A simulated telescope's view of the sky in the presence of a Schwarzschild blackhole

Steven Dorsher

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1 Introduction

With the LIGO detection, many people are interested in the question of how blackholes make the sky appear to the external observer. Understanding the question of how they will lens the presence of stars behind them could help make blackholes without accretion disks or partners indirectly observable. The problem of a Schwarzschild blackhole in a vaccuum with light passing by it from a static field of stars is not a particularly challenging problem and appears to have been solved numerically in the 1970's or 1980's, though I coudn't find a specific source. Many people since then have solved more difficult problems for rotating, perturbed, or asymptotically cosmological spacetimes.

In this project, I numerically integrate the eight coupled ordinary differential equations governing light rays (null geodesics) backward from a telescope plane near an ordinary (no spin or charge) in a vaccuum. I iterate over many pixels in the telescope plane, and determine their origin on the "sky" at some finite distance away. From that, I reconstruct a png file of what the sky looks like in the presence of a Schwarzschild black hole, at the location of the Earth, and a telescope, $700M$ away. Henceforth, the mass of the blackhole will simply be referred to as M. I will use units of $c = G = 1$.

2 The geodesic equations

2.1 Overview

In general relativity, the differential equations governing the path matter or light takes in a vacuum are called the geodesic equations. They are second order rank four tensor equations, which are separable, resulting in eight coupled ordinary differential equations. The geodesic equation states that a free particle, which feels no forces other than the curvature of spacetime that causes gravity, follows a path that is the shortest path between two points.

2.2 Computation

The geodesic equation is given by

$$\frac{d^2x^\mu}{d\lambda^2} = -\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} \tag{1}$$

Here x^μ is the μth component of a spacetime four-vector, $\Gamma^\mu_{\alpha\beta}$ are the Christoffel connections, and λ is an affine parameter that plays the role of proper time for a null geodesic, since a null geodesic has no rest frame. There is an implicit summation convention over repeated indices, where upper and lower indices are connected by the metric $g_{\mu\nu}$ and the inverse metric $g^{\mu\nu}$.

The metric determines the distance between two points in a curved spacetime. In a flat spacetime like the one near Earth (far away from relativistic sources), the Minkowski metric is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2}$$

In matrix form this can be written:

$$\eta_{\mu\nu} = \eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

In the Schwarzschild metric,

2.3 Result

3 The adaptive fourth order Runga-Kutta method

3.1 Method

3.2 Testing using a simple harmonic oscillator ODE

4 Testing the geodesic equations with orbits with elipticity

4.1 Expected characteristics of these orbits

4.2 Initial data

4.3 Results

5 Null geodesic initial data

5.1 Definition of a null geodesic

5.2 Perpendicularity to the origin plane

5.3 The resulting constraints

5.4 End conditions: inside the horizon or outside the sky radius

5.5 The resulting orbits

6 Generating a telescope image

6.1 Flat row flat pixel png format

6.2 Connecting theta and phi to pixels on the sky image

6.3 Making the blackhole red

6.4 Serial results

7 Profiling

7.1 Serial code profile

7.2 What to parallelize?