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ORIGINAL PAPER

Area-level analysis of forest inventory variables

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Abstract Small-area estimation is a subject area of growing importance in forest inventories. Modelling the link between a study variable Y and auxiliary variables $\mathbf{X}$—in pursuit of an improved accuracy in estimators—is typically done at the level of a sampling unit. However, for various reasons, it may only be possible to formulate a linking model at the level of an area of interest (AOI). Area-level models and their potential have rarely been explored in forestry. This study demonstrates, with data ($Y =$ stem volume per ha) from four actual inventories aided by aerial laser scanner data (3 cases) or photogrammetric point clouds (1 case), application of three distinct models representing the currency of area-level modelling. The studied AOIs varied in size from forest management units to forest districts, and municipalities. The variance explained by $\mathbf{X}$ declined sharply with the average size of an AOI. In comparison with a direct estimate mean of Y in an AOI, all three models achieved practically important reduction in the relative root-mean-squared error of an AOI mean. In terms of the reduction in mean-squared errors, a model with a spatial location effect was overall most attractive. We recommend the pursuit of a spatial model component in area-level modelling as promising within the context of a forest inventory.

Keywords Direct estimators · Empirical best linear unbiased prediction · Hierarchical Bayes empirical best linear unbiased prediction · Non-stationary spatial effects · Shrinkage factor · Variance smoothing

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Introduction

Forest inventories typically have a plethora of objectives and multiple study-variables (Köhl and Magnussen 2014). Some objectives, but not all, can be met by direct design-based estimators (Kangas and Maltamo 2006, Chap. 1; Köhl et al. 2006, Chap. 1). An inventory supported by one or more auxiliary variables ($\mathbf{X}$) known for all population units and correlated with a study variable (Y) provides the opportunity to formulate a model for predicting Y from $\mathbf{X}$ and using this model to produce spatially explicit predictions for each unit in an area of interest (AOI) (Brosofske et al. 2014; Mauro et al. 2016; Tomppo 2006). The map portrays the expected value of Y given the model linking Y to $\mathbf{X}$ (Gregoire et al. 2016; McRoberts 2011; Opsomer et al. 2007; Ståhl et al. 2016). If the model is correctly specified for all AOIs, the mapped unit-level values provide a model-unbiased estimate of the mean (total) of Y in these areas (Mandallaz 2013; McRoberts 2010). An approximately unbiased design-based variance estimator for this mean (total) is possible as long as there is at least two sampled units from an AOI (Mandallaz 2013; Pfeffermann 2013; Rai and Pandey 2013). Model-based estimators are always feasible (Chambers and Clark 2012, ch. 15).

In inference about Y in an AOI, the key question becomes whether an entertained model is correctly specified for the AOI. With a sufficient sample size from an AOI, an analyst can accommodate local effects by refitting a linking model to data from the AOI or a group of 'similar' AOIs (Lehtonen and Veijanen 2009; Magnussen et al. 2014; Mandallaz et al. 2013). Alternatively one can formulate a model that includes random (unobservable) area effects for a set of AOIs with a combined sample size that supports the estimation objectives (Breidt 2004; Rao 2005).

The model for linking Y to $\mathbf{X}$ is typically—and most effectively—formulated at the level of collocated congruent units of observation of Y and $\mathbf{X}$. In forestry, a unit is, in most cases, a sample plot composed of one or more subplots (Brosofske et al. 2014; Ohmann et al. 2014). However, the spatial support of $\mathbf{X}$ and Y may not be compatible, the location of a sample unit may not be known with a desired accuracy, or the exact location of a sample unit may be confidential due to regulatory decisions or matters of privacy (Bechtold and Patterson 2005). To exploit the predictive power of $\mathbf{X}$ in these situations, a less powerful modelling approach at the level of an AOI is open to the analyst (Fay and Herriot 1979; Rao and Yu 1994). In an area-level model, the area sample mean (total) of Y is linked to auxiliary information at the AOI level (i.e. area mean (total) of $\mathbf{X}$ within the AOI) and the model may or may not include random area effects. Although practically relevant, there has only been a few forestry-related applications of area-level models for small-area estimation problems (Boubeta et al. 2015; Goerndt et al. 2011).

Recent developments in area-level analyses of AOIs suggest a potential of a greater efficiency by exploring possible spatial effects (Chandra et al. 2007; Marhuenda et al. 2013; Molina et al. 2009; Pereira and Coelho 2012; Petrucci et al. 2005; Pfeffermann 2013; Pratesi and Salvati 2008; Salvati et al. 2012), or conversely, pursue a more efficient shrinkage of area means (total) of Y towards model-based expectations (Datta et al. 2011, 2005; Wang and Fuller 2003; Wanjoya et al. 2012).

Our objective for this study is a demonstration of three relatively recent area-level estimators that we believe to be of interest to forestry when a unit-level modelling and analysis is no longer feasible.

We apply the three novel methods to four study sites that are examples from actual inventories with forest stands, forest districts, or municipal forests in the role of AOIs. We have used the software Mathematica version 11.1 for our analyses, but links to code written in R can be found in the references cited under each method.

Materials and Methods

A basic area-level model

For a group of m 'small' AOIs each with a direct estimate $\bar{y}_i$ of the mean (total) of a study variable Y and a set of p auxiliary variables $\mathbf{X}$ correlated with $\bar{y}_i$, Fay and Herriot (1979) introduced a generic area-level model

$$\bar{y}_i = \theta_i + e_i, \quad i = 1, \ldots, m \tag{1}$$

where $\bar{y}_i$ is a direct estimate of the mean of Y in the ith AOI derived from $n_i \geq 2$ sample units (plots) with observations of Y; θ_i is the true mean of Y in the ith AOI; and e_i is the sampling error in $\bar{y}_i$. The sampling errors $e_1, \ldots, e_m$ are assumed independent (by design) and normally distributed with $e_i \sim N\left(0, \sigma_{ei}^2\right)$ and σ_{ei}^2 assumed known. The true mean θ_i is assumed to be a function of the vectors of auxiliary variables. In this study, we specifically assume that θ_i follows a linear model

$$\theta_i = \bar{\mathbf{x}}_i^T \boldsymbol{\beta} + v_i, \quad i = 1, \ldots, m \tag{2}$$

where $\bar{\mathbf{x}}_i^T$ is the transpose of a length $(p + 1)$ row vector of p auxiliary variables plus an intercept in the ith AOI, $\boldsymbol{\beta}$ is a vector of regression coefficients to be estimated; and v_i is a random area effect for the ith AOI. The random effects $v_1, \ldots, v_m$ are assumed independent of $e_1, \ldots, e_m$ and *iid* $N\left(0, \sigma_v^2\right)$. Traditionally, estimators of θ_i have been either empirical best linear unbiased predictions, EBLUP (Rao and Molina 2015, p. 123) or empirical Bayes EB (Rao and Molina 2015, p. 270). Mean-squared errors (MSE) for these types of estimators are in Rao and Molina (2015) on page 136 and 273, respectively.

After combining Eq. 1 and 2 and some rearrangements, the desired estimator of θ_i can be written as the sum of a synthetic (model-based) estimate $\bar{\mathbf{x}}_i^T \hat{\boldsymbol{\beta}}$ and a scaled residual error $\hat{\gamma}_i\left(\bar{y}_i - \bar{\mathbf{x}}_i^T \hat{\boldsymbol{\beta}}\right)$. Specifically

$$\tilde{\theta}_i = \hat{\gamma}_i \bar{y}_i + (1 - \hat{\gamma}_i) \bar{\mathbf{x}}_i^T \hat{\beta} \tag{3}$$

where $\hat{\gamma}_i = \hat{\sigma}_v^2 \left(\hat{\sigma}_v^2 + \sigma_{ei}^2\right)^{-1}$. The scaling factor $0 < \hat{\gamma}_i < 1$ 'shrinks' the estimated residual errors $\left(\bar{y}_i - \bar{\mathbf{x}}_i^T \hat{\boldsymbol{\beta}}\right)$ towards zero as a function of the reliability measure $\hat{\gamma}_i$ of the area effects v_i in Eq. 2 (Donner and Eliasziw 1987; Lin et al. 2002).

A baseline area-level model (WF)

The basic Fay–Herriot area-level model (Fay and Herriot 1979) was given in Eq. 3. Traditional EBLUP estimates of θ_i have assumed that the variance σ_{ei}^2 of a direct estimate of an area mean is known. With relatively small sample sizes in forest inventory, we think this assumption is questionable. In order to recognize the uncertainty in σ^2_{ei}, we adopted the area-level estimators of Wang and Fuller (2003). Accordingly, the Wang–Fuller EBLUP of θ_i was

$$\tilde{\theta}_i^{WF} = \bar{\mathbf{x}}_i^T \tilde{\boldsymbol{\beta}} + \hat{\gamma}_i^{WF} \left(\bar{y}_i - \bar{\mathbf{x}}_i^T \tilde{\boldsymbol{\beta}} \right), \quad i = 1, \ldots, m \tag{4}$$

In the computations, we used their Eq. 21 as the estimator of σ^2_v, Eqs. 26 and 27 for the estimates of $\boldsymbol{\beta}$ and their covariance, and Eqs. 32–34 for the calculation of the MSE of $\tilde{\theta}_i^{WF}$.

Area-level models applied in this study

Two area-level model variants, to be introduced later, both seek a reduction in MSE of an estimate of the true area mean θ_i. The first assumes that the regression model in Eq. 2 has one or more spatially varying coefficients (Fotheringham et al. 2003). Chandra et al. (2015) developed the modelling framework and used a restricted maximum likelihood estimation of the model parameters. Chandra et al. (2015) used the acronym NSEBLUP for their estimators which stands for non-stationary spatial empirical best linear unbiased predictor. Here, we use the label CH for the sake of brevity. In forestry, an assumption of spatially varying coefficients seems intuitive (Molina et al. 2009; Opsomer et al. 2008; Salvati et al. 2012).

The second area-level variant regards area effects (v_i) as nuisance parameters as they increase the uncertainty in a point and interval estimator of a small-area mean (Datta et al. 2011). If the null hypothesis of zero valued small-area effects is not rejected, a MSE of $\tilde{\theta}_i$ conditional on $\hat{\sigma}^2_v = 0$ will be smaller than a MSE conditional on $\hat{\sigma}^2_v > 0$. The same authors also observed that a statistically significant area effect is often caused by a few areas with larger than usual empirical residual errors $\left(\text{i.e.}\bar{y}_i - \bar{\mathbf{x}}_i^T \hat{\boldsymbol{\beta}}\right)$. The idea of the second area-level model is therefore to increase the shrinkage of $\bar{y}_i$ towards the synthetic mean $\bar{\mathbf{x}}_i^T \hat{\boldsymbol{\beta}}$ based on the strength of area-specific evidence of a non-null area effect. For areas with strong evidence of an area effect, the additional shrinkage will be zero or near zero. The increased shrinkage is achieved by multiplying γ_i by a (posterior) probability $0 < p_i \leq 1$ that the area effect in the ith area is non-zero. A hierarchical Bayes method is applied in what they called small-area estimation with uncertain random effects (Datta and Mandal 2015). We shall use the abbreviation DM for this area-level model.

Full details of CH and DM are given in the above-cited references; only the most important features are detailed here. Results with CH and DM are profiled against results from the baseline area-level model WF by Wang and Fuller (2003). For AOIs with less than 2 plots, estimates are derived using synthetic estimators (SYN).

The CH model

For the ith AOI the spatially non-stationary EBLUP model by Chandra et al. (2012) combines the models in Eqs. 1 and 2 with a spatially varying term to give

$$\bar{y}_i = \bar{\mathbf{x}}_i^T \boldsymbol{\beta} + \boldsymbol{\psi}_i^T \boldsymbol{\kappa}_i + v_i + e_i, \quad i = 1, \ldots, m \tag{5}$$

where $\boldsymbol{\psi}_i$ is a length q row vector of the subset $(q \leq p)$ of the auxiliary variables in $\mathbf{x}_i$ with an assumed spatially varying regression coefficient, and $\boldsymbol{\kappa}_i$ is a length q row vector of random spatial 'location' effects in the corresponding regression coefficients, the remaining terms are as defined in Eqs. 1 and 2. A unit-level variant of the model in (4) was proposed by (Chandra et al. 2012). The spatial effects $\boldsymbol{\kappa}_i$ are assumed independent of the random area effect (v_i) and error term (e_i). The expected value of $\boldsymbol{\kappa}_i$ over all areas is $\mathbf{0}$ which means that the expected value of $\bar{y}_i$ across the m areas is $\mathbf{x}^T \boldsymbol{\beta}$ where $\mathbf{x}$ is the area-weighted mean of $\bar{\mathbf{x}}$. According to Eq. 5, the covariance among the direct estimates of $\bar{y}_i$ given $\bar{\mathbf{x}}_i^T$ and the location (loc = centroid) of an area becomes

$$\hat{cov}\left(\bar{y}_i, \bar{y}_j | \mathbf{x}_i, \mathbf{x}_j, \text{loc}_i, \text{loc}_j\right) = \boldsymbol{\psi}_{i||j}^T \boldsymbol{\Sigma}_{ij} \boldsymbol{\psi}_{i||j} + \delta_{i=j}\left(\sigma^2_v + \sigma^2_{ei}\right), \quad i, j = 1, \ldots, m \tag{6}$$

where $\boldsymbol{\psi}_{i||j}$ is the length $2q$ row vector obtained after concatenating $\boldsymbol{\psi}_i$ and $\boldsymbol{\psi}_j$, and $\boldsymbol{\Sigma}_{ij}$ is a $2q \times 2q$ matrix of distance-weighted 'intensities' of the area-specific correlation among the random effects in the q regression coefficients, and $\delta_{i=j}$ is a binary indicator variable taking the value of 1 when $i = j$ and 0 elsewhere. With, for example, $q = 2$, the composition of $\boldsymbol{\Sigma}_{ij}$ follows the separable covariance structure (Cressie and Wikle 2011)

$$\boldsymbol{\Sigma}_{ij} = \begin{pmatrix} \lambda_1 & 0 & w_{ij}\lambda_1 & 0 \\ 0 & \lambda_2 & 0 & w_{ij}\lambda_2 \\ w_{ji}\lambda_1 & 0 & \lambda_1 & 0 \\ 0 & w_{ji}\lambda_2 & 0 & \lambda_2 \end{pmatrix} \tag{7}$$

in that $\boldsymbol{\Sigma}_{ij}$ can be written as $\boldsymbol{\Sigma}_{ij} = \mathbf{W}_{ij} \otimes \boldsymbol{\Gamma}_{\boldsymbol{\kappa}}$ where $\mathbf{W}_{ij}$ is a 2×2 matrix of spatial contiguities between areas i and j, $\boldsymbol{\Gamma}_\lambda$ is a $q \times q$ diagonal matrix of random spatial autocorrelation parameters with diagonal $\boldsymbol{\lambda}_q = (\lambda_1, \lambda_2)$. The elements of $\mathbf{W}_{ij}$ are, as in Chandra et al. (2015), $w_{ij} = \left(1 + \text{distance}(\text{centroid}_i, \text{centroid}_j)\right)^{-1}, \; i, j = 1, \ldots, m$. Here, we use Euclidean distances among standardized (mean zero and variance of one) centroids. The covariance expression in Eq. 6 is easily converted to a correlation coefficient. When combined with the known among-area distances, the analyst obtains a distance-dependent correlation function.

Accordingly, under the CH model, the EBLUP of the true area mean (θ_i) is

$$\tilde{\theta}_i^{CH} = \mathbf{x}_i^T\tilde{\boldsymbol{\beta}} + \boldsymbol{\psi}_i^T\tilde{\kappa}_i + \tilde{v}_i \tag{8}$$

where $\tilde{\boldsymbol{\beta}} = \left(\mathbf{X}^T\hat{\mathbf{V}}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^T\hat{\mathbf{V}}^{-1}\bar{\mathbf{y}}$, $\hat{\mathbf{V}}$ is the estimated covariance matrix of the m area means (cf. Eq. 6) with estimates of σ_v^2 and sampling variances σ_{ei}^2, the m-length vector of area effects is $\tilde{\mathbf{v}} = \tilde{\sigma}_v^2\hat{\mathbf{V}}^{-1}\left(\bar{\mathbf{y}} - \mathbf{X}^T\tilde{\boldsymbol{\beta}}\right)$, and $(\tilde{\kappa}_1||\ldots\tilde{\kappa}_i||\ldots\tilde{\kappa}_m)^T_{1\times mq} = \hat{\Sigma}_{mq\times mq}\boldsymbol{\Psi}^t_{m\times mq}\hat{\mathbf{V}}^{-1}_{m\times m}\ \left(\bar{\mathbf{y}} - \bar{\mathbf{x}}_i^T\tilde{\boldsymbol{\beta}}\right)_{m\times 1}$ with $\|$ indicating a concatenation of column vectors (matrices).

A distinct attraction of the CH model is the possibility to estimate the random effects in the regression coefficients for areas with a sample size of one or zero. This affords a local 'calibration' of an otherwise default synthetic estimator (Rao and Molina 2015) for these areas. Let $\bar{\mathbf{x}}_o$ denote the vector of auxiliary census values for an area o with a sample size less than 2. With an obvious extension of notation, the CH prediction of θ_o becomes $\tilde{\theta}_o^{CH} = \bar{\mathbf{x}}_o^T\tilde{\boldsymbol{\beta}} + \boldsymbol{\psi}_o^T\tilde{\kappa}_o$ (the estimator is on the top of page 117 in Chandra et al. (2015)).

All model parameters $\left(\boldsymbol{\beta}, \sigma_v^2, \sigma_e^2, \boldsymbol{\lambda}_q\right)$ were estimated via a restricted maximum likelihood with assumed Gaussian distributions of the random effects. Details are given in Eqs. (21)–(24) in Chandra et al. (2015) which also provides a reference to a computer code in R.

To compute MSEs of $\tilde{\theta}_i^{CH}$, we opted for the parametric bootstrap suggested by González-Manteiga et al. (2007). We judged assumptions underpinning analytical approximations (Opsomer et al. 2008) rather restrictive; in particular the requirement of a cap on the largest eigenvalue of $\mathbf{V}$. In simulations, the parametric bootstrap was found to perform well (Chandra et al. 2015). The parametric bootstrap is, of course, time consuming but the provision of quantile-based confidence intervals, and an estimate of the distribution of the likelihood ratio test under the null hypothesis $\boldsymbol{\lambda} = \mathbf{0}$ speaks to its advantage. In the bth parametric bootstrap $(b = 1, \ldots, B)$, independent random realizations are drawn from the assumed Gaussian distributions of the random location effects in the vector of regression coefficients, the area effects $\tilde{\mathbf{v}}$, and the residual errors $(\mathbf{e}^*)$. Conditional on these draws, a new vector of m pseudo small-area direct estimates of the mean of Y was generated $(\bar{\mathbf{y}}^*)$ and in turn used to re-estimate the random effects with the restricted ML procedure, and a bootstrap replicate CH estimate of θ_i. Then, a log-likelihood ℓ_1^b of the data given the estimated parameters was calculated. A second log-likelihood $\left(\ell_2^b\right)$ under the null hypothesis $\boldsymbol{\kappa} = \mathbf{0}$ was also computed along with $2\left(\ell_1^b - \ell_2^b\right)$ We repeated this process $B = 600$ times to obtain an approximation to the distribution of the log-likelihood ratio for testing $\boldsymbol{\kappa} = \mathbf{0}$ and determined the percentile in this distribution of the log-likelihood ratio computed from the data to gauge the significance of $\boldsymbol{\kappa}$.

The DM model

A key feature of the DM model is the generation of a posterior distribution of the probability of a non-zero area effect in the ith AOI. In accordance with the notion of a probability of a non-zero area effect, the estimated area means of Y_i is set to follow a two-component mixture of normal distributions [see (3) in Datta and Mandal 2015] where, with probability p_i, there is a random area effect (v_i) in the ith AOI and with probability $1 - p_i$ there is no such effect. Accordingly

$$\left(\bar{y}_i|\beta, \bar{\mathbf{x}}_i, \sigma_v^2, p_i\right) = \frac{1}{2\sqrt{\pi}}\left[\frac{(1-p_i)}{\sqrt{\sigma_{ei}^2}}\exp\left(-\frac{\left(\bar{y}_i - \bar{\mathbf{x}}_i^T\boldsymbol{\beta}\right)^2}{2\sigma_{ei}^2}\right) + \frac{p_i}{\sqrt{\sigma_v^2 + \sigma_{ei}^2}}\exp\left(-\frac{\left(\bar{y}_i - \bar{\mathbf{x}}_i^T\boldsymbol{\beta}\right)^2}{2\left(\sigma_v^2 + \sigma_{ei}^2\right)}\right)\right] \tag{9}$$

With the priors detailed below on σ_v^2, p_i, and $\boldsymbol{\beta}$, a Monte-Carlo estimate of the joint posterior predictive distribution of v_i, p_i, $i = 1, \ldots, m$, and $\boldsymbol{\beta}$ was generated via Gibbs sampling (Robert and Casella 1999, p. 245) from fully specified posterior conditional distributions. The last of 600 iterations of the Gibbs sampler was accepted as a draw from the posterior predictive distribution. A total of 2000 draws entered the final estimate of the joint posterior multivariate distribution.

Let $\boldsymbol{\delta}$ denote a length m row vector of binary indicator variables taking the value of 1 in the positions for the AOIs with a non-zero area effect, and 0 in positions for the AOIs with a zero area effect. The probability that $\delta_i = 1$ is p_i, $i = 1, \ldots, m$. With an inverse gamma distribution with parameters a (rate) and b (shape) as a prior on σ_v^2, a beta distribution with parameters c and d on the prior probability p of a non-zero area effect, and a uniform prior on $\boldsymbol{\beta}$, the posterior distribution (π) is proportional to

$$\begin{aligned}\pi\left(\mathbf{v}, \boldsymbol{\delta}, p, \sigma_v^2, \boldsymbol{\beta}|\bar{\mathbf{x}}, \bar{\mathbf{y}}\right) &\propto \exp\left[-\frac{1}{2}\sum_{i=1}^{m}\frac{\left(\bar{y}_i - \bar{\mathbf{x}}_i^T\boldsymbol{\beta} - \delta_i v_i\right)^2}{\sigma_{ei}^2}\right] \\ &\times \prod_{i=1}^{m}\left\{\left(\sigma_v^2\right)^{-0.5}\exp\left(\frac{v_i^2}{2\sigma_v^2}\right)\right\}^{\delta_i}\left\{I(v_i = 0)\right\}^{1-\delta_i} \\ &\times p^{\sum_{i=1}^{m}\delta_i}(1-p)^{m-\sum_{i=1}^{m}\delta_i}\left(\sigma_v^2\right)^{-(b+1)} \\ &\times \exp\left\{-\frac{a}{\sigma_v^2}\right\}p^{c-1}(1-p)^{d-1}\end{aligned} \tag{10}$$

As in Datta and Mandal (2015), the parameters a, and b were determined so that: (1) the mean of the inverse gamma distribution is equal to the (area-weighted) mean of the sampling variances σ^2_{ei}; and (2) ab^2 equals the square of this mean. For the beta prior, we also have $c = 1$ and $d = 4$ to reflect a prior expectation that a non-zero area effect is only present in $1/(c + d)$ or 20% of the AOIs. The five conditional distributions used for the Gibbs sampling are detailed on page 1738 in Datta and Mandal (2015).

In the model by Datta and Mandal (2015), a random draw from the conditional joint multivariate predictive posterior distribution (conditional on the observed data) of $\left(\tilde{\boldsymbol{\beta}}, \tilde{\sigma}^2_v, \text{ and } \tilde{\mathbf{p}}\right)$—where $\tilde{\mathbf{p}}$ is the length m row vector of area-specific probabilities of a random area effect in the residual errors $\left(\bar{\mathbf{y}} - \bar{\mathbf{x}}^T\tilde{\boldsymbol{\beta}}\right)$ and used as an 'extra' shrinkage factor in addition to $\hat{\gamma}_i$—provides a posterior predictive estimate of θ_i

$$\tilde{\theta}^*_i = \bar{\mathbf{x}}^T_i\tilde{\boldsymbol{\beta}}^* + \tilde{p}^*_i\hat{\gamma}^*_i\left(\bar{y}_i - \bar{\mathbf{x}}^T_i\tilde{\boldsymbol{\beta}}^*\right), \quad i = 1, \ldots, m \tag{11}$$

The mean and variance of the 2000 draws of $\tilde{\theta}^*_i$ are used as the DM estimators of the (posterior) area means of Y and their variances. Clearly, this variance is conditional on the observed area-level sample means. To recognize that the directly estimated area means $(\bar{y}_i)$ are but sample estimate, we generated—in all iterations of the Gibbs sampler—m independent random realizations of these means from a normal distribution with a mean and variance consistent with the observed data.

Synthetic estimators

In areas with no or just a single observation of Y, we recommend a synthetic estimator of an area mean (Rao and Molina 2015, Chap. 6.2.2) even though DM and WF could still be used. Specifically, the synthetic mean estimator for the oth area in this category is

$$\tilde{\theta}^{SYN}_o = \begin{cases} \bar{\mathbf{x}}^T_o\tilde{\boldsymbol{\beta}} & \text{if } \hat{\lambda} = \mathbf{0} \\ \bar{\mathbf{x}}^T_o\tilde{\boldsymbol{\beta}} + \boldsymbol{\psi}^T_o\tilde{\kappa}_o & \text{if } \hat{\lambda} \neq \mathbf{0} \end{cases} \tag{12}$$

The expected variance of $\tilde{\theta}^{SYN}_o$ must recognize the sampling variance within in the m areas with $n_{\mathrm{i}} \geq 2$, the uncertainty in $\tilde{\sigma}^2_v$, and in $\tilde{\kappa}_o$. Under the assumption that directly estimated means $\hat{\bar{y}}_i$ follow a Gaussian distribution with a mean and variance equal to their direct estimates, an approximation to the variance of $\tilde{\theta}^{SYN}_i$ can be obtained by a bootstrap procedure as $E\left[\hat{V}\left(\tilde{\theta}^{SYN}_o\right)\right] + V\left(E\left[\tilde{\theta}^{SYN}_o\right]\right)$ where the first term is the average of the bootstrap estimates of the error variance (or mean-squared error) in $\tilde{\theta}^{SYN}_o$, and $V\left(E\left[\tilde{\theta}^{SYN}_o\right]\right)$ is the variance of the bootstrap estimates of $\tilde{\theta}^{SYN}_o$ (Wolter 2007, p. 197). The choice of $\hat{V}\left(\tilde{\theta}^{SYN}_o\right)$ depends on the random effects included in the model for $\tilde{\theta}^{SYN}_i$. If area and spatial location effects are both assumed absent, a simple model-based estimator of variance apply, i.e. $\hat{V}\left(\tilde{\theta}^{SYN}_o\right) = \bar{\mathbf{x}}^T_o\hat{\Sigma}_\beta\bar{\mathbf{x}}_o$. When it can be assumed that $\tilde{\sigma}^2_v > 0$ and $\tilde{\lambda}_1 = 0$ we used the estimator provided by Rao and Molina (2015). Lastly, under the assumption $\tilde{\sigma}^2_v > 0$ and $\tilde{\lambda}_1 > 0$, we used the estimator by Chandra et al. (2012). Our bootstrap estimates of variance are based on 600 replications. Rao and Molina (2015) also recommend a bootstrap procedure to approximate the mean-squared error of a synthetic mean (Rao and Molina 2015, p. 139).

Smoothed sampling variances

To gauge the sensitivity of the presented estimators to *the direct estimates of sampling variance*, we also obtained results with smoothed estimates of variance (Goerndt et al. 2011, 2013). Specifically, $\sigma^{2\,\text{smooth}}_{ei} = \left(\sum_{i=1}^m \mathrm{A}_i\sigma^2_{ei}\right) \times \left(\sum_{i=1}^m \mathrm{A}_i\right)^{-1}$ where A_i is the forest area in the ith AOI. A smoothed variance is more strongly correlated with the inverse of n_i than σ^2_{ei}. Results with smoothed variances are not shown as they did not reveal any consistent difference to the results obtained with the observed variance.

T test of area effects

Datta et al. (2011) proposed a test to gauge the strength of random area effects. The test statistics (T) is the sum of the ratios of the squared residual errors to the expected weighted variance of the synthetic model prediction $\bar{\mathbf{x}}^T_i\tilde{\boldsymbol{\beta}}$. With an assumed simple random sampling within the AOIs, the area weights (w_i) are equal to the inverse of the relative sample sizes, i.e.$w_i = n_i^{-1}\left(\sum_{i=1}^m n_i^{-1}\right)^{-1}$, and the test statistic becomes

$$T_m = \sum_{i=1}^m \frac{\left(\bar{y}_i - \bar{\mathbf{x}}^T_i\tilde{\boldsymbol{\beta}}\right)^2}{w_i\bar{\mathbf{x}}^T_i\hat{\Sigma}_\beta\bar{\mathbf{x}}_i} \tag{13}$$

where $\hat{\Sigma}_\beta$ is the estimate of the covariance matrix of the regression coefficients in the entertained linear model. The probability, of obtaining T_m under the null hypothesis of no random area effects, was estimated via a with-replacement bootstrap resampling of T_m under the null hypothesis. That is, for each area, n_i random unit-level realizations y^*_{ij} were generated under the null hypothesis $(v_i = 0)$ and conditional on the observed area mean and variances, averaged to $\bar{y}^*_i$, and then used with $\bar{\mathbf{x}}_i$ (fixed) to refit a weighted least squares linear model to obtain a bootstrap estimate of $\tilde{\boldsymbol{\beta}}^*$ and a bootstrap T^*_m. This process was repeated 1000 times and the proportion of T_m^* greater than T_m was taken as the estimate of the probability of no random area effects in the ensemble of m AOIs. We also estimated the probability that the ith area effect is zero by repeating the above test at the area level (i.e. no summation over areas).

Case studies

Result with CH and DM is presented for four case studies and compared to results with the WF estimators. For AOIs with less than two plots SYN estimators based on CH, DM and WF are used. For each case, we recommend an estimator. Our recommendation is based on estimates of MSE of an area mean θ_i. Specifically, we focus on the average reduction in the uncertainty of an area mean whereby a direct estimate of uncertainty serves as a benchmark. An implicit assumption behind our criteria is that the fidelity of CH, DM, and WF estimators of MSE is not materially different. We do acknowledge, however, that there is no objective best way for choosing among CH, DM, and WF estimators (Pfeffermann 2002, 2013).

The AOIs in the four cases represent, in order of size of an AOI: forest management units; forest districts; and municipalities. Forest management units were chosen to represent small scale surveys, while forest districts and municipalities represent larger-scale surveys.

Forest inventories furnished area-level summary statistics of Y, while area-level census summaries of two auxiliary variables (**X**) were derived from data captured by remote sensing (airborne laser scanning and aerial photography). The study variable Y is the area-level mean stem volume (VOL) in units of $\mathrm{m}^3\ \mathrm{ha}^{-1}$. The two auxiliary variables represent the mean canopy height (xCH, unit m) and the mean of squared canopy heights ($xCHQ$, unit m^2). The rationale for including $xCHQ$ is that it provides information about the variance in xCH which is an important predictor of canopy roughness (Magnussen and Boudewyn 1998). A linear model as in Eq. 1 is assumed in all cases. Note, due to collinearity between xCH and $xCHQ$ and the fact that the CH model includes a random area effect (intercept), we restricted, for reasons of computational stability and identifiability, the CH model to a single ($q = 1$) random spatial effect in the regression coefficient to xCH.

A brief summary of the data used in this study is given next in order of the average size of an AOI.

Burgos

The AOIs are $m = 54$ forest management units within a 1365.57 ha *Pinus pinaster* (Ait.) forest located in the Valle de las Caderechas (Burgos) in central Spain. The forest covers a steep valley surrounded by high plateaus and elevation ranges from 600 to 1100 m above sea level. The mean area is 25.3 ha (range 6.3–74.2 ha, SD 13.1 ha). Direct estimates $\bar{y}_i$ (VOL $\mathrm{m}^3\ \mathrm{ha}^{-1}$) and their variances ($\hat{\sigma}_{ei}^2 \equiv \sigma_{ei}^2$) were supplied by 200 forest inventory field plots located on a regular grid. Two units have a field sample size of 0 and four a sample size of 1. Hence, for the analyses $m = 54 - 2 - 4 = 48$ units with an average sample size of $n_i = 4$ per AOI (range 2–11). The auxiliary census means of canopy height (xCH) and squared canopy heights ($xCHQ$) were obtained from an airborne laser scanner survey (LiDAR). Additional details are in Mauro et al. (2016).

Rastatt

A collection of $m = 28$ forest compartments (management units) with a total area of 1033 ha, a mean of 36.9 ha (range 5.9–53.9 ha, SD 10.8 ha) located in the Rastatt state forest in the northern parts of the Black Forest in Germany constitutes the AOIs. The forest is dominated by spruce. A forest inventory conducted between January 2014 and March 2015 with 496 circular field plots located on a 100 m × 200 m grid furnished the direct area estimates $\bar{y}_i$ (VOL $\mathrm{m}^3\ \mathrm{ha}^{-1}$) and their variances $\hat{\sigma}_{ei}^2$. Of the 28 AOIs, 26 had at least 6 field plots and two had just one. For units with at least 6 field plots, the mean was 17 and the maximum was 26. The auxiliary census mean canopy height (xCH m) and the mean of squared canopy heights ($xCHQ$ m^2) were derived from aerial stereo photographs acquired between July 17 and 19. Further details of the data and the inventory procedures are in Magnussen et al. (2016) and Kublin et al. (2013).

Jura

The AOIs are $m = 29$ forest districts (FD) in the Jura region with a total forest area of 205,200 ha, a mean of 70,800 ha (range 660–25,070 ha, SD 4770 ha). The SW–NE elongated shape of this production region, with known precipitation and elevational gradients, suggested a predisposition for a spatial covariance structure. A total of 940 field plots from the Swiss National Forest inventory and located on a regular $\sqrt{2} \times \sqrt{2}$ km national grid (Brändli 2010) provided area estimates of $\bar{y}_i$ (VOL $\mathrm{m}^3\ \mathrm{ha}^{-1}$) and their variances ($\hat{\sigma}_{ei}^2$). The number of plots in an AOI varied from 2 to 122 with a mean of 32. Census auxiliary means of xCH and $xCHQ$ came from on an airborne laser scanning survey at a resolution of 2 × 2 m conducted by the Swiss Federal Office of Topography. Additional details on the design and data are in Kaufmann (1999).

Vestfold

Municipalities ($m = 14$) in the county of Vestfold in the south-eastern parts of Norway serve as AOIs. The total county area is 218,400 ha of which 123,000 ha is productive forest dominated by spruce and pine. The mean (total) area of a municipality is 14,970 ha (range 3110–5,2730 ha, SD 12,800 ha). Direct AOI estimates of $\bar{y}_i$ (VOL m^3 ha^{-1}) and their variances ($\hat{\sigma}^2_{ei}$) were derived from 154 forested circular 250 m^2 permanent sample plots of the Norwegian National Forest Inventory located on a 3 × 3 km grid. Municipal sample sizes (n_i) varied from 1 in three AOIs to 35 with a mean of 10. AOI-level auxiliary census means of *xCH* and *xCHQ* in forested areas were obtained from digital aerial images. Additional details can be obtained from Breidenbach and Astrup (2012).

Results

Burgos

Direct estimates of mean stem volume per ha varied from 82 to 302 m^3 ha^{-1} with a mean of 162 m^3 ha^{-1} and a standard deviation of 59 m^3 ha^{-1}. With an average of four sample plots per unit, the accuracy of a direct estimate of stem volume per ha was, expectedly, not great. To wit, the average relative error was estimated at 22% (range 6–61%) but with a trend to increase by approximately 9 percentage points for an increase in the mean volume by 100 m^3 ha^{-1}. The direct regression of $\bar{y}_i$ on canopy height and its square explained approximately 60% of the variation in $\bar{y}_i$.

A summary of the estimated MSEs with CH, DM, and WF are in Table 1. For this site, the CH estimator would be our recommendation as it achieves the best average reduction of $(1 - 0.29) \times 100\% = 71\%$ in the uncertainty of an area mean. The DM estimator is a close runner-up.

The random area variance was manifest regardless of estimator and estimated at 624 m^6 ha^{-2} with CH. The spatial effect parameter (λ_1) was estimated at 6.2 ($P = 0.08 | H_0 : \lambda_1 = 0$). The spatial location effect in *xCH* increased the explained variance in area means ($\bar{y}_i$) by 7 or 30% when combined with $\tilde{v}_i$. Spearman's rank correlation between $\bar{y}_i$ and $\tilde{\theta}_i^{CH}$ was strong (0.94). The bootstrap *T* test suggested that most of the area variance originated from stands 5, 7, 33, 40, and 50.

Figure 1 provides a summary of pertinent results. In Fig. 1a, the direct and synthetic area means are plotted against the mean canopy height. In Fig. 1b, the CH and SYN area estimates are plotted against the direct estimates and we see that CH estimates are, on average, closer to the indicated one-to-one line of a perfect agreement. Finally,

Table 1 Summary of direct (DI) area-level estimates of variance (V), and CH, DM, and WF estimators of mean-squared errors (MSE) in Burgos

	DI	CH	DM	WF
Min.	77	32	41	83
Max.	22,400	726	4086	1030
Mean	1780	288	537	645
Mean of ratios MSE_{xx}/V_{DI}, XX = {CH, DM, WF}				
Min.	n.a.	0.03	0.05	0.05
Max.	n.a.	0.49	0.53	1.25
Mean	n.a.	0.29	0.32	0.71

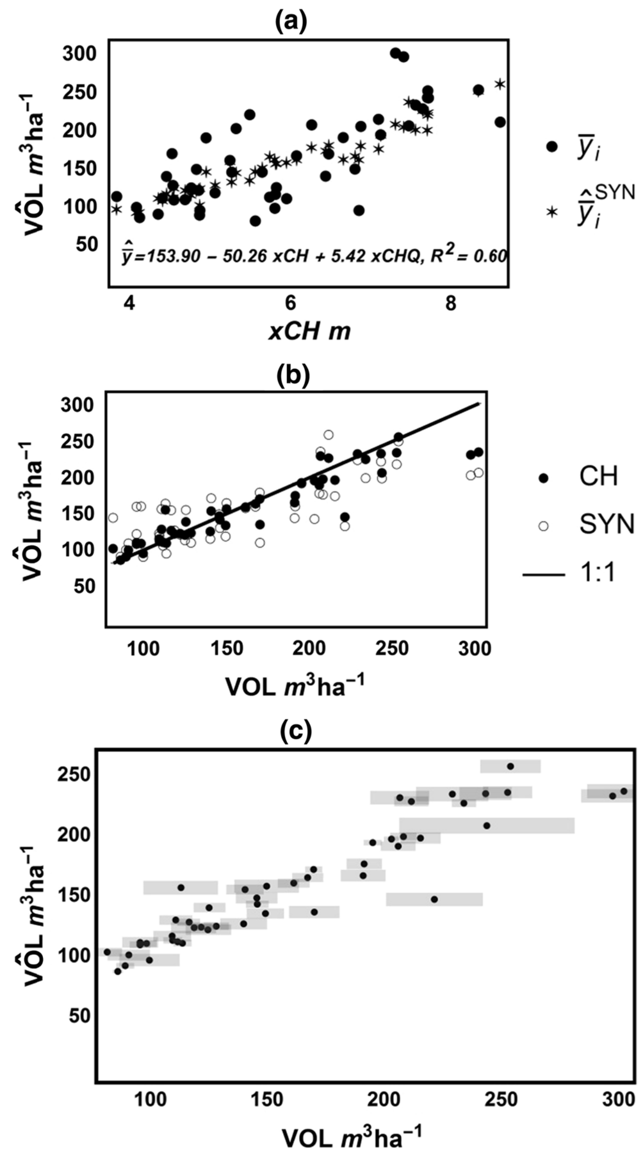

Fig. 1 Summary of results from Burgos. **a** Direct ($\bar{y}_i$) and synthetic ($\hat{\bar{y}}_i^{SYN}$) estimates of area means of stem volume (m^3 ha^{-1}) plotted against the area census mean of canopy height (*xCH*). **b** CH- and SYN estimates of area means of stem volume (m^3 ha^{-1}) plotted against direct estimates ($\bar{y}_i$). **c** Root-mean-squared errors (scaled) of CH estimates (vertical axis) and direct estimates (horizontal axis). The shaded box has width and height equal to 2 standard deviations and is centred on the point estimates

Fig. 1c visualizes a considerable reduction in the uncertainty of an estimate of an area mean achieved by CH *vis-à-vis* the uncertainty in a direct estimate.

Maps of scaled (0–1) mean canopy heights x_iCH, direct estimates of volume $\bar{y}_i$, and the spatial location effect $\tilde{\kappa}_i xCH_i$ (cf. Eq. 7) are in Fig. 2a–c, respectively. A spatial clustering of x_iCH and $\bar{y}_i$ is apparent, but $\tilde{\kappa}_i xCH_i$ exhibits a different pattern than does xCH_i. In Fig. 2d, predicted correlations among area means $\bar{y}_i$ (derived via Eq. 5) plotted against scaled (0–1) distances suggest a weak correlation among neighbouring AOIs. Trends in the spatial correlation among location effects $\tilde{\kappa}_i xCH_i$ plotted against distances in Fig. 2e suggest a Gaussian model with a slow decay of correlations as a function of distance.

Rastatt

Direct estimates of a compartment mean stem volume per ha varied from 84 to 565 m^3 ha^{-1} with a mean of 334 m^3 ha^{-1} and a standard deviation of 83 m^3 ha^{-1}. The relative error of a direct estimate varied from 8 to 54% with a mean of 16%. The direct regression of $\bar{y}_i$ on canopy height and its square explained approximately 45% of the variation in $\bar{y}_i$.

A summary of the estimated MSEs with CH, DM, and WF is in Table 2. According to our criterion, DM would hold a small edge over CH, but we nevertheless recommend CH for this as it provides for considerably narrower range of estimated MSEs. With CH the average reduction in an area MSE is $(1 - 0.28) \times 100\% = 72\%$.

Table 2 Summary of direct (DI) area-level estimates of variance, and CH, DM, and WF estimators of mean-squared errors (MSE) in Rastatt

	DI	CH	DM	WF
Min.	433	179	220	466
Max.	39,300	2110	3630	6970
Mean	3750	624	768	1550
Mean of ratios MSE_{xx}/V_{DI}, XX = {CH, DM, WF}				
Min.	n.a.	0.05	0.08	0.18
Max.	n.a.	0.41	0.83	1.10
Mean	n.a.	0.28	0.27	0.66

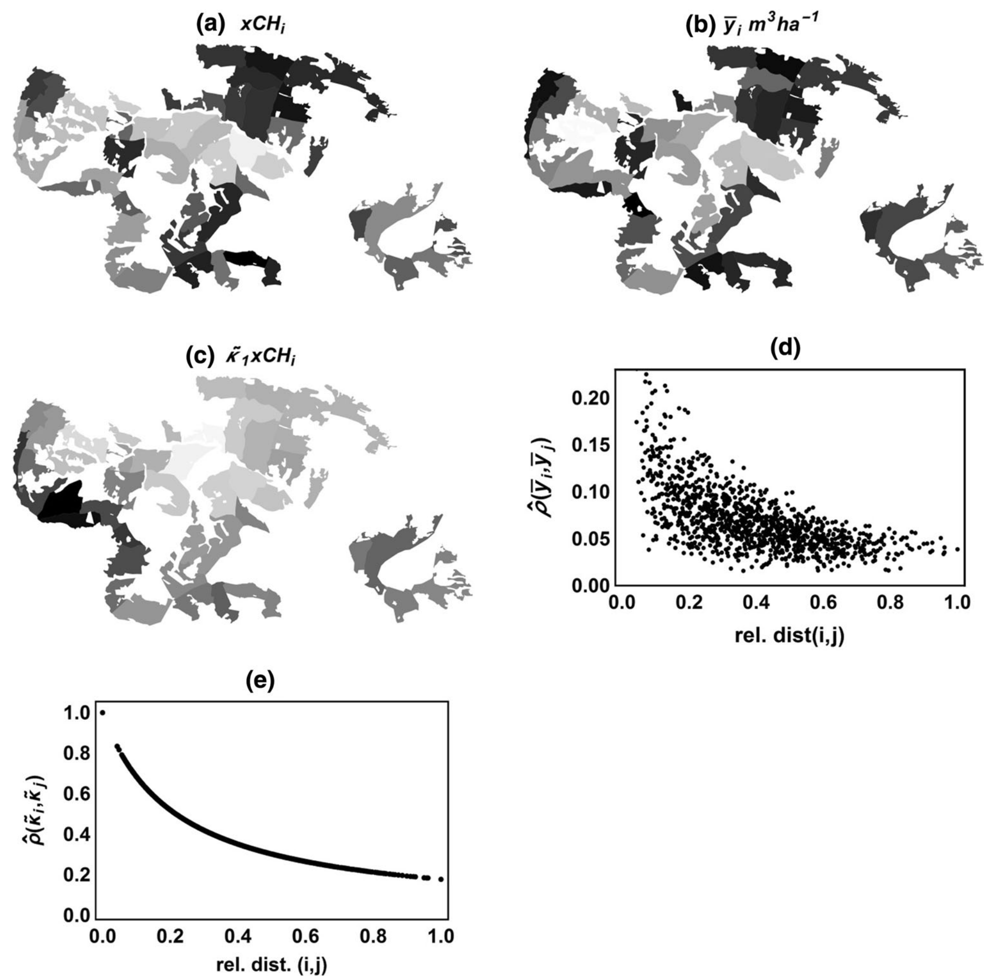

Fig. 2 Maps of Burgos management units: **a** area census means of canopy height (xCH_i); **b** direct estimates of stem volume $(\hat{\bar{y}}_i)$; and **c** CH estimates of the spatial location effect in xCH. A darker grey-tone is associated with a greater numerical value. A scatter plot of CH predictions of the correlation among direct estimates versus the relative (0–1) distance between area centroids is in subplot (**d**). Correlations of random location effects in xCH plotted against relative (binned) distances are in subplot (**e**)

The random area variance was manifest regardless of estimator and estimated at 160.0 m^6 ha^{-2} with CH. The spatial effect parameter (λ_1) was estimated at 272.0 $(P = 0.29|H_0 : \lambda_1 = 0)$. The spatial location effect in xCH increased the explained variance in area means $(\bar{y}_i)$ by 9%. The sum of location effects and area effects accounted for 77% of the variance in area means. Spearman's rank correlation between $\bar{y}_i$ and $\tilde{\theta}_i^{CH}$ was a moderate 0.62. The bootstrap T test suggested that most of the area variance originated from compartments 4102, 4112, 4303, 4305, and 4308.

Figure 3 provides a summary of pertinent results. In Fig. 3a, the direct and synthetic area means are plotted against the mean canopy height. In Fig. 3b, the CH and SYN area estimates are plotted against the direct estimates and we see that CH estimates are, on average, closer to the indicated one-to-one line of a perfect agreement. Finally, Fig. 3c visualizes a considerable reduction in the uncertainty of an estimate of an area mean achieved by CH vis-à-vis the uncertainty in a direct estimate.

The low significance of the spatial effects does not warrant a display.

Jura

Direct estimates of forest district mean stem volume varied from 256 to 535 m^3 ha^{-1} with a mean of 368 and a standard deviation of 64 m^3 ha^{-1}. The relative error of a direct estimate varied from 7 to 19% with a mean of 11%. The direct regression of $\bar{y}_i$ on canopy height and its square explained only 17% of the variation in $\bar{y}_i$.

A summary of the estimated MSEs with CH, DM, and WF are in Table 3. Once more, the CH estimator would be our recommendation as it achieves the best average reduction of $(1 - 0.32) \times 100\% = 68\%$ in the uncertainty of an area mean. The DM estimator is also a runner-up here, but a more distant one.

The combined random area effects were manifest regardless of estimator and estimated at 1570 m^6 ha^{-2} with CH. The spatial effect parameter (λ_1) was estimated at 5225 $(P = 0.01|H_0 : \lambda_1 = 0)$. The spatial location effect in xCH increased the explained variance in area means $(\bar{y}_i)$ by 57 or 91% when combined with $\tilde{v}_i$. Spearman's rank correlation between $\bar{y}_i$ and $\tilde{\theta}_i^{CH}$ was relatively weak (0.24). The bootstrap T test suggested that most of the area variance originated from districts 208 2404 and 2601.

Figure 4 provides a summary of pertinent results. In Fig. 4a, the direct and synthetic area means are plotted against the mean canopy height. In Fig. 4b, the CH and SYN area estimates are plotted against the direct estimates, and we see that CH estimates are consistently closer to the indicated one-to-one line of a perfect agreement. Finally,

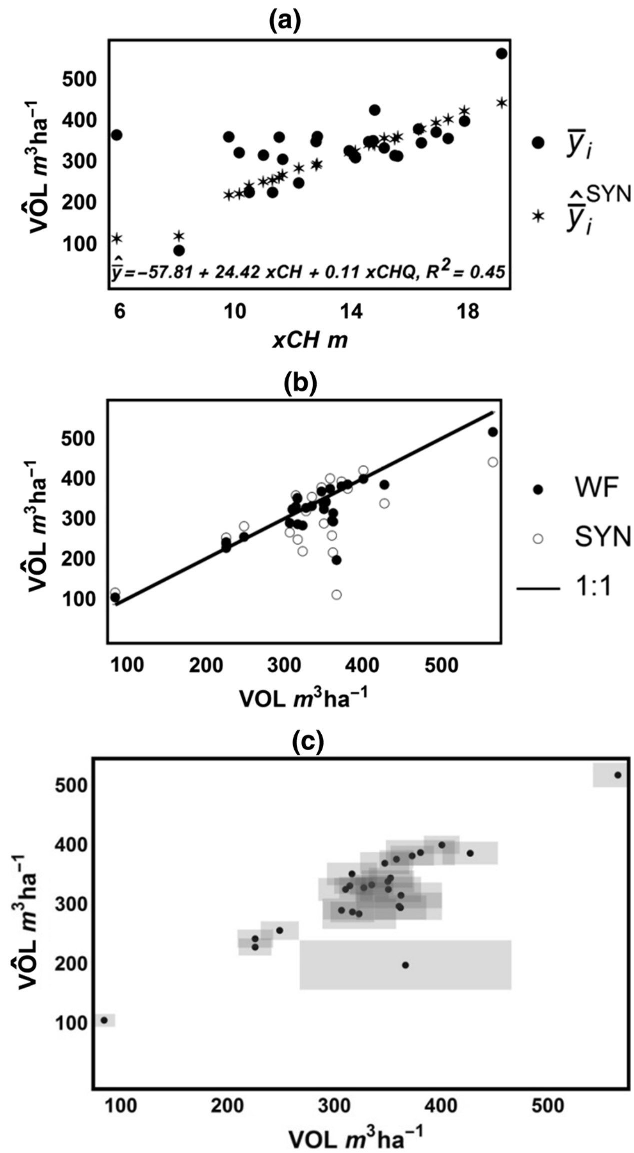

Fig. 3 Summary of results from Rastatt. **a** Direct $(\bar{y}_i)$ and synthetic $(\hat{\bar{y}}_i^{SYN})$ estimates of area means of stem volume (m^3 ha^{-1}) plotted against the area census mean of canopy height (xCH). **b** CH- and SYN estimates of area means of stem volume (m^3 ha^{-1}) plotted against direct estimates $(\bar{y}_i)$. **c** Root-mean-squared errors (scaled) of CH estimates (vertical axis) and direct estimates (horizontal axis). The shaded box has width and height equal to 2 standard deviations and is centred on the point estimates

Fig. 4c visualizes a considerable reduction in the uncertainty of an estimate of an area mean achieved by CH vis-à-vis the uncertainty in a direct estimate.

Maps of scaled (zero to one) mean canopy heights x_iCH, direct estimates of volume $\bar{y}_i$, and the spatial location effect $\tilde{\kappa}_i xCH_i$ (cf. Eq. 7) are in Fig. 5a–c, respectively. A spatial clustering of x_iCH and $\bar{y}_i$ is apparent with $\tilde{\kappa}_i xCH_i$ resembling a smoothed version of $\bar{y}_i$. In Fig. 5d, predicted correlations among area means $\bar{y}_i$ (derived via Eq. 5) plotted against scaled (0–1) distances suggest both weak and

Table 3 Summary of direct (DI) area-level estimates of variance, and CH, DM, and WF estimators of mean-squared errors (MSE) in Jura

	DI	CH	DM	WF
Min.	345	149	264	330
Max.	7440	1390	3400	1070
Mean	1960	546	709	1300
Mean of ratios MSE_{xx}/V_{DI}, XX = {CH, DM, WF}				
Min.	n.a.	0.07	0.10	0.33
Max.	n.a.	0.55	1.35	1.17
Mean	n.a.	0.32	0.51	0.77

relatively strong correlation among neighbouring AOIs. The upper limit of predicted correlations, however, declined as—more or less—expected for a Gaussian process. Trends in the spatial correlation among location effects $\tilde{\kappa}_i xCH_i$ plotted against distances in Fig. 5e suggest a trend very similar to the trend in the results from Rastatt.

Vestfold

Direct estimates of municipality forest mean stem volume varied from 64 to 225 m^3ha^{-1} with an average of 143 m^3ha^{-1} and a standard deviation of 46 m^3 ha^{-1}. The relative error of a direct estimate varied from 14 to 64% with a mean of 26%. The regression of $\bar{y}_i$ on canopy height and its square explained only 13% of the variation in $\bar{y}_i$.

A summary of the estimated MSEs with CH, DM, and WF are in Table 4. Again, CH would be our recommended estimator as it achieves the best average reduction of $(1 - 0.34) \times 100\% = 66\%$ in the uncertainty of an area mean. WF is a close runner-up.

A random area variance was manifest regardless of estimator and estimated at 859 m^6 ha^{-2} with CH. The spatial effect parameter (λ_1) was estimated at 4716 ($P = 0.15 | H_0 : \lambda_1 = 0$). The spatial location effect in xCH increased the explained variance in area means ($\bar{y}_i$) by 74%. Combined the location and area effects explained 76% of the variation in area means. Spearman's rank correlation between $\bar{y}_i$ and $\tilde{\theta}_i^{CH}$ was 0.9. The bootstrap T test suggested that most of the area variance originated from municipalities 706 and 713.

Figure 6 provides a summary of pertinent results. In Fig. 6a, the direct and synthetic area means are plotted against the mean canopy height. In Fig. 6b, the CH and SYN area estimates are plotted against the direct estimates and we see that CH estimates are somewhat closer to the indicated one-to-one line of a perfect agreement. Finally, Fig. 6c visualizes a sizeable reduction in the uncertainty of an estimate of an area mean achieved by CH vis-à-vis the uncertainty in a direct estimate.

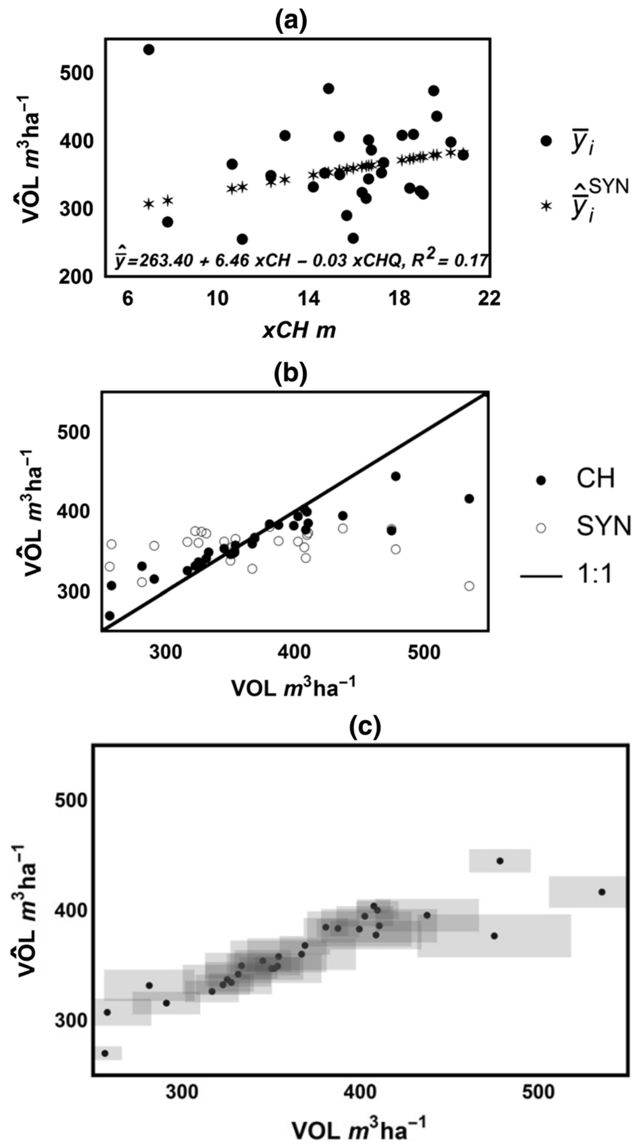

Fig. 4 Summary of results from Jura. **a** Direct ($\bar{y}_i$) and synthetic ($\hat{\bar{y}}_i^{SYN}$) estimates of area means of stem volume (m^3 ha^{-1}) plotted against the area census mean of canopy height (xCH). **b** CH- and SYN estimates of area means of stem volume (m^3 ha^{-1}) plotted against direct estimates ($\bar{y}_i$). **c** Root-mean-squared errors (scaled) of CH estimates (vertical axis) and direct estimates (horizontal axis). The shaded box has width and height equal to 2 standard deviations and is centred on the point estimates

Maps of scaled (zero to one) mean canopy heights x_iCH, direct estimates of volume $\bar{y}_i$, and the spatial location effect $\tilde{\kappa}_i xCH_i$ (cf. Eq. 7) are in Fig. 7a–c, respectively. A spatial clustering of x_iCH and $\bar{y}_i$ is apparent with $\tilde{\kappa}_i xCH_i$ being very similar to $\bar{y}_i$. In Fig. 7d, predicted correlations among area means $\bar{y}_i$ (derived via Eq. 5) plotted against scaled (0–1) distances do not indicate any trend with distance. Trends in the spatial correlation among location effects $\tilde{\kappa}_i xCH_i$ plotted against distances in Fig. 7e resemble those reported for Rastatt and Jura.

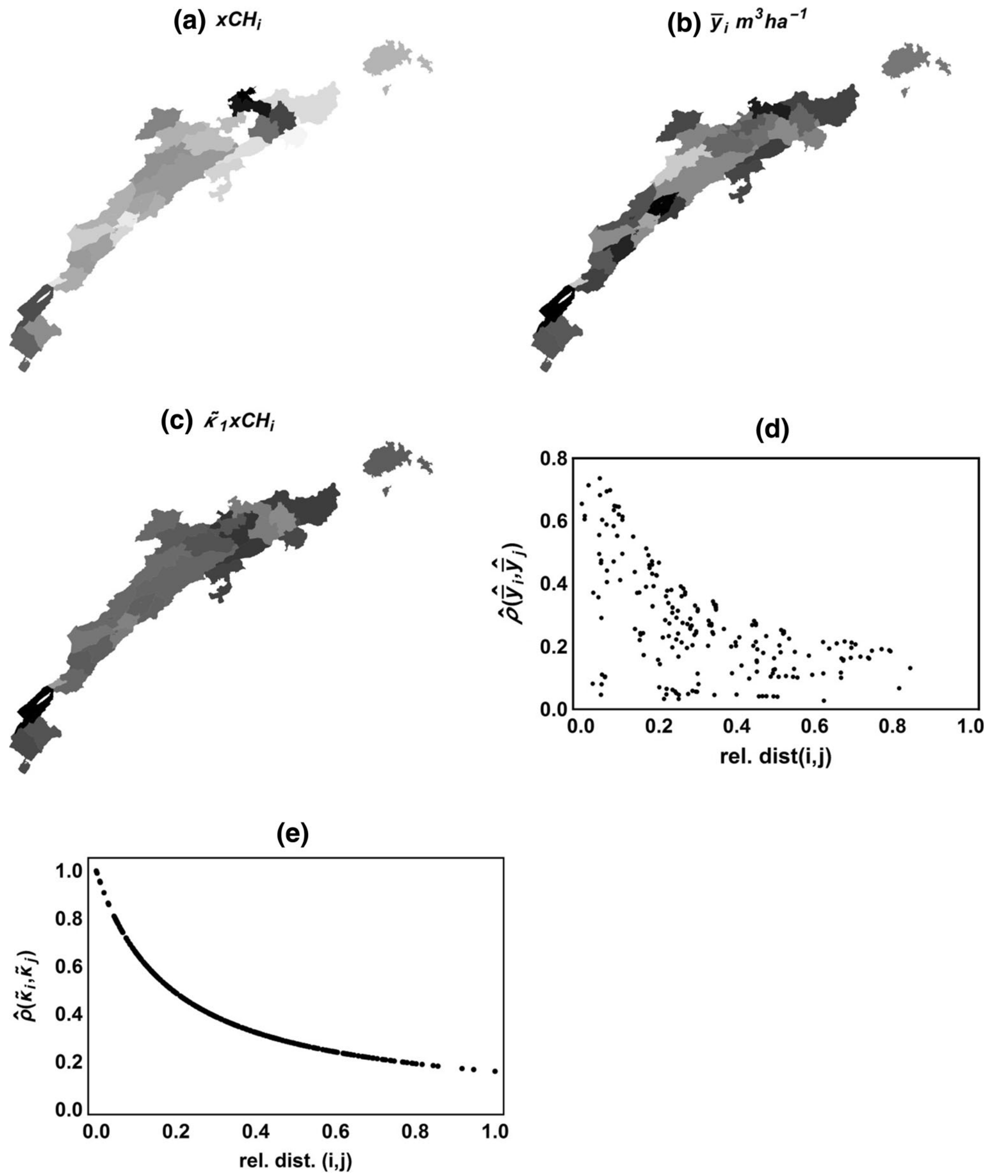

Fig. 5 Maps of Jura forest districts: **a** area census means of canopy height (xCH_i); **b** direct estimates of stem volume ($\bar{y}_i$); and **c** CH estimates of the spatial location effect in *xCH*. A darker grey-tone is associated with a greater numerical value. A scatter plot of CH predictions of the correlation among direct estimates versus the relative (zero to one) distance between area centroids is in subplot (**d**). Correlations of random location effects in *xCH* plotted against relative (binned) distances are in subplot (**e**)

Table 4 Summary of direct (DI) area-level estimates of variance, and CH, DM, and WF estimators of mean-squared errors (MSE) in Vestfold

	DI	CH	DM	WF
Min.	306	153	306	250
Max.	2940	1530	3312	1100
Mean	1410	555	971	558
Mean of ratios MSE_{xx}/V_{DI}, XX = {CH, DM, WF}				
Min.	n.a.	0.20	0.16	0.11
Max.	n.a.	0.49	1.09	0.90
Mean	n.a.	0.34	0.80	0.44

The random spatial location effect of *xCH* affords an opportunity to improve upon the default SYN estimator for AOIs with a sample size of 0 or 1. AOIs 701, 722, and 723 had only one sample point. The WF estimates of $\tilde{\theta}_o$ were 172, 139 and 115 m^3 ha^{-1} compared to direct estimates of 94, 32, and 158, m^3 ha^{-1}, respectively. After inclusion of spatial location effects in *xCH*, the estimates came to 186, 149, and 119 m^3 ha^{-1}. When compared to the direct estimates and with a view to the census means of *xCH* and *xCHQ*, the synthetic estimates appear reasonable. Bootstrap estimates of relative root-mean-squared errors were 36, 24, and 35% under the assumption of no area and no spatial location effects. They rose to 45, 39, and 58% when using the WF (bootstrap) estimate of σ_v^2, and to 52, 43, and 52% with inclusion of CH (bootstrap) estimates of the random area effects and random spatial location effects in *xCH*.

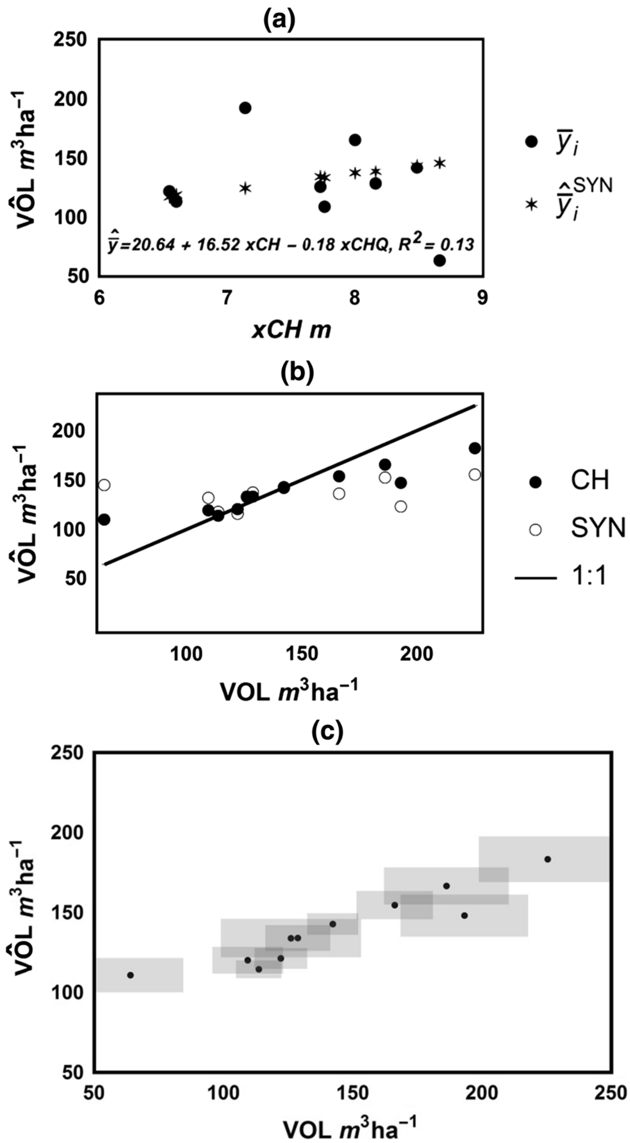

Fig. 6 Summary of results from Vestfold. **a** Direct $(\bar{y}_i)$ and synthetic $\left(\hat{\bar{y}}_i^{SYN}\right)$ estimates of area means of stem volume (m^3 ha^{-1}) plotted against the area census mean of canopy height (*xCH*). **b** CH- and SYN estimates of area means of stem volume (m^3 ha^{-1}) plotted against direct estimates $(\bar{y}_i)$. **c** Root-mean-squared errors (scaled) of CH estimates (vertical axis) and direct estimates (horizontal axis). The shaded box has width and height equal to 2 standard deviations and is centred on the point estimates

Discussion

Area-level modelling and inference has not been widely used in forest inventories, yet our examples demonstrated the potential of auxiliary variables (**X**) to improve the precision of area-level means when there is a strong relationship between the auxiliaries and the study variable (*Y*) or a significant exploitable spatial covariance structure in at least one *X*. Our examples with per ha stem volume suggest that area-level means (totals) with a relative sampling error no larger than 20% is conducive for achieving a high likelihood of consistent (across AOIs) and important improvements in precision over and above the precision of directly estimated area means.

In forestry, area-level modelling and inference opens for a wider use of data from national forest inventories without a need to know the exact plot location. Cruise plot data and stand level inventory data (Haara and Leskinen 2009; Holmgren 2004; Næsset 2002) are also candidates for use in area-level analyses as it is often difficult to ensure a correct spatial alignment (co-registration) between the field observations and the auxiliary information as required for a more powerful unit-level analysis. The use of variable radius plots (Flewelling and Thomas 1984), and plot-layouts with shapes and sizes that do not match the spatial units of one or more of the auxiliary variables (McRoberts and Tomppo 2007), also points to the utility of area-level analyses. Finally, in an area-level analysis, it will also be easier to explore and make use of a wider array of auxiliary variables with different spatial resolutions than in a unit-level analysis.

In forestry, with a traditionally low sampling intensity, direct estimates of an area mean have a variance that result in an attenuation of the estimated area-level model coefficients, and a lower coefficient of determination (Carroll et al. 1995, Chap. 2). In unit-level models, observations are assumed as a ground truth (not estimates), and therefore this attenuation problem does not exist in a unit-level context. We saw a clear demonstration of this effect in form of a reduction in the amount of explained variance in *Y* as the average size of an AOI increased.

Inference from area-level models is clearly model-based (Gregoire 1998) which may be a deterrent to agencies and institutions committed to design-based estimators (Magnussen 2015). While an area-level model linking a direct estimate of *Y* to an auxiliary **X** may appear similar to a unit-level model, sampling errors in *Y* and their dependence on area-specific sample sizes and intrinsic area-specific variances sets it apart from a unit-level model.

In a basic Fay-Herriot model (Rao and Molina 2015, p. 76), the assumption of a known sampling variance in direct estimates is necessary but unrealistic. Considering that (for a Gaussian variable) it may take a sample size of 50 or more to bring the relative error of a direct estimate of the sampling variance below 20% (Box 1953), the assumption would be untenable in most forestry applications. In our analyses we used the direct (design-based) estimates of sampling variance in a direct estimate of the area mean of *Y*. We explored a functional area-level relationship between **X** and the sampling variance of *Y* similar to the relationship between **X** and the residual variance that often exist in unit-level applications (Breidenbach et al. 2015; Mauro et al. 2016), but failed to identify any

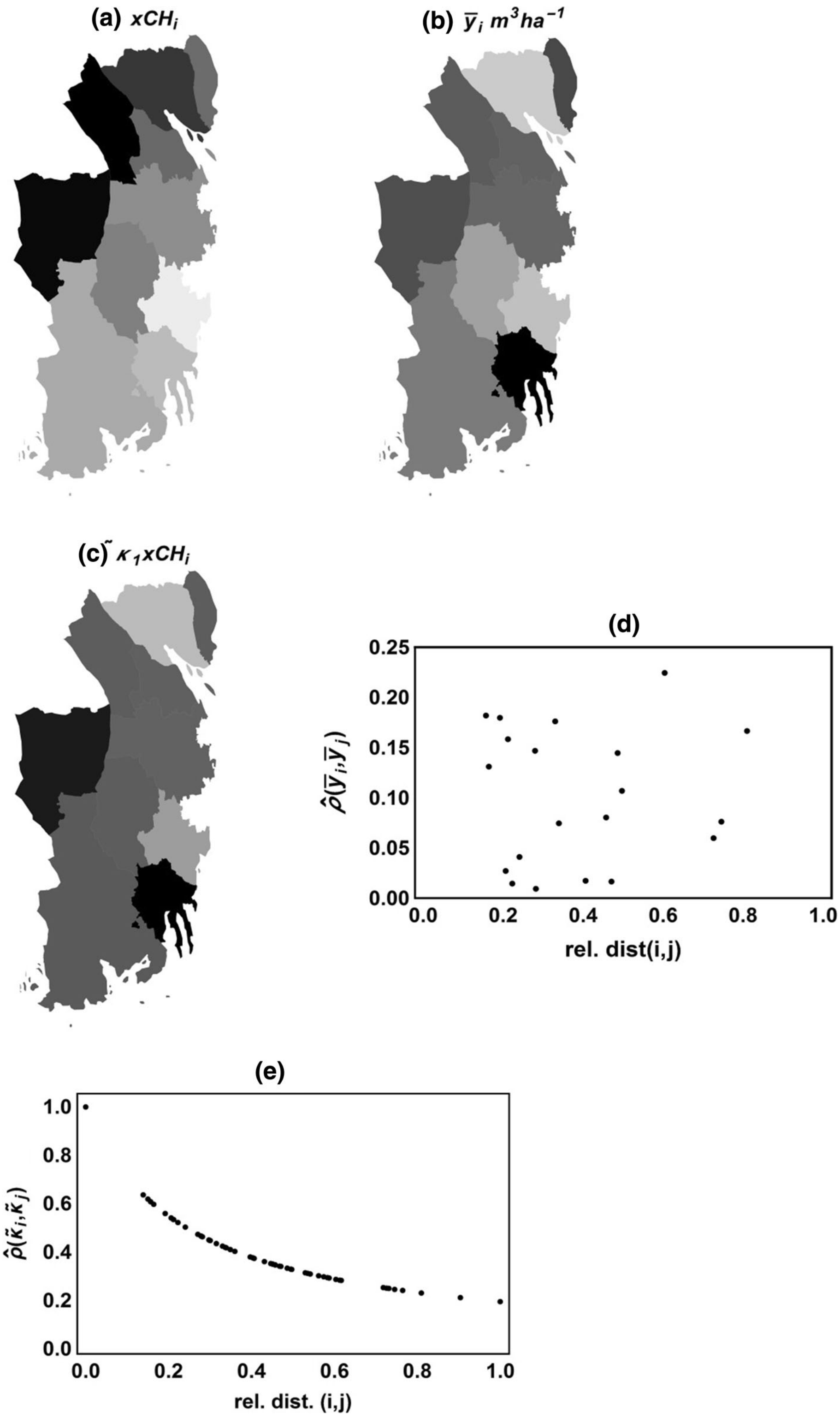

Fig. 7 Maps of Vestfold municipalities: **a** area census means of canopy height (xCH_i); **b** direct estimates of stem volume ($\bar{y}_i$); and **c** CH estimates of the spatial location effect in *xCH*. A darker grey-tone is associated with a greater numerical value. A scatter plot of CH predictions of the correlation among direct estimates versus the relative (zero to one) distance between area centroids is in subplot (**d**). Correlations of random location effects in *xCH* plotted against relative (binned) distances are in subplot (**e**)

meaningful relationship. At the area-level, a smoothing of variances has been recommended (Goerndt et al. 2013; Rao and Molina 2015, p. 76). However, in forestry, one can argue that a difference in direct estimates of variance has more to do with the actual variation within an area than sample size per se. For this reason, we hesitate to recommend a smoothing of variances, but a repeat of area-level analyses with smoothed variances may reveal an undue sensitivity of the results to observed direct estimates of variance.

For AOIs not identified in a sampling design, the realized sample size in an area is a random variable (Särndal et al. 1992, Chap. 10.4) which means that a variance estimated conditional on a realized sample ignores the contribution from a variance in realized sample sizes (Gregoire et al. 2015). An issue ignored in model-based inference. At least estimators like the WF (Wang and Fuller 2003) and the DM (Datta and Mandal 2015) incorporate the uncertainty in a direct (conditional) estimate of variance. As a consequence their estimates of the random area effects are smaller than if the sampling variances were taken as known. For the same reason, one should consider the possibility that a CH-based estimate of the among-area variance is upwardly biased. However, in presence of a random spatial location effect in an auxiliary variable, the CH estimate of the random (residual) area effect is reduced and the issue of a possible bias becomes less important.

There is a real potential in forest inventories to exploit non-trivial spatial processes which influence and shape spatial trends (Finley et al. 2011; Freeman and Moisen 2007; Koistinen et al. 2008; Meng et al. 2009). A previous study with the Swiss NFI data, and a unit-level mixed linear model with spatially correlated random effects (Pratesi and Salvati 2008) already confirmed this potential (Magnussen et al. 2014). To us, the CH model proposed by Chandra et al. (2015) is attractive. It avoids hard-to-justify assumptions about stationary covariance processes, exemplified in CAR and SAR models (Cressie 1993, Chap. 6.3) and several others (Marhuenda et al. 2013; Molina et al. 2009; Petrucci et al. 2005; Pratesi and Salvati 2008). This avoidance demands de jure separable covariance matrices of spatial and non-spatial random effects (Johannesson et al. 2007; Quick et al. 2015) which meets with a different set of non-verifiable assumptions. A practical (computational) issue with CH is that there does not seem to be a way to ascertain the significance of λ without an implementation of a time-consuming parametric bootstrap. A map-based study can help identify candidate explanatory variable for a spatial covariance structure, but cannot be used to ascertain statistical significance. While programming the CH model and the parametric bootstrap was straightforward in the Mathematica® programming language, we noticed that λ and σ_v^2 were strongly and negatively correlated setting the stage for unstable derivatives.

In forestry, area-level analyses invariably include one or more AOIs with no or just one direct observed value of Y as was the case in Burgos and Vestfold. Application of area-level models that exploits random spatial covariance structures, like the CH model, offers an important opportunity to improve upon a default synthetic estimator (Rao and Molina 2015, Chap. 6.2.2).

In our examples, the DM model (Datta et al. 2011) took a backseat to CH. Nevertheless, we consider the DM model as an attractive area-level model as its basic tenet that statistically significant random (residual) area effects may be triggered by a few AOIs was largely confirmed in our examples. In applications without a strong spatial covariance structure in the auxiliary variables and with an emphasis on area-level inference and hypothesis testing, the prospect of an improved precision in estimated area means for areas with little or no support of a random (residual) area effect seems worthwhile to pursue. Our inference was guided more towards averages across the set of AOIs as opposed to estimators for individual AOIs. For this reason, we recommended CH for the data considered here. However, interests and hypotheses directed towards a single or a few AOIs could very well have favoured DM. Although the specification of a Hierarchical Bayesian model and a Gibbs sampler may seem complicated, the actual implementation and execution was straightforward (programming software: Mathematica® (Wolfram 2016)).

The WF variant of the basic Fay-Herriot model was not recommended for any of our case studies. However, it is computationally simple and fast, which, to some, may sway the choice of estimators.

We have based our recommendation on estimates (predictions) of mean-squared errors. In practical applications, this is often the accepted coinage for choosing a model. We do acknowledge that there is no single superior criterion for choosing among CH, DM, and WF (Pfeffermann 2002, 2013). In a forestry context, one should intuitively expect a spatial structure in the distribution of a forest resource of interest within area(s) of interest (Babcock et al. 2015; Massey and Mandallaz 2015; Montes et al. 2005; Salas et al. 2010). A structure formed by growing conditions, tree species composition, and possibly forest management. An area-level model that accommodates a spatial structure and exploits it effectively in an effort to reduce prediction errors would seem a logical first choice.

Our study included just three but distinct area-level models considered as potentially attractive and worth exploring in the context of a forest inventory. Combined, they do capture the main variants of currently available area-level models (Chandra et al. 2013; Datta et al. 2005; Namazi-Rad and Steel 2015; Rao and Molina 2015, Chap. 5.4). We are therefore comfortable in our recommendation of a spatial area-level modelling. It is of course possible that alternative spatial area-level models could have outperformed the model by Chandra et al. (2012), but we consider it unlikely that it will be by a practically important margin.

We do not think it is realistic to expect an analyst to explore a large set of models or to engage in a model selection process or model averaging procedure (Burnham and Anderson 2002; Claeskens and Hjort 2008), that in any rate would be questionable (Pfeffermann 2013). Our recommendation, in the context of a forest inventory, is to begin the analyses with an area-level model that accommodates a spatial covariance structure in one or more auxiliary variables, like the CH model (Chandra et al. 2012). If there is little or no support for a spatial covariance structure and the MSEs obtained with CH are in line with those of WF, then we suggest the use of the WF or one of its many variants when interest centres on averages across the AOI under study. Finally, the DM model appears most suitable when there is a clear focus on individual area-level results and only a minority of areas—as in this study—contributes significantly to the random among-area effect(s). DM affords, in principle, a more balanced approach by weighing, on a case-by-case basis, the 'evidence' for and against a non-zero area effect. We recognize, however, that acceptance of the Bayesian DM estimator may be low in agencies analysing forest inventory data (Burk and Ek 1982; Finley et al. 2008; Melville et al. 2015).

The SYN estimator is only recommended when there no or very little support for random area effects or when an AOI is not supported by a direct estimate of Y. Our result suggests that area effects may be quite common in forest inventory applications.

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