to a spatial mean. Since the local density function is in practice always unknown, one estimates  $Y$  by collecting a sample  $s_2$  of all local density values by the conduction of a terrestrial inventory at  $n_2$  uniformly and independently distributed sample points over  $F$ . This procedure is often referred to as *one-phase sampling* (OPS). Opposed to the one-phase approach, *two-phase* or *double-sampling* procedures use information from two nested samples (phases). Practically speaking, the terrestrial inventory  $s_2$  is embedded in a large phase  $s_1$  comprising  $n_1$  sample locations that each provide a set of explanatory variables described by the column vector  $\mathbf{Z}(x) = (z(x)_1, z(x)_2, \dots, z(x)_p)^\top$  at each point  $x \in s_1$ . These explanatory variables are derived from auxiliary information that is available in high quantity within the forest  $F$ . For every  $x \in s_1$ ,  $\mathbf{Z}(x)$  is transformed into a prediction  $\hat{Y}(x)$  of  $Y(x)$  using the choice of some prediction model. The basic idea of this method is to boost the sample size by providing a large sample of less precise but cheaper predictions of  $Y(x)$  in  $s_1$  and to correct any possible model bias, i.e.,  $\mathbb{E}(Y(x) - \hat{Y}(x))$ , using the subsample of terrestrial inventory units where the value of  $Y(x)$  is observed.

## 4 Estimators

### 4.1 Design-based SRS estimator for cluster sampling

The simple random sampling (SRS) estimator for cluster sampling constitutes the *status quo* that is currently applied under the existing one-phase sampling design of the German NFI in order to obtain a point and variance estimate for the mean timber volume of a given estimation unit. In order to provide all estimators in the infinite population framework and ensure a consistent terminology with the two-phase estimators in Section 4.2, we will introduce the SRS estimator that is applied in the BWI3 algorithms ([Schmitz et al, 2008](#)) in the form given in [Mandallaz \(2008\)](#); [Mandallaz et al \(2016\)](#).

In order to calculate the local density  $Y_c(x)$  at the cluster level, a cluster is defined as consisting of  $M$  sample locations (in the BWI3, we have  $M = 4$ ) where  $M - 1$  sample locations  $x_2, \dots, x_M$  are created close to the cluster origin  $x_1$  by adding a fixed set of spatial vectors  $e_2, \dots, e_M$  to  $x_1$ . The actual number of plots per cluster,  $M(x)$ , is a random variable due to the uniform distribution of  $x_l$  ( $l = 1, \dots, M$ ) in the forest  $F$  and forest/non-forest decision for each sample location  $x_l$ :

$$M(x) = \sum_{l=1}^M I_F(x_l) \quad \text{where} \quad I_F(x_l) = \begin{cases} 1 & \text{if } x_l \in F \\ 0 & \text{if } x_l \notin F \end{cases} \quad (1)$$

The local density on cluster level  $Y_c(x)$ , in our case the timber volume per hectare, is then defined as the average of the individual sample plot densities  $Y(x_l)$ :

$$Y_c(x) = \frac{\sum_{l=1}^M I_F(x_l) Y(x_l)}{M(x)} \quad (2)$$

The local density  $Y(x_l)$  on individual sample plot level was calculated according to the description in [Mandallaz \(2008\)](#), which can be rewritten for angle-count sampling technique applied in the BWI3. The general form of  $Y(x)$  in [Mandallaz \(2008\)](#) is given as the Horwitz-Thompson estimator

$$Y(x_l) = \sum_{i \in s_2(x_l)} \frac{Y_i}{\pi_i \lambda(F)} \quad (3)$$

where  $Y_i$  is in our case the predicted timber volume of the tree  $i$  recorded at sample location  $x$  in  $\text{m}^3$ . Each tree has an inclusion probability  $\pi_i$  that is well defined as the proportion of its inclusion circle area  $\lambda(K_i)$  within the forest area  $\lambda(F)$ , i.e. via their geometric intersection:

$$\pi_i = \frac{\lambda(K_i \cap F)}{\lambda(F)} \quad (4)$$

The radius  $R_i$  of the tree-individual inclusion circle  $K_i$  is given by  $R_i = bhd_i/cf_{i,corr}$  (also referred to as *limiting distance*), where  $cf_{i,corr}$  is the counting factor corrected for potential boundary effects at the forest border. In case of angle-count sampling, we can rewrite  $\pi_i$  as

$$\pi_i = \frac{G_i}{cf_{i,corr}\lambda(F)} \quad (5)$$

since the intersection area  $\lambda(K_i \cap F)/\lambda(F)$  can be expressed using the trees basal area  $G_i$  (in  $\text{m}^2$ ) and the corrected counting factor:

$$\lambda(K_i \cap F) = \frac{G_i}{cf_{i,corr}} \quad \text{where} \quad cf_{i,corr} = cf \frac{\lambda(K_i)}{\lambda(K_i \cap F)} \quad (6)$$

Using Eq. 5 in Eq. 3 yields the rewritten form of  $Y(x_l)$  for angle count sampling that conforms to the definition used in the BWI3 algorithms ([Schmitz et al, 2008](#)):

$$Y(x_l) = \sum_{i \in s_2(x_l)} \frac{cf_{i,corr} Y_i}{G_i} = \sum_{i \in s_2(x_l)} nha_i Y_i \quad (7)$$

where  $nha_i$  is the number of trees per hectare represented by tree  $i$ . The local densities on cluster level can then be used to derive the estimated spatial mean  $\hat{Y}_c$  and its estimated variance  $\hat{\mathbb{V}}(\hat{Y}_c)$  for any given spatial unit for which  $n_2 \geq 2$  ( $n_2$  denoting the number of sample units, i.e. clusters):

$$\hat{Y}_c = \frac{\sum_{x \in s_2} M(x) Y_c(x)}{\sum_{x \in s_2} M(x)} \quad (8a)$$

$$\hat{\mathbb{V}}(\hat{Y}_c) = \frac{1}{n_2(n_2 - 1)} \sum_{x \in s_2} \left( \frac{M(x)}{\bar{M}_2} \right)^2 (Y_c(x) - \hat{Y}_c)^2 \quad (8b)$$

## 4.2 Design-based small area regression estimators for cluster sampling

All three considered small area estimators have in common that they use ordinary least square (OLS) regression models to produce the predictions of the local density  $Y_c(x)$  directly on the cluster level  $c$ . We consider the *internal model approach*, where the vector of estimated regression coefficients on the cluster level is found by "borrowing strength" from the entire terrestrial sample  $s_2$  of the current inventory:

$$\hat{\boldsymbol{\beta}}_{c,s_2} = \mathbf{A}_{c,s_2}^{-1} \left( \frac{1}{n_2} \sum_{x \in s_2} M(x) Y_c(x) \mathbf{Z}_c(x) \right) \quad (9a)$$

$$\mathbf{A}_{c,s_2} = \frac{1}{n_2} \sum_{x \in s_2} M(x) \mathbf{Z}_c(x) \mathbf{Z}_c^\top(x) \quad (9b)$$

$\mathbf{Z}_c(x)$  is the vector of explanatory variables on the cluster level, which is calculated as the weighted average of the explanatory variables  $\mathbf{Z}(x_l)$  on the individual plot levels  $x_1, \dots, x_l$  (Eq. 10). The weight  $w(x_l)$  is the proportion of the support-area within the forest  $F$  used to derive the explanatory variables from the raw auxiliary information.

$$\mathbf{Z}_c(x) = \frac{\sum_{l=1}^M I_F(x_l) w(x_l) \mathbf{Z}(x_l)}{\sum_{l=1}^M I_F(x_l) w(x_l)} \quad (10)$$

The estimated design-based variance-covariance matrix  $\hat{\Sigma}_{\hat{\boldsymbol{\beta}}_{s_2}}$  accounts for the fact that the regression model is internal by reflecting the dependency of the estimated regression coefficients on the realized sample  $s_2$ . It is defined as

$$\hat{\Sigma}_{\hat{\boldsymbol{\beta}}_{s_2}} = \mathbf{A}_{c,s_2}^{-1} \left( \frac{1}{n_2^2} \sum_{x \in s_2} M^2(x) \hat{R}_c^2(x) \mathbf{Z}_c(x) \mathbf{Z}_c^\top(x) \right) \mathbf{A}_{c,s_2}^{-1} \quad (11)$$

with  $\hat{R}_c = Y_c(x) - \mathbf{Z}_c^\top(x) \hat{\boldsymbol{\beta}}_{c,s_2}$  being the empirical model residuals at the cluster level, which by construction of OLS satisfy the important *zero mean residual property*, i.e.  $\frac{\sum_{x \in s_2} M(x) \hat{R}_c(x)}{\sum_{x \in s_2} M(x)} = 0$ .

In the following, we will give a short description of each small area estimator and refer to [Mandalaz \(2013a\)](#); [Mandalaz et al \(2016, 2013\)](#) if the reader requires additional details or proofs. The estimators have also been implemented in the R-package *forestinventory* ([Hill](#)

and [Massey, 2017](#)) which was used to compute all estimates in this study.

#### 4.2.1 Pseudo Small Area Estimator (PSMALL)

All point information used for small area estimation is now restricted to that available at the sample locations  $s_{1,G}$  or  $s_{2,G}$  in the small area  $G$ , with exception of  $\hat{\beta}_{c,s_2}$  and  $\hat{\Sigma}_{\hat{\beta}_{c,s_2}}$  which are always based on the entire sample  $s_2$ . We thus first define the following quantities on the small area level:

$$\hat{\mathbf{Z}}_{c,G} = \frac{\sum_{x \in s_{1,G}} M_G(x) \mathbf{Z}_{c,G}(x)}{\sum_{x \in s_{1,G}} M_G(x)} \quad \text{where } \mathbf{Z}_{c,G}(x) = \frac{\sum_{l=1}^L I_G(x_l) \mathbf{Z}(x_l)}{M_G(x)} \quad (12a)$$

$$Y_{c,G}(x) = \frac{\sum_{l=1}^L I_G(x_l) \mathbf{Y}(x_l)}{M_G(x)} \quad \text{and } \hat{Y}_{c,G}(x) = \hat{\mathbf{Z}}_{c,G}^\top \hat{\beta}_{c,s_2} \quad (12b)$$

$$\bar{\hat{R}}_{2,G} = \frac{\sum_{x \in s_{2,G}} M_G(x) \hat{R}_{c,G}(x)}{\sum_{x \in s_{2,G}} M_G(x)} \quad \text{where } \hat{R}_{c,G}(x) = Y_{c,G}(x) - \hat{Y}_{c,G}(x) \quad (12c)$$

Note that the restriction to  $G$ , i.e.  $I_G(x_l) = \{0, 1\}$ , is made on the individual sample plot level  $x_l$ , and  $M_G(x) = \sum_{l=1}^L I_G(x_l)$  thus is the number of sample plots per cluster within the small area. The asymptotically design-unbiased point estimate of *PSMALL* is then defined according to Eq. 13a. The first term estimates the small area population mean of  $G$  by applying the globally derived regression coefficients to the small area cluster means of the explanatory variables  $\hat{\mathbf{Z}}_{c,G}$ . The second term then corrects for a potential bias of the regression model predictions in the small area  $G$  by adding the mean of the empirical residuals  $\bar{\hat{R}}_{2,G}$  in  $G$ . This correction insures that the *zero mean residual property* in  $F$  also holds within the small area  $G$ , which is per se not ensured by fitting the regression coefficients with data outside  $G$ .

$$\hat{Y}_{c,G,PSMALL} = \hat{\mathbf{Z}}_{c,G}^\top \hat{\beta}_{c,s_2} + \bar{\hat{R}}_{2,G} \quad (13a)$$

$$\begin{aligned} \hat{\mathbb{V}}(\hat{Y}_{c,G,PSMALL}) &= \hat{\mathbf{Z}}_{c,G}^\top \hat{\Sigma}_{\hat{\beta}_{c,s_2}} \hat{\mathbf{Z}}_{c,G} + \hat{\beta}_{c,s_2}^\top \hat{\Sigma}_{\hat{\mathbf{Z}}_{c,G}} \hat{\beta}_{c,s_2} \\ &\quad + \frac{1}{n_{2,G}(n_{2,G}-1)} \sum_{x \in s_{2,G}} \left( \frac{M_G(x)}{\bar{M}_{2,G}} \right)^2 (\hat{R}_{c,G}(x) - \bar{\hat{R}}_{2,G})^2 \end{aligned} \quad (13b)$$

The estimated design-based variance of  $\hat{Y}_{c,G,PSMALL}$  is given by Eq. 13b. Basically, the first term constitutes the variance introduced by the uncertainty in the regression coefficients, whereas the second term expresses the variance caused by estimating the exact auxiliary mean in  $G$  using a non-exhaustive sample  $s_{1,G}$ . The third term is the variance of the model residuals and thus accounts for the inaccuracies of the model predictions. Note that the first term can also be rewritten using g-weights ([Mandallaz et al, 2016](#), pg.14) which ensure calibration properties of the auxiliary variables on the terrestrial sample.

The variance-covariance matrix of the auxiliary vector  $\hat{\Sigma}_{\hat{\mathbf{Z}}_{c,G}}$  is thereby defined as

$$\hat{\Sigma}_{\hat{\mathbf{Z}}_{c,G}} = \frac{1}{n_{1,G}(n_{1,G}-1)} \sum_{x \in s_{1,G}} \left( \frac{M_G(x)}{\bar{M}_{1,G}} \right)^2 (\mathbf{Z}_{c,G}(x) - \hat{\mathbf{Z}}_{c,G})(\mathbf{Z}_{c,G}(x) - \hat{\mathbf{Z}}_{c,G})^\top \quad (14)$$

$$\text{with } \bar{M}_{1,G} = \frac{\sum_{x \in s_{1,G}} M_G(x)}{n_{1,G}}.$$

#### 4.2.2 Pseudo Synthetic Estimator (PSYNTH)

The PSYNTH estimator is commonly applied when no terrestrial sample is available within the small area  $G$  (i.e.  $n_{2,G} = 0$ ). The point estimate (Eq. 15a) is thus only based on the predictions generated by applying the globally derived regression coefficients to the small area cluster means of the explanatory variables  $\hat{\mathbf{Z}}_{c,G}$ . Note that the bias correction term using the empirical residuals (Eq. 13a) can no longer be applied. The PSYNTH estimator thus has a potential unobservable design-based bias.

$$\hat{Y}_{c,G,PSYNTH} = \hat{\mathbf{Z}}_{c,G}^\top \hat{\boldsymbol{\beta}}_{c,s_2} \quad (15a)$$

$$\hat{\mathbb{V}}(\hat{Y}_{c,G,PSYNTH}) = \hat{\mathbf{Z}}_{c,G}^\top \hat{\Sigma}_{\hat{\boldsymbol{\beta}}_{c,s_2}} \hat{\mathbf{Z}}_{c,G} + \hat{\boldsymbol{\beta}}_{c,s_2}^\top \hat{\Sigma}_{\hat{\mathbf{Z}}_{c,G}} \hat{\boldsymbol{\beta}}_{c,s_2} \quad (15b)$$

The uncertainty of the model predictions can also no longer be considered in the variance estimation (Eq. 15b). The synthetic estimator will therefore usually have a smaller variance than estimators incorporating the model uncertainties, but at the cost of a potential bias. Note that the PSYNTH estimator is still design-based, but one purely has to rely on the validity of the regression model within the small area as it is the case in the model-dependent framework.

#### 4.2.3 Extended Pseudo Synthetic Estimator (EXTPSYNTH)

The EXTPSYNTH estimator (Eq. 16) has been proposed by [Mandalaz \(2013a\)](#) as a transformed version of the PSMALL estimator that has the form of the PSYNTH estimator but remains asymptotically design unbiased. It has the advantage that the mean of the empirical model residuals of the OLS regression model for the entire area  $F$  and the small area  $G$  are by construction both zero at the same time, i.e.  $\tilde{R}_c = \tilde{R}_{c,G} = 0$ . This is realized by *extending* the auxiliary vector  $\mathbf{Z}_c(x)$  by the indicator variable  $I_{c,G}$  which takes the value 1 if the entire cluster lies within the small area  $G$  and 0 if the entire cluster is outside  $G$ , i.e.  $I_{c,G}(x) = \frac{M_G(x)}{M(x)}$ . The extended auxiliary vector thus becomes  $\mathbf{Z}_c^\top(x) = (\mathbf{Z}_c^\top(x), I_{c,G}(x))$  and the new regression coefficient using  $\mathbf{Z}_c(x)$  instead of  $\mathbf{Z}_c(x)$  in Eq. 9 is denoted as  $\hat{\boldsymbol{\theta}}_{s_2}$ . All remaining components are calculated by plugging in  $\mathbf{Z}_c(x)$  in Eq. 12. A decomposition of  $\hat{\boldsymbol{\theta}}_{s_2}$  reveals that the residual correction term is now included in the regression coefficient  $\hat{\boldsymbol{\theta}}_{s_2}$ .

$$\hat{Y}_{c,G,EXTPSYNTH} = \hat{\mathbf{Z}}_{c,G}^\top \hat{\boldsymbol{\theta}}_{c,s_2} \quad (16a)$$

$$\hat{\mathbb{V}}(\hat{Y}_{c,G,EXTPSYNTH}) = \hat{\mathbf{Z}}_{c,G}^\top \hat{\boldsymbol{\Sigma}}_{\hat{\boldsymbol{\theta}}_{c,s_2}} \hat{\mathbf{Z}}_{c,G} + \hat{\boldsymbol{\theta}}_{c,s_2}^\top \hat{\boldsymbol{\Sigma}}_{\hat{\mathbf{Z}}_{c,G}} \hat{\boldsymbol{\theta}}_{c,s_2} \quad (16b)$$

However, it is important to note that  $\hat{R}_{c,G} = 0$  under the extended regression model only holds if the sample plots  $x_1, \dots, x_l$  of a cluster are *all* either inside or outside the small area, i.e.  $M_G(x) \equiv M(x)$ , and thus  $I_{c,G}(x) = \frac{M_G(x)}{M(x)}$  can only take the values 1 or 0. [Mandalaz et al \(2016\)](#) assumed that the effects on the estimates should be negligible as the number of occasions where  $M_G(x) < M(x)$  was considered to be small in practical implementations. It was thus a further objective of this study to investigate the actual occurrences and effects of this phenomenon by comparing the estimates of EXTPSYNTH to those of PSMALL.

### 4.3 Measures of estimation accuracy

The estimation accuracies were quantified by the *estimation error*, which is the ratio of the standard error and the point estimate:

$$error(\hat{Y})[\%] = \frac{\sqrt{\hat{\mathbb{V}}(\hat{Y})}}{\hat{Y}} * 100 \quad (17)$$

We further calculated the 95% confidence interval for each estimate for visualization purpose. The confidence intervals can also be used for hypothesis testing whether the point estimates of the three estimators for a given small area are statistically different. The confidence intervals under SRS can be obtained as:

$$CI_{1-\alpha}(\hat{Y}) = \left[ \hat{Y} - t_{n_2-1,1-\frac{\alpha}{2}} \sqrt{\hat{\mathbb{V}}(\hat{Y})}, \hat{Y} + t_{n_2-1,1-\frac{\alpha}{2}} \sqrt{\hat{\mathbb{V}}(\hat{Y})} \right] \quad (18)$$

The confidence intervals for the PSMALL and EXTPSYNTH estimates are calculated as:

$$CI_{1-\alpha}(\hat{Y}) = \left[ \hat{Y} - t_{n_{2,G}-1,1-\frac{\alpha}{2}} \sqrt{\hat{\mathbb{V}}(\hat{Y})}, \hat{Y} + t_{n_{2,G}-1,1-\frac{\alpha}{2}} \sqrt{\hat{\mathbb{V}}(\hat{Y})} \right] \quad (19)$$

For the PSYNTH estimates, the confidence intervals are

$$CI_{1-\alpha}(\hat{Y}) = \left[ \hat{Y} - t_{n_2-p,1-\frac{\alpha}{2}} \sqrt{\hat{\mathbb{V}}(\hat{Y})}, \hat{Y} + t_{n_2-p,1-\frac{\alpha}{2}} \sqrt{\hat{\mathbb{V}}(\hat{Y})} \right] \quad (20)$$

In order to address the potential benefits of the small area estimators compared with the SRS approach, we calculated the *relative efficiency* (Eq. 21) which can be interpreted as the relative sample size under SRS needed to achieve the variance under the double sampling estimators.

$$rel.eff = \frac{\hat{\mathbb{V}}_{SRS}(\hat{Y})}{\hat{\mathbb{V}}_{SAE_{2phase}}(\hat{Y})} \quad (21)$$

## 5 Case study

### 5.1 Study area and small area units

The German federal state Rhineland-Palatinate (*RLP*) is located in the western part of Germany and borders Luxembourg, France and Belgium. With 42.3% (appr. 8400 km<sup>2</sup>) of the entire state area (19850 km<sup>2</sup>) covered by forest, RLP is one of the two states with the highest forest coverage among all federal states of Germany ([Thünen-Institut, 2014](#)). The forests of RLP are further characterized by a pronounced diversity in bioclimatic growing conditions that have strong influence on the local growth dynamics as well as tree species composition ([Gauer and Aldinger, 2005](#)) and are further characterized by large variety of forest structures ranging from characteristic oak coppices (Moselle valley), pure spruce, beech and scots pine forests (i.a. Hunsrück and Palatinate forest) up to mixed forests comprising variable proportions of oak, larch, spruce, Scots pine and beech. Around 82% of the forest area in RLP are mixed forest stands and 69% of the forest area exhibit a multi-layered vertical structure. The forest area of RLP are divided into 3 ownership classes, i.e. state forest (27%), communal forest (46%) and privately owned forest (27%). The forest service of RLP has the legal mandate to sustainably manage the state and communal forest area (73% of the entire forest area), including forest planning, harvesting and the sale of wood ([LWaldG, 2000](#)). For this reason, the entire forest area has been spatially organised in 3 main hierarchical management units (Figure 1). On the upper level, RLP has been divided into 45 Forstämter (*FA*), which are further divided into a total number of 405 Forstreviere (*FR*). The next level are the forest stands (104'184 in total) for which expert judgements are conducted by SFIs in a 5- to 10 year period in order to set up management strategies for the upcoming 10 years. The FAs and FRs constituted the small area (*SA*) units for which design-based small area estimations of the mean standing timber volume were calculated by incorporating the available terrestrial inventory data of the BWI3 in the estimators described in Section 4. The average area of the SA units were 43'777 ha on the FA-level, and 4624 ha on the FR level.

### 5.2 Terrestrial sample

Rhineland-Palatinate (*RLP*) is covered by a 2x2 km inventory grid of the German NFI. In the last inventory (BWI3) conducted in the year 2013, timber volume information was derived for 2810 cluster (8092 plots) in the field survey. The local timber volume density on the plot and cluster level for this sample was consequently calculated according to Section 4.1. In the frame of this survey, the plot center coordinates were re-measured with differential global satellite navigation system (DGPS) technique. Knowledge about the exact plot positions were considered crucial to provide optimal comparability between the terrestrially observations and the information derived from the auxiliary information. A comparison of the DGPS coordinates with the so-far used target coordinates revealed that 90% of all horizontal deviations lay in the range of 25 meters. A detailed analysis of horizontal DGPS errors in

## 5. CASE STUDY

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RLP by Lamprecht et al (2017) indicated that 80% of the plots should not exceed horizontal DGPS errors of 8 meters. For 162 plots, the DGPS coordinates were replaced by their target coordinates due to missingness or implausible values. The terrestrial sample size  $n_{2,G}$  within the FA units was 46 cluster on average and ranged between 11 and 64. Within the FR units,  $n_{2,G}$  was considerably smaller with an average of 5 cluster and a range between 0 and 13.

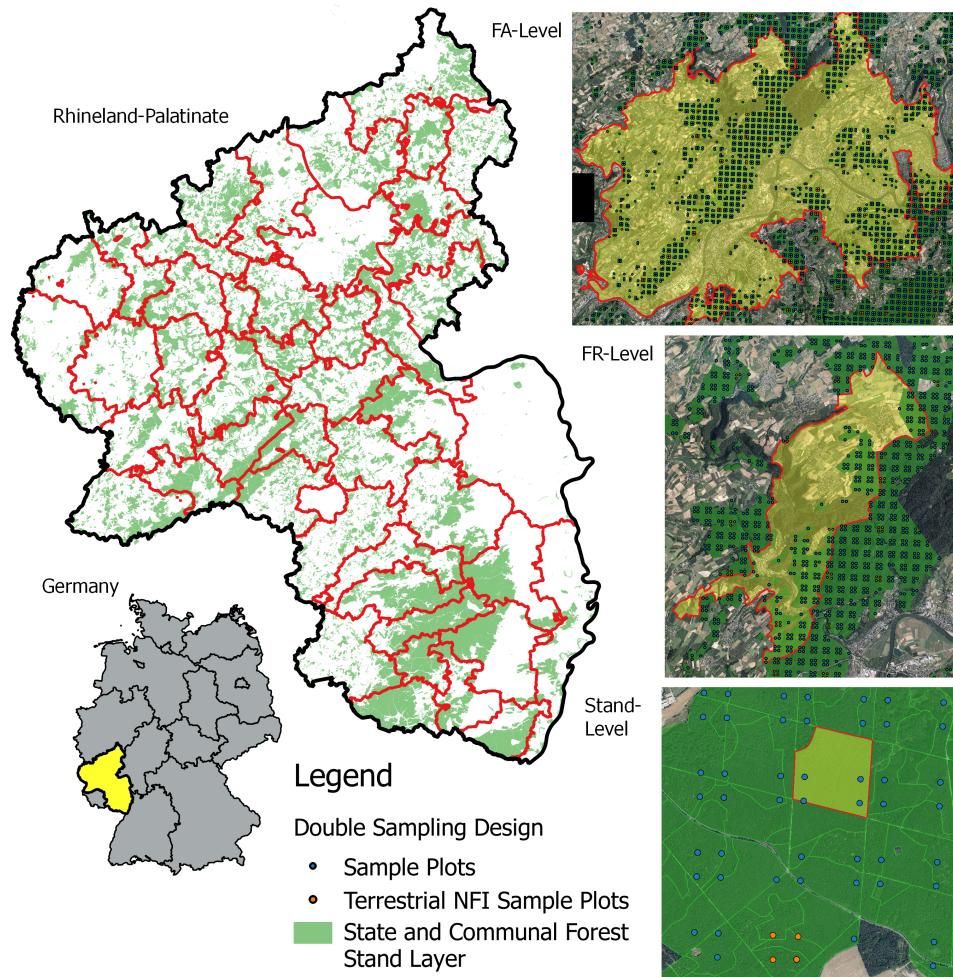


Figure 1: *Left:* Study Area with delineated FA forest management units. *Right:* Example for each of the three management units (from top to bottom): FA, FR and forest stand unit with overlayed the extended double sampling cluster design. *Green:* Forest stand polygon layer defining the forest area of this study.

### 5.3 Extension to double sampling design

In order to apply the small area estimators (Section (4.2)), the existing NFI design was extended to a double sampling design by densifying the existing systematic 2x2 kilometer grid to a grid size of 500x500 meters that constituted the large first phase  $s_1$  in accordance to

Section 3 (Figure 1, right). The existing terrestrial phase  $s_2$  was consequently integrated by replacing the target coordinates of the respective  $s_1$  cluster by the terrestrially measured DGPS coordinates. For our study, we restricted the *sampling frame* to the communal and state forest. The forest/non-forest decision for each plot was thereby made by a spatial intersection of the plot center coordinates with a polygon layer of the communal and state forest stands provided by the forest service. Using this stand layer provided the advantage to consistently apply the same forest/non-forest definition to the entire sample  $s_1$  in order to decide about excluding or including a plot in the sampling frame. The terrestrial sample size  $n_2$  was thus reduced to 2055 cluster (5791 plots). Table 1 provides a short descriptive summary about the volume densities and the main attributes of the NFI plots located in the state and communal forest sampling frame. The densification led to an average sample size  $n_{1,G}$  of 759 cluster (range: 246 – 1022) in the FA units, and 88 cluster (range: 1 – 194) in the FR units.

Table 1: Descriptive statistics of the forest observed on NFI sample plots located within communal and state forest area (n=5791).

Variable	Mean	SD	Maximum
Timber Volume (m <sup>3</sup> /ha)	300.86	195.55	1375.31
Mean DBH (mm)	354.90	137.22	1123.20
Mean height (dm)	239.60	72.43	497.43
Mean stem density per hectare	101.00	114.01	1010.31

## 5.4 Auxiliary information

### 5.4.1 LiDAR canopy height model

A prerequisite for the application of the suggested two-phase small area estimators was the identification of suitable auxiliary information that is available over the entire study area. Between the year 2003 and 2013, the topographic survey institution of RLP conducted an airborne laser scanning (LiDAR) acquisition over the entire federal state at leaf-off condition in order to derive a countrywide digital terrain and surface model. For this study, the recorded LiDAR data was used to create a canopy height model (CHM) in raster format, providing discrete information about the canopy surface height of the forest area in a spatial resolution of 5 meters (Figure 2, top). The CHM was calculated as the difference between the digital terrain model (DTM) and the digital surface model (DSM) that were derived by a Delauney interpolation of the two separated LiDAR point clouds (ground pulses and first pulses from non-ground objects). Based on numerous studies (e.g. Maack et al (2016)), the CHM was considered to provide the most valuable information to be used in the OLS regression model for predicting the timber volume on sample plot level. However, during the extended acquisition period, airborne laser scanning technology evolved significantly, which lead to severe quality variations in the CHM. In particular, the LiDAR acquisition recorded in 2002 and 2003 exhibited a rather poor quality with about only 1 point per 5x5 m<sup>2</sup>, while more recently acquired

datasets contained more than 125 points per 5x5 m<sup>2</sup> raster cell. Furthermore, due to sensor failures, CHM information was not available at 16 sample locations (none of them considered terrestrial sample plots of the  $s_2$  phase though). These plots were deleted from the sampling frame and its non-available information thus treated as missing at random. This assumption was considered to be reasonable as the respective sample locations did not exclude specific forest structures.

#### 5.4.2 Tree species map

The second source of auxiliary information was a countrywide satellite-based classification map of the five main tree species (European beech, Sessile and Pedunculate oak, Norway spruce, Douglas fir, Scots pine) described in [Stoffels et al \(2015\)](#) (Figure 2, bottom). The classified tree species map has a grid size of 5 meters and was calculated from 22 bi-temporal satellite images (SPOT5 and RapidEye) by application of a spatially adaptive classification algorithm described in [Stoffels et al \(2012\)](#). As timber volume estimations on the tree level are often based on species-specific biomass and volume equations, the use of tree species information - especially for timber volume estimation - has often been stated as a key factor for improving the estimation precision [White et al \(2016\)](#). In this respect, incorporating the tree species map information was particularly attractive as it predicts five of the seven tree species that are used in the BWI3 taper functions ([Kublin et al, 2013](#)) to calculate the timber volume of a sample tree. However, due to unavailable satellite data for the classification, the tree species map excluded one large patch with an area of 415 km<sup>2</sup> in the south-west part of RLP covering an entire FA-unit (10 FR units respectively). In 9 additional FR units, the tree species information was also missing for a subset of the sample locations due to two additional patches with an area of 76 km<sup>2</sup> and 100 km<sup>2</sup> in the northern part of RLP. For these parts, small area estimation was thus restricted to using only the available CHM information in the regression model. With respect to model fitting, the tree species information was missing for 407 (7%) of the 5791 sample locations. A summary of the sample sizes and missing auxiliary information is again provided in Table 2.

Table 2: Sample size for each phase in entire study area.  $n_{\{1,2\},plots}$ : number of plots.  $n_{\{1,2\}}$ : number of cluster

<i>Sampling frame</i>	$n_{1,plot}$	$n_1$	$n_{2,plot}$	$n_2$
communal and state forest	96'854	33'365	5791	2055
missing CHM	18	10	0	0
missing TSPEC	7060	3587	414	385
missing CHM and TSPEC	3	2	0	0
missing CHM or TSPEC	7075	3595	414	385

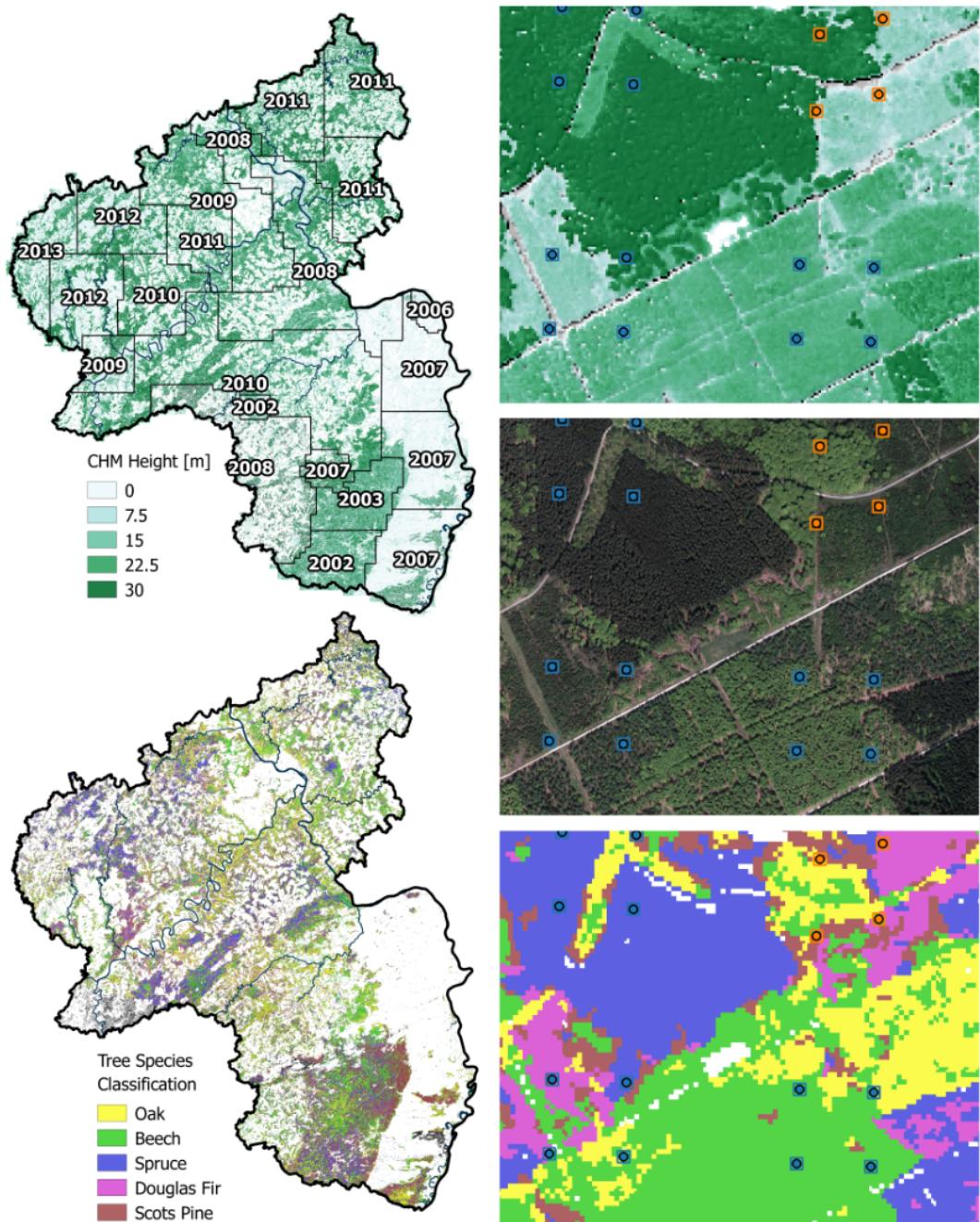


Figure 2: Left: CHM (top) and tree species classification map (bottom) available on the federal state level were used as auxiliary information. Right: Magnified illustration of the rectangular supports used to derive the explanatory variables from the auxiliary information.

## 5.5 Calculation of the auxiliary variables

### 5.5.1 LiDAR canopy height model

From the CHM, the mean canopy height (*meanheight*) and the standard deviation (*stddev*) was calculated by evaluating the raster values around each sample location within a predefined rectangular square (support) with a side length of 12 meters that was centered around the sample plot centers. Each support was previously intersected with the forest polygon layer in order to evaluate auxiliary information only within the forest definition. The percentage of the non-intersected support within the forest layer was used as the weight  $w(x_l)$  introduced in Eq. 10. Furthermore, the year of the LiDAR acquisition (*lidaryear*) was derived as an additional categorical variable to account for the time-lag between the CHM information and the terrestrial survey as well as to explain heterogeneity in the data introduced by the varying LiDAR quality. Adjustments were made to the original acquisition years by introducing an additional factor level *2008\_I* for a subset of the 2008 acquisition where the quality turned out to be considerably poor due to a sensor error. In addition, the years 2006 and 2007 as well as 2012 and 2013 were pooled in order to increase the number of observations per factor level, resulting in nine categories in total (2002, 2003, 2007, *2008\_I*, 2008, 2009, 2010, 2011 and 2012).

### 5.5.2 Tree species map

The tree species map information was used to predict the main tree species of a sample plot (*treespecies*). This implied two consecutive processing steps. In the *first* step, one of the five tree species was assigned to a sample location if 100% of the raster values within the same support as used for the CHM were classified as that species. Otherwise, the sample location was assigned the value 'mixed'. It should be mentioned that the support size and the percentage threshold are parameters that can be optimised in order to achieve the best variance decomposition and thus improvement in model accuracy when using the categorical *treespecies* variable in a regression model. [Hill et al \(2018\)](#) conducted a detailed analysis of how to optimally derive the auxiliary variables for the present data set and identified the above described settings as the most suitable. It was additionally shown that using the random forest algorithm by [Breiman \(2001\)](#) for a calibration of the predicted main plot species on the actual main taper-function species derived from the terrestrial inventory data could boost the classification accuracy in the classes *Scots pine* and *Douglas Fir*, which performed considerably worse than the remaining species (Table 3). The calibration was able to considerably mitigate the effects of misclassification on the regression coefficients and thus allowed for an increase in model accuracy. In a *second* step, we thus provided the random forest algorithm in the statistical software R ([R Core Team, 2017](#)) with the actual main taper-function species derived from the terrestrial inventory data as response variable as well as a full set of  $p$  predictor variables that comprised the initial prediction of the main plot tree species (*treespecies*), the mean canopy height (*meanheight*) and standard deviation (*stddev*) derived from the CHM, the proportion of coniferous trees estimated from the tree species classification map (*prop.conif*)

and the bioclimatic growing region ( $wgb$ ) at the sample location. The algorithm was grown with 2000 trees, considering  $\sqrt{p} \approx 3$  of the predictors for each split. The resulting random forest model was consequently applied at all sample locations  $s_1$  to provide the *treespecies* variable.

Table 3: Classification accuracies of the *treespecies* variable before and after calibration.  $n_{ref}$ : number of terrestrial reference plots.  $n_{class}$ : number of classified plots.

Main plot species	Producer's accuracy[%]		User's accuracy[%]		nref	nclass	
	uncalib	calib	uncalib	calib		uncalib	calib
Beech	29.56	22.31	46.11	47.02	883	566	419
Douglas Fir	33.04	24.78	47.50	48.72	230	160	117
Oak	47.75	11.07	24.34	48.48	289	567	66
Spruce	43.63	53.15	60.81	61.13	651	467	566
Scots Pine	55.87	22.91	29.59	46.07	179	338	89
Mixed	66.40	84.49	63.69	64.53	3152	3286	4127
Overall accuracy: 54.83% (61.96%)					5384	5384	

## 5.6 Regression Model

## 6 Results

### 6.1 General estimation results

An application of the SRS, PSMALL and EXTPSYNTH estimator was not feasible for 17 of all 405 FR-units due to an insufficient terrestrial sample size of  $n_{2,G} < 2$ . We further restricted the calculation of the PSMALL and EXTPSYNTH estimator to small area units with a minimum terrestrial sample size of  $n_{2,G} \geq 4$  to avoid unstable estimates. This affected 65 additional FR units and limited unbiased two-phase estimations to 321 (79%) of the 405 FR units. It should be noted that also the PSYNTH estimator could not be applied for 2 FR-units since  $n_{1,G} < 2$ . Due to substantial larger sample sizes, all estimators could however be applied to all 45 FA units. The average value and the range of the mean timber volume estimates over the evaluated FA and FR units turned out to be very similar between all estimators (table 4). An additional pairwise comparison of the 95% confidence intervals revealed that the four estimators did in fact not produce statistically different point estimates for all FA and FR units. This confirmed that the differences between the estimators are solely found in the precision which they provide for the point estimates.

Table 4: Descriptive summary of point estimates and estimation errors on the two forest district levels.  $N_u$ : number of evaluated small area units.

District level	Estimator	Point estimates			Errors [%]		
		mean	min	max	mean	min	max
FA	SRS ( $N_u=45$ )	300.16	215.91	392.84	6.69	3.87	13.21
	PSMALL ( $N_u=45$ )	307.29	209.26	417.10	5.16	3.46	14.33
	EXTPSYNTH ( $N_u=45$ )	307.27	209.01	415.02	4.78	3.25	13.88
	PSYNTH ( $N_u=45$ )	306.90	223.51	409.92	2.34	1.54	3.95
FR	SRS ( $N_u=388$ )	301.83	99.89	612.13	18.32	0.34	104.97
	PSMALL ( $N_u=321$ )	308.15	159.64	568.67	12.24	3.48	44.94
	EXTPSYNTH ( $N_u=321$ )	308.38	154.07	544.34	11.34	3.60	40.91
	PSYNTH ( $N_u=403$ )	307.82	166.01	444.29	4.65	2.56	62.51

### 6.2 Estimation errors

On both small area levels, the design-unbiased estimators PSMALL and EXTPSYNTH led to a substantial reduction in the estimation errors compared to the SRS estimator (fig. 3). On the FA level, the SRS estimator yielded an estimation error of 6.7% on average compared to 5.2% and 4.8% under the EXTPSYNTH and PSMALL estimator (table 4). The cumulative error distribution (fig. 3, left) reveals that under the SRS estimator, errors  $\leq 5\%$  were achieved for 17% of the FA units (8 of 45). This proportion could be increased to 62% (28 FA units) and

## 6. RESULTS

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73% (33 FA units) by application of the PSMALL and EXTPSYNTH estimator. 95% of all estimates exhibited errors  $\leq 9.5\%$  under the SRS estimator and  $\leq 6.6\%$  when using PSMALL or EXTPSYNTH. Estimation errors higher than 10% only appeared twice for each of the three estimators.

The error reduction by the PSMALL and EXTPSYNTH estimator was even more pronounced on the FR level (fig. 3, right), although the overall error niveau was substantially higher than on FA level. The average error under the SRS estimator was 18.3%, while it was 11.3% and 12.2% under the PSMALL and EXTPSYNTH estimator (table 4). Errors smaller than 10% were achieved for 15% of the FR units by the SRS estimator, and for 46% by the PSMALL and PSYNTH estimator. 95% of the 321 FR units where PSMALL and EXTPSYNTH could be applied exhibited errors  $\leq 20\%$ . In comparison, the SRS estimates resulted in errors  $\leq 36.6\%$  for 95% of the 388 FR units.

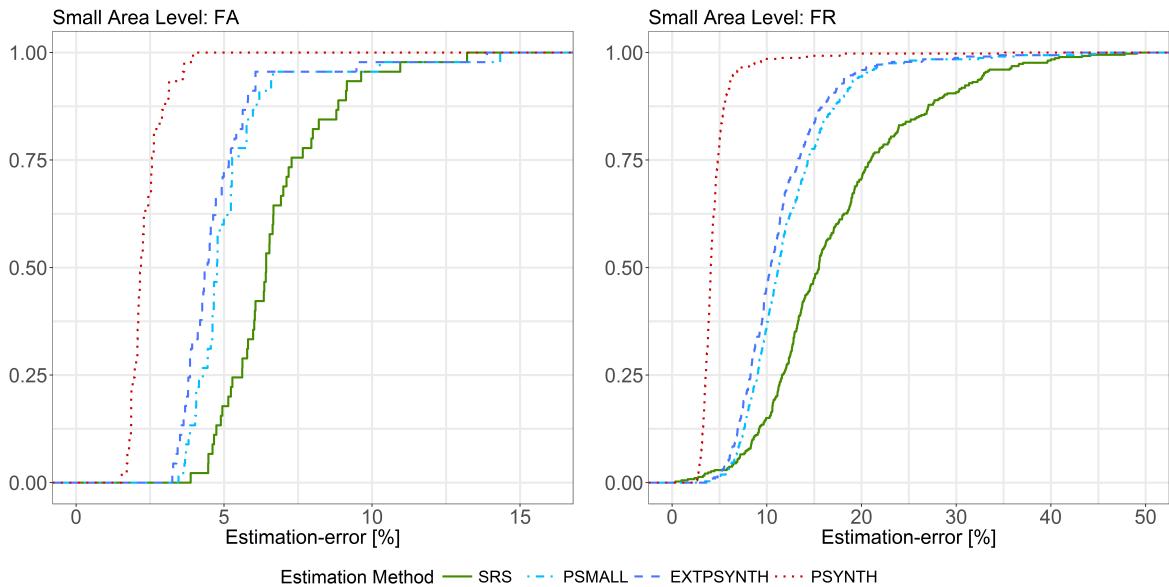


Figure 3: Cumulative distribution of estimation errors under the simple random sampling (SRS), the pseudo small (PSMALL), the extended pseudo synthetic (EXTPSYNTH) and the pseudo synthetic (PSYNTH) estimator. *Left:* Results for the 45 FA units. *Right:* Results for the 388 (SRS), 321 (PSMALL / EXTPSYNTH) and 403 (PSYNTH) FR units.

On both small area levels, the PSYNTH estimator resulted in much smaller estimation errors compared to PSMALL and EXTPSYNTH. This was as expected, since the PSYNTH variance estimate does not take the residual variation in each small area unit into account (section 4.2.2). Compared to the asymptotically design-unbiased estimators PSMALL and EXTPSYNTH, the estimation errors produced by PSYNTH thus seem to be too optimistic. One should also recall that the estimates of the PSYNTH estimator are potentially design-biased.

### 6.3 Comparison of PSMALL and EXTPSYNTH

Figure 3 reveals that the error distribution of PSMALL and EXTPSYNTH are very similar, with PSMALL showing marginally higher estimation errors. In order to investigate the differences between PSMALL and EXTPSYNTH, we compared the g-weight variances of both estimators for all 321 FR units (fig. 4, left). As obvious, PSMALL yielded slightly larger variances for the vast majority of the estimates. As addressed in section 4.2.3, one possible explanation for such differences was the effect of one or more cluster not entirely being included in a small area unit, as this would constitute a violation of the EXTPSYNTH estimator. This violation was actually observed in 155 of the 321 FR units (48%). However, the affected FR units (depicted in red diamonds, fig. 4) did not show a significant divergence from the PSMALL variances with respect to the remaining unaffected FR units. The variance differences between the two estimators were thus due to the mathematical formulations of the PSMALL and EXTPSYNTH estimator, which are asymptotically equivalent only under large terrestrial sample sizes  $n_{2,G}$  within the small area (Mandallaz et al, 2016, pp.17–18). An additional comparison of the absolute differences in the g-weight variance (fig. 4, right) revealed that large divergences did in fact particularly occur for small area units with small terrestrial sample sizes ( $n_{2,G} \leq 5$ ). The differences decreased with increasing sample size and thus confirmed the asymptotic relationship between the two estimators. However, a comparison of the confidence intervals of PSMALL and EXTPSYNTH revealed that the variance differences did not lead to statistically significant point estimates. The results indicate that the adjustment of the intercept term to the data within a small area used in the EXTPSYNTH estimator will in general yield slightly smaller variances than the PSMALL estimator.

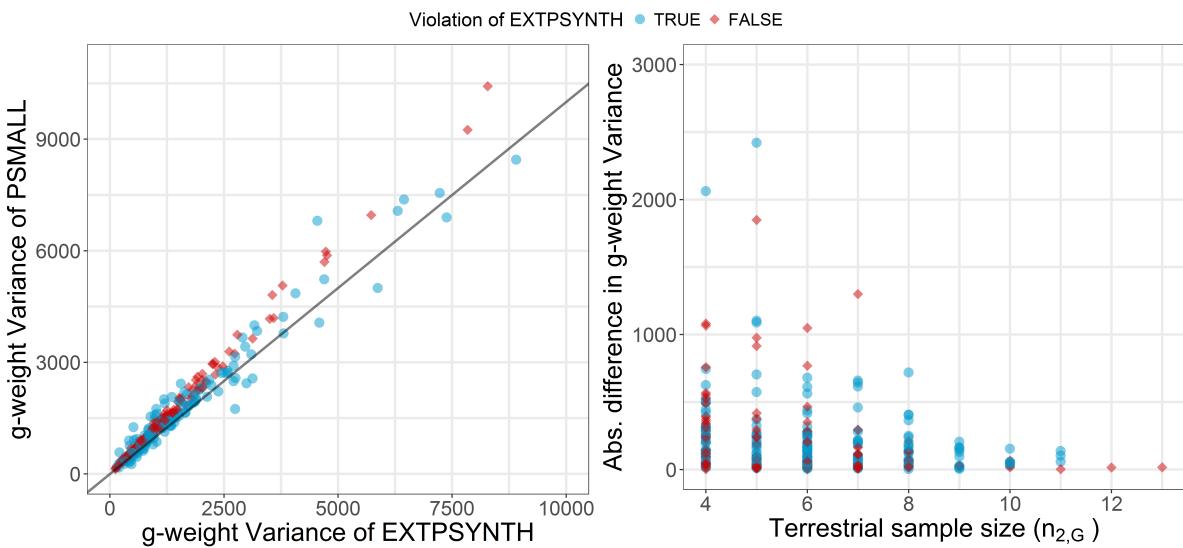


Figure 4: *Left:* Comparison of the g-weight variance between the PSMALL and the EXTPSYNTH estimator for the 321 FR units. *Right:* Difference in g-weight variance between the PSMALL and the EXTPSYNTH estimator in dependence of the terrestrial data ( $n_{2G}$ ) in the FR unit.

## 6.4 Reduction of SRS variance by PSMALL and EXTPSYNTH

A direct comparison of the achieved variances within the small area units revealed that the application of the design-unbiased estimators (PSMALL and EXTPSYNTH) led to a reduction of the SRS variance in all FA units. In 75% of the FA units, the EXTPSYNTH estimator was able to reduce the SRS variance by up to 54.1% (fig. 5). The reduction in variance was also expressed in the relative efficiency values, which were 2.02 on average and ranged between 1.18 and 4.13 on the FA level. On FR-level, the reduction in variance and the relative efficiencies reached even higher values (table 5). The PSMALL estimator again yielded slightly lower variance reductions and relative efficiencies due to the smaller variances produced by the EXTPSYNTH estimator (section 6.3). However, cases also occurred on the FR level where one or both two-phase estimators produced larger variance values than under the SRS estimator. This particularly happened in 19% (61) of the FR units under the EXTPSYNTH, and in 24% (76) of the FR units under the PSMALL estimator.

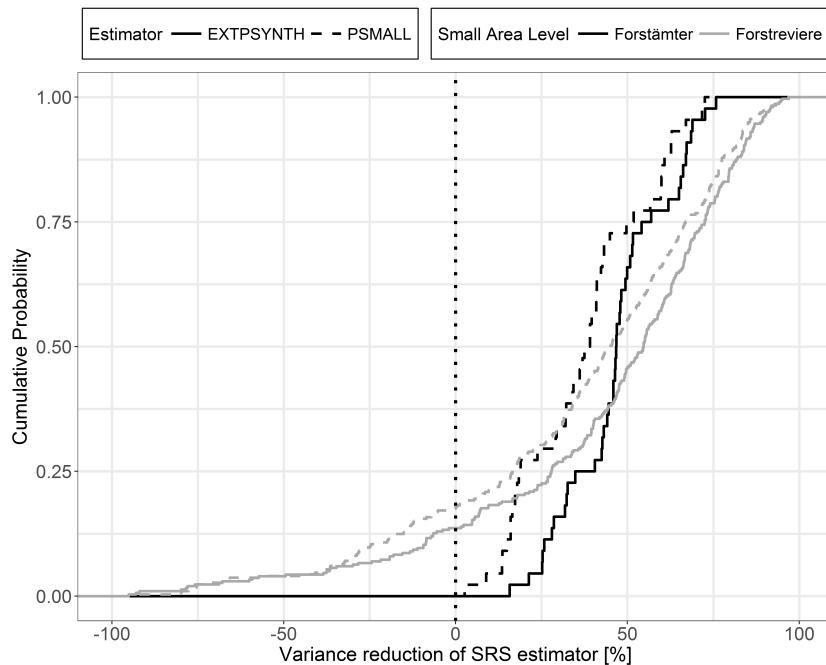


Figure 5: Cumulative distribution of variance reduction by the PSMALL and EXTPSYNTH compared to the SRS estimator for the 45 FA and 321 FR units.

One possible reason for an increased variance by the two-phase estimators was supposed to be a poor performance of the regression model within the small area unit. In order to investigate this hypothesis, we analyzed the three variance terms of the PSMALL estimator (eq. 13b), i.e. the variance introduced by the uncertainty of the regression coefficients (term 1), the variance caused by estimating the auxiliary means (term 2), and the variance of the model residuals that expresses the model performance within the small area (term 3). Figure 6 illustrates the reduction or increase of the SRS variance by PSMALL for all FR units in dependence of the residual variance term proportion on the g-weight variance. Two issues

## 6. RESULTS

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become obvious: firstly, the residual term generally constitutes the dominating part of the g-weight variance for all PSMALL estimates (around 84% on average). FR units for which the PSMALL estimator produced larger variances than the SRS estimator did not differ distinctly from the pattern of the remaining FR units. The same was also found when looking at the absolute residual term values (fig. 7). Secondly, FR units with the highest variance increases compared to SRS were particularly observed under small terrestrial sample sizes  $n_{2,G} = 4$ . The average increase in variance for  $n_{2,G} = 4$  was 272% compared to 62% for  $n_{2,G} > 4$ . In comparison, the average reduction in SRS variances was around 50% for both sampling sizes  $n_{2,G} = 4$  and  $n_{2,G} > 4$ . These findings suggest that moderate or poor model fits within a small area can cause the PSMALL and EXTPSYNTH estimator to produce higher variances than the SRS estimator particularly under small terrestrial sample sizes. However, small terrestrial sample sizes can also have the effect that the SRS estimator might not reflect the actual variation of the local density within a small area. In this case, the two-phase variance estimate might be larger but more realistic. Whereas a visual analysis of aerial images, remote sensing data or stand maps might give some evidence for this hypothesis, a definite proof is practically infeasible.

Table 5: Descriptive summary of SRS variance reduction and relative efficiencies on the two forest district levels.  
 $N_u$ : number of evaluated small area units.

District level	Estimator	Reduction of SRS variance [%]			relative efficiency		
		mean	min	max	mean	min	max
FA	PSMALL ( $N_u=45$ )	33.51	2.6	72.5	1.74	1.03	3.64
	EXTPSYNTH ( $N_u=45$ )	43.30	15.7	75.8	2.03	1.18	4.13
FR	PSMALL ( $N_u=321$ )	12.48	-1203.9	96.8	2.54	0.08	31.61
	EXTPSYNTH ( $N_u=321$ )	24.75	-892.7	97.0	2.95	0.10	33.70

## 6. RESULTS

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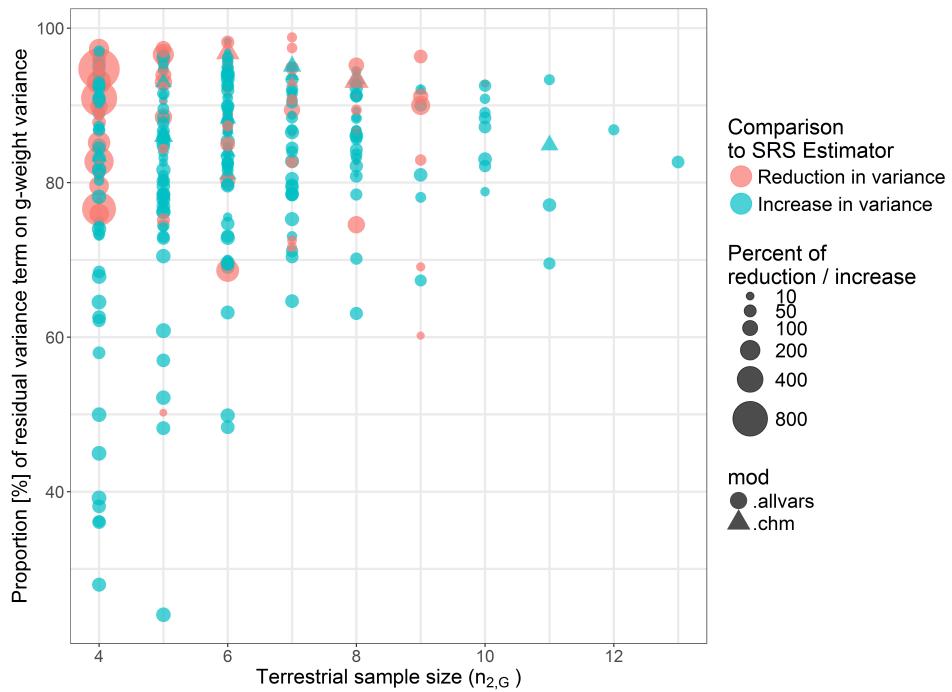


Figure 6

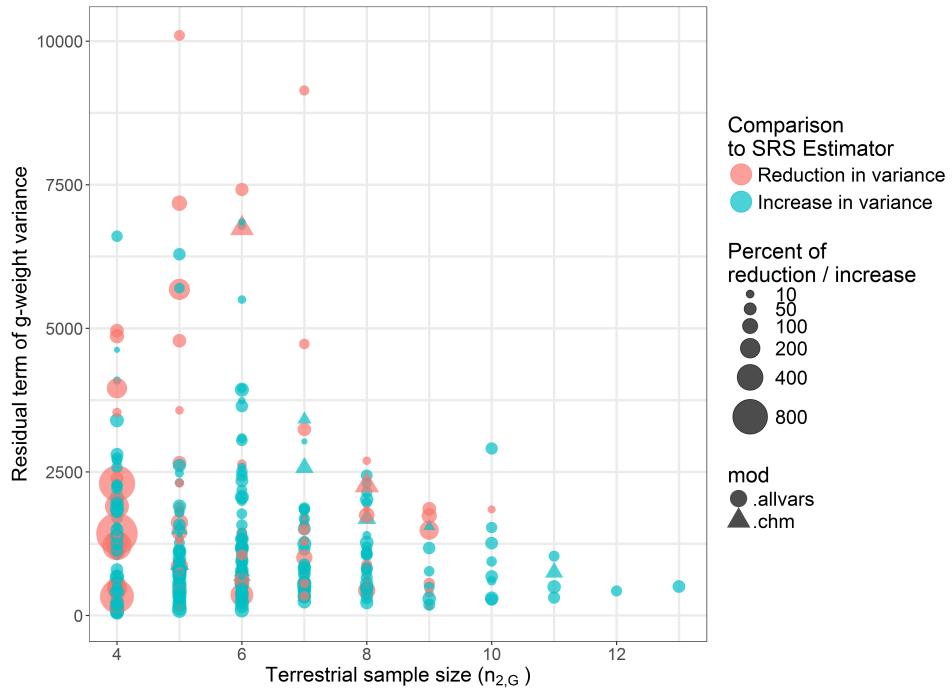


Figure 7

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