Using Monte Carlo Method to calculate Pi

1 Theory

We will calculate an approximation to π using the Monte Carlo method.

We know the formula for the area of a circle:

$$A_{circle} = \pi r^2$$

Also, we know the area formula for a square with side length d:

$$A_{square} = d^2$$

If we choose d so that $2r = d$ we get the ratio of the areas:

$$\frac{A_{circle}}{A_{square}} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$

Since we don't know π we cannot calculate A_{circle}. But we can take advantage of random numbers and the implicit formula of the circle $x^2 + y^2 = 1$ in a Cartesian coordinate system.

We draw pairs of random numbers from 0 to 1 representing its position on the x- and the y-axis.Then we evaluate whether the (x, y)-pair is in the inner circle. For a large enough number of pairs, we get useful approximation to the above ratio. If we multiply it with 4, we get an approximation of π.

2 Calculation in R

```r
set.seed(3141)
n <- 1000
x <- runif(n)
y <- runif(n)
inner <- x^2 + y^2 <= 1
MC_pi <- 4*sum(inner)/length(inner)

MC_pi
```

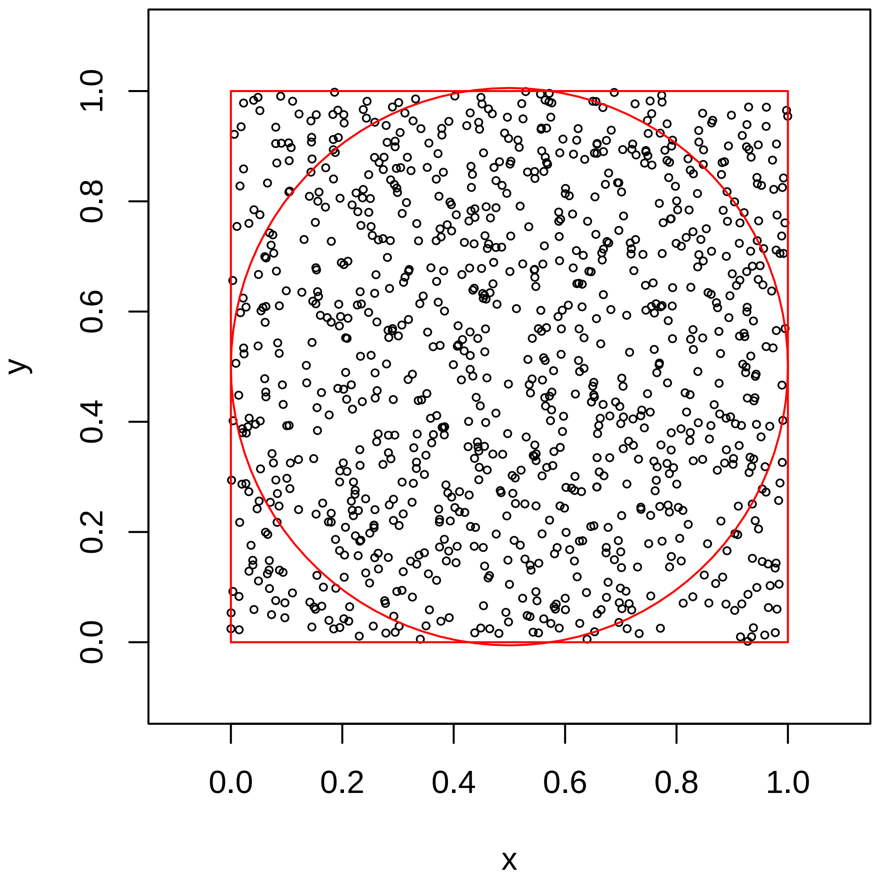
```
[1] 3.108
```

We will plot only a sample:

```
library("plotrix")
plot(x, y, xlim=c(-0.1, 1.1), ylim=c(-0.1, 1.1), pch = "°")

draw.circle(0.5, 0.5, 0.5, border="red")

lines(x=c(0,1), y=c(0,0), col="red")
lines(x=c(0,1), y=c(1,1), col="red")
lines(x=c(0,0), y=c(0,1), col="red")
lines(x=c(1,1), y=c(1,0), col="red")
```



3 How many pairs do we need?

```
steps <- 1:1000
MC_pi.2 <- lapply(steps, function(x)
  4*sum(runif(x)^2+runif(x)^2 <= 1)/x)

plot(steps, MC_pi.2, type="l", ylab="approximation for Pi",
     xlab="number of simulated (x,y)-pairs")
abline(h=pi, col="red")
```