# The Laplace Transform LAPLACE OPERATIONS\*

	F(t)	f(s)
1	F(t)	$\int_0^\infty e^{-st} F(t) dt$
2	AF(t) + BG(t)	Af(s) + Bg(s)
3	F'(t)	sf(s) - F(+0)
4	$F^{(n)}(t)$	$   s^{n} f(s) - s^{n-1} F(+0) - s^{n-2} F'(+0) - \cdots - F^{(n-1)}(+0) $
5	$\int_0^t F(\tau) d\tau$	$\frac{1}{s} f(s)$
	$\int_0^1 \int_0^1 F(\lambda)  d\lambda  d\tau$	$\frac{1}{s^2}f(s)$
7	$\int_0^t F_1(t-\tau) F_2(\tau) d\tau = F_1 * F_2$	$f_1(s) f_2(s)$
8	tF(t)	-f'(s)
9	$t^n F(t)$	$(-1)^n f^{(n)}(s)$
10	$\frac{1}{t} F(t)$	$\int_{s}^{\infty} f(x) dx$
11	$e^{at}F(t)$	f(s-a)
12	F(t - b), where $F(t) = 0when t < 0$	$e^{-bs}f(s)$
13	$\frac{1}{c} F\left(\frac{t}{c}\right)$	f(cs)
14	$\frac{1}{c} e^{(bt)/c} F\left(\frac{t}{c}\right)$	f(cs-b)
15	F(t+a)=F(t)	$\frac{\int_0^a e^{-st} F(t) dt}{1 - e^{-as}}$
16	F(t+a)=-F(t)	$\frac{\int_0^a e^{-st} F(t) dt}{1 + e^{-as}}$
17	$F_1(t)$ , the half-wave rectification of $F(t)$ in No. 16	$\frac{f(s)}{1-e^{-as}}$
18	$F_2(t)$ , the full-wave rectification of $F(t)$ in No. 16	$f(s) \coth \frac{as}{2}$
19	$\sum_{1}^{m} \frac{p(a_n)}{q'(a_n)} e^{a_n t}$	$\frac{p(s)}{q(s)}, q(s) = (s - a_1)(s - a_2) \cdots (s - a_m)$
20	$e^{at}\sum_{n=1}^{r}\frac{\phi^{(r-n)}(a)}{(r-n)!}\frac{t^{n-1}}{(n-1)!}+\cdots$	$\frac{p(s)}{q(s)} = \frac{\phi(s)}{(s-a)'}$

<sup>&</sup>quot;These tables of Laplace Operations, Laplace Transforms, and Finite Fourier sine and cosine transforms were taken from "Modern Operational Mathematics in Engineering", by permission of the author, R. V. Churchill, and the publisher, McGraw-Hill Book Company, Inc.

## LAPLACE TRANSFORMS

	f(s)	F(t)
1	<u>l</u> <u>s</u>	$\mu(t)$ , unit step function
2	$\frac{1}{s^2}$	t
3	$\frac{1}{s^n} (n = 1, 2, \ldots)$	$\frac{t^{n-1}}{(n-1)!}$
4	$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$
5	$s^{-3/2}$	$2\sqrt{\frac{t}{\pi}}$
6	$s^{-[n+(1/2)]}$ $(n = 1, 2,)$	$\frac{2^{n}t^{n-(1/2)}}{1\cdot 3\cdot 5\cdots (2n-1)\sqrt{\pi}}$
7	$\frac{\Gamma(k)}{s^k} (k > 0)$	t <sup>k-1</sup>
8	$\frac{1}{s-a}$	e <sup>ai</sup>
9	$\frac{1}{(s-a)^2}$	te <sup>at</sup>
10	$\frac{1}{(s-a)^n} (n=1,2,\ldots)$	$\frac{1}{(n-1)!} t^{n-1} e^{at}$
11	$\frac{\Gamma(k)}{(s-a)^k} (k > 0)$	$t^{k-1}e^{at}$
12*	$\frac{1}{(s-a)(s-b)}$	$\frac{1}{a-b}\left(e^{ai}-e^{bi}\right)$
13*	$\frac{s}{(s-a)(s-b)}$	$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$
14*	$\frac{1}{(s-a)(s-b)(s-c)}$	$-\frac{(b-c)e^{at}+(c-a)e^{bt}+(a-b)e^{ct}}{(a-b)(b-c)(c-a)}$
15	$\frac{1}{s^2 + a^2}$	$\frac{1}{a}\sin at$
16	$\frac{s}{s^2 + a^2}$	cos at
17	$\frac{1}{s^2-a^2}$	$\frac{1}{a}$ sinh $at$
18	$\frac{1}{s^2 + a^2}$ $\frac{s}{s^2 + a^2}$ $\frac{1}{s^2 - a^2}$ $\frac{s}{s^2 - a^2}$	cosh at

<sup>\*</sup>Here a, b, and (in 14) c represent distinct constants.

LAPLACE TRANSFORMS (Continued)				
	f(s)	F(t)		
19	$\frac{1}{s(s^2+a^2)}$	$\frac{1}{a^2}(1-\cos at)$		
20	$\frac{1}{s^2(s^2+a^2)}$	$\frac{1}{a^3}(at-\sin at)$		
21	$\frac{1}{(s^2+a^2)^2}$	$\frac{1}{2a^3}\left(\sin at - at\cos at\right)$		
22	$\frac{s}{(s^2+a^2)^2}$	$\frac{t}{2a}\sin at$		
23	$\frac{s^2}{(s^2+a^2)^2}$	$\frac{1}{2a}\left(\sin at + at\cos at\right)$		
24	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$	t cos at		
25	$\frac{s}{(s^2+a^2)(s^2+b^2)}(a^2\neq b^2)$	$\frac{\cos at - \cos bt}{b^2 - a^2}$		
26	$\frac{1}{(s-a)^2+b^2}$	$\frac{1}{b}e^{at}\sin bt$		
27	$\frac{s-a}{(s-a)^2+b^2}$	e° cos bi		
27.1	$\frac{1}{[(s+a)^2+b^2]^n}$	$\frac{-e^{-at}}{4^{n-1}b^{2n}}\sum_{r=1}^{n}\binom{2n-r-1}{n-1}(-2t)^{n-1}\frac{d^{r}}{dt^{r}}\left[\cos(bt)\right]$		
27.2	$\frac{s}{[(s+a)^2+b^2]^n}$	$\frac{e^{-at}}{4^{n-1}b^{2n}} \left\{ \sum_{r=1}^{n} \binom{2n-r-1}{n-1} \frac{1}{(r-1)!} \right\}$		
		$(-2t)^{r-1}\frac{d^r}{dt^r}\left[a\cos(bt)+b\sin(bt)\right]$		
		$-2b\sum_{r=1}^{n-1}\frac{1}{(r-1)!}\binom{2n-r-2}{n-1}$		
	9,500	$(-2t)^{r-1}\frac{d^r}{dt^r}\left[\sin bt\right]$		
	$\frac{3a^2}{s^3+a^3}$	$e^{-at} - e^{(at)/2} \left( \cos \frac{at \sqrt{3}}{2} - \sqrt{3} \sin \frac{at \sqrt{3}}{2} \right)$		
29	$\frac{4a^3}{s^4+4a^4}$	sin at cosh at - cos at sinh at		
30	$\frac{s}{s^4 + 4a^4}$	$\frac{1}{2a^2} \sin at \sinh at$		

LAFLACE TRANSPORIVIS (CONTINUED)			
11	f(s)	F(t)	
31	$\frac{1}{s^4-a^4}$	$\frac{1}{2a^3}\left(\sinh at - \sin at\right)$	
32	$\frac{s}{s^4-a^4}$	$\frac{1}{2a^2} \left( \cosh at - \cos at \right)$	
33	$\frac{8a^{3}s^{2}}{(s^{2}+a^{2})^{3}}$	$(1 + a^2t^2)\sin at - \cos at$	
34*	$\frac{1}{s}\left(\frac{s-1}{s}\right)^n$	$L_n(t) = \frac{e^t}{n!} \frac{d^n}{dt^n} (t^n e^{-t})$	
35	$\frac{s}{(s-a)^{3/2}}$	$\frac{1}{\sqrt{\pi t}}e^{at}(1+2at)$	
36	$\sqrt{s-a}-\sqrt{s-b}$	$\frac{1}{2\sqrt{\pi t^3}}\left(e^{bt}-e^{at}\right)$	
37	$\frac{1}{\sqrt{s}+a}$	$\frac{1}{\sqrt{\pi t}} - ae^{a^2t} \operatorname{erfc}(a\sqrt{t})$	
38	$\frac{\sqrt{s}}{s-a^2}$	$\frac{1}{\sqrt{\pi t}} + ae^{a't} \operatorname{erf}(a\sqrt{t})$	
39	$\frac{\sqrt{s}}{s+a^2}$	$\frac{1}{\sqrt{\pi t}} - \frac{2a}{\sqrt{\pi}} e^{-a^{\prime}t} \int_0^{a\sqrt{t}} e^{\lambda^{\prime}} d\lambda$	
40	$\frac{1}{\sqrt{s}(s-a^2)}$	$\frac{1}{a} e^{a^{i}t} \operatorname{erf} (a \sqrt{t})$	
41	$\frac{1}{\sqrt{s(s+a^2)}}$	$\frac{2}{a\sqrt{\pi}}e^{-a't}\int_0^{a\sqrt{t}}e^{\lambda'}d\lambda$	
42	$\frac{b^2-a^2}{(s-a^2)(b+\sqrt{s})}$	$e^{a't}[b-a\operatorname{erf}(a\sqrt{t})]-be^{b't}\operatorname{erfc}(b\sqrt{t})$	
43	$\frac{1}{\sqrt{s}(\sqrt{s}+a)}$	$e^{a^{i}t}$ erfc $(a\sqrt{t})$	
	$\frac{1}{(s+a)\sqrt{s+b}}$	$\frac{1}{\sqrt{b-a}} e^{-at} \operatorname{erf} \left( \sqrt{b-a} \sqrt{t} \right)$	
	$\frac{b^2-a^2}{\sqrt{s}(s-a^2)(\sqrt{s}+b)}$	$e^{a't}\left[\frac{b}{a}\operatorname{erf}(a\sqrt{t})-1\right]+e^{b't}\operatorname{erfc}(b\sqrt{t})$	
46†	$\frac{(1-s)^n}{s^{n+(1/2)}}$	$\frac{n!}{(2n)!\sqrt{\pi t}}H_{2n}(\sqrt{t})$	
47	$\frac{(1-s)^n}{s^{n+(3/2)}}$	$-\frac{n!}{\sqrt{\pi}(2n+1)!}H_{2n+1}(\sqrt{t})$	

<sup>\*</sup> $L_n(t)$  is the Laguerre polynomial of degree n. † $H_n(x)$  is the Hermite polynomial,  $H_n(x) = e^{x^2} \frac{d^n}{dx^n} (e^{-x^2})$ .

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	f(s)	F(t)
48*	$\frac{\sqrt{s+2a}}{\sqrt{s}}-1$	$ae^{-at}[I_1(at) + I_0(at)]$
49	$\frac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{-(1/2)(a+b)t}I_0\left(\frac{a-b}{2}t\right)$
50	$\frac{\Gamma(k)}{(s+a)^k(s+b)^k} (k \ge 0)$	$\sqrt{\pi} \left( \frac{t}{a-b} \right)^{k-(1/2)} e^{-(1/2)(a+b)t} I_{k-(1/2)} \left( \frac{a-b}{2} \right)^{t}$
1	$\frac{1}{(s+a)^{1/2}(s+b)^{3/2}}$	$te^{-(1/2)(a+b)t}\left[I_0\left(\frac{a-b}{2}t\right)+I_1\left(\frac{a-b}{2}t\right)\right]$
52	$\frac{\sqrt{s+2a}-\sqrt{s}}{\sqrt{s+2a}+\sqrt{s}}$	$\frac{1}{t} e^{-at} I_1(at)$
53	$\frac{(a-b)^k}{(\sqrt{s+a}+\sqrt{s+b})^{2k}}(k>0)$	$\frac{k}{t} e^{-(1/2)(a+b)t} I_k \left( \frac{a-b}{2} t \right)$
54	$\frac{(\sqrt{s+a}+\sqrt{s})^{-2\nu}}{\sqrt{s}\sqrt{s+a}}(\nu>-1)$	$\frac{1}{a''} e^{-(1/2)(at)} I_{\nu} \left(\frac{1}{2} at\right)$
55	$\frac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
56	$\frac{(\sqrt{s^2 + a^2} - s)^{\nu}}{\sqrt{s^2 + a^2}} (\nu > -1)$	$a''J_{r}(at)$
57	$\frac{1}{\left(s^2+a^2\right)^k}\left(k>0\right)$	$\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a}\right)^{k-(1/2)} J_{k-(1/2)}(at)$
58	$(\sqrt{s^2+a^2}-s)^k(k>0)$	$\frac{ka^k}{t}J_k(at)$
59	$\frac{(s-\sqrt{s^2-a^2})'}{\sqrt{s^2-a^2}}(\nu > -1)$	a"l,(at)
60	$\frac{1}{\left(s^2-a^2\right)^k}\left(k>0\right)$	$\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a}\right)^{k-(1/2)} I_{k-(1/2)}(at)$
61	e <sup>-ks</sup>	$S_k(t) = \begin{cases} 0 \text{ when } 0 < t < k \\ 1 \text{ when } t > k \end{cases}$
62	$\frac{e^{-ks}}{s^2}$	$\begin{cases} 0 \text{ when } 0 < t < k \\ t - k \text{ when } t > k \end{cases}$
63	$\frac{e^{-ks}}{s^2}$ $\frac{e^{-ks}}{s^{\mu}} (\mu > 0)$ $\frac{1 - e^{-ks}}{s}$	$\begin{cases} 0 \text{ when } t > k \\ t - k \text{ when } t > k \end{cases}$ $\begin{cases} 0 & \text{when } 0 < t < k \\ \frac{(t - k)^{\mu - 1}}{\Gamma(\mu)} \text{ when } t > k \end{cases}$ $\begin{cases} 1 \text{ when } 0 < t < k \\ 0 \text{ when } t > k \end{cases}$
64	$\frac{1-e^{-ks}}{s}$	$\begin{cases} 1 \text{ when } 0 < t < k \\ 0 \text{ when } t > k \end{cases}$

	f(s)	F(t)
65	$\frac{1}{s(1-e^{-ks})} = \frac{1+\coth\frac{1}{2}ks}{2s}$	S(k,t) = n  when (n-1)k < t < nk(n = 1, 2,)
66	$\frac{1}{s(e^{ks}-a)}$	$\begin{cases} 0 \text{ when } 0 < t < k \\ 1 + a + a^2 + \dots + a^{n-1} \\ \text{when } nk < t < (n+1)k(n=1,2,\dots) \end{cases}$
67	$\frac{1}{s}$ tanh $ks$	$M(2k,t) = (-1)^{n-1}$ when $2k(n-1) < t < 2kn$ (n = 1, 2,)
68	$\frac{1}{s(1+e^{-ks})}$	$\frac{1}{2}M(k,t) + \frac{1}{2} = \frac{1 - (-1)^n}{2}$ when $(n-1)k < t < nk$
69*	$\frac{1}{s^2}$ tanh $ks$	H(2k,t)
70	1 s sinh ks	2S(2k, t + k) - 2 = 2(n - 1) when $(2n - 3)k < t < (2n - 1)k$ $(t > 0)$
71	$\frac{1}{s \cosh ks}$	$M(2k, t + 3k) + 1 = 1 + (-1)^n$ when $(2n - 3)k < t < (2n - 1)k$ $(t > 0)$
72	$\frac{1}{s}$ coth $ks$	2S(2k,t) - 1 = 2n - 1 when $2k(n-1) < t < 2kn$
73	$\frac{k}{s^2 + k^2} \coth \frac{\pi s}{2k}$	sin kt
74	$\frac{1}{(s^2+1)(1-e^{-\pi s})}$	$\begin{cases} \sin t \text{ when } (2n-2)\pi < t < (2n-1)\pi \\ 0 \text{ when } (2n-1)\pi < t < 2n\pi \end{cases}$
75	$\frac{1}{s} e^{-k/s}$ $\frac{1}{\sqrt{s}} e^{-k/s}$	$J_0(2\sqrt{kt})$
76	$\frac{1}{\sqrt{s}} e^{-k/s}$	$\frac{1}{\sqrt{\pi t}}\cos 2\sqrt{kt}$
77	$\frac{1}{\sqrt{s}} e^{k/s}$	$\frac{1}{\sqrt{\pi t}}\cosh 2\sqrt{kt}$
78	$\frac{1}{s^{3/2}} e^{-k/s}$ $\frac{1}{s^{3/2}} e^{k/s}$	$\frac{1}{\sqrt{\pi k}} \sin 2 \sqrt{kt}$ $\frac{1}{\sqrt{\pi k}} \sinh 2 \sqrt{kt}$
79	$\frac{1}{s^{3/2}} e^{k/s}$	$\frac{1}{\sqrt{\pi k}}$ sinh 2 $\sqrt{kt}$
80	$\frac{1}{s^{\mu}} e^{-k/s} (\mu > 0)$	$\left(\frac{t}{k}\right)^{(\mu-1)/2}J_{\mu-1}(2\sqrt{kt})$
*H(2)	$(x,t) = k + (r - k)(-1)^n$ where $t = 2kn$	$+ r; 0 \le r < 2k; n = 0, 1, 2, \dots$

$87  \frac{e^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})} (k \ge 0)$ $88  \frac{e^{-k\sqrt{s}(s+a)}}{\sqrt{s}(s+a)}$ $89  \frac{e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}}$ $90  \frac{e^{-k\sqrt{s^{2}-a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $91  \frac{e^{-k(\sqrt{s^{2}+a^{2}-s})}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $92  e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}$ $93  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $94  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $95  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $96  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $97  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $98  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $99  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $90  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $90  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $90  \frac{e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $90  \frac{e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $91  \frac{e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$ $92  \frac{e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} (k \ge 0)$		$\int (s)$	F(t)
83 $\frac{1}{s} e^{-k\sqrt{t}}(k \ge 0)$ $\text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$ 84 $\frac{1}{\sqrt{s}} e^{-k\sqrt{t}}(k \ge 0)$ $\frac{1}{\sqrt{\pi t}} \exp\left(-\frac{k^2}{4t}\right)$ 85 $s^{-3/2}e^{-k\sqrt{t}}(k \ge 0)$ $2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{k^2}{4t}\right) - k \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$ 86 $\frac{ae^{-k\sqrt{t}}}{s(a+\sqrt{s})}(k \ge 0)$ $-e^{ak}e^{a^{t}t}\operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$ 87 $\frac{e^{-k\sqrt{t}}}{\sqrt{s(a+\sqrt{s})}}(k \ge 0)$ $e^{ak}e^{a^{t}t}\operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$ 88 $\frac{e^{-k\sqrt{t(s+a)}}}{\sqrt{s(s+a)}}$ $\left\{0\frac{\sinh 0 < t < k}{e^{-(1/2)(at)}I_0(\frac{1}{2}a\sqrt{t^2-k^2}) \text{ when } t > k}\right\}$ 90 $\frac{e^{-k\sqrt{t^2-a^2}}}{\sqrt{s^2-a^2}}$ $\left\{0\frac{\sinh 0 < t < k}{J_0(a\sqrt{t^2-k^2}) \text{ when } t > k}\right\}$ 91 $\frac{e^{-k(\sqrt{t^2+a^2}-s)}}{\sqrt{s^2+a^2}}(k \ge 0)$ $\left\{0\frac{\sinh 0 < t < k}{J_0(a\sqrt{t^2-k^2}) \text{ when } t > k}\right\}$ 92 $e^{-ks} - e^{-k\sqrt{t^2+a^2}}$ $\left\{0\frac{\sinh 0 < t < k}{J_0(a\sqrt{t^2-k^2}) \text{ when } t > k}\right\}$	81	$\frac{1}{s^{\mu}} e^{k/s} (\mu > 0)$	$\left(\frac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}(2\sqrt{kt})$
$ \frac{1}{\sqrt{s}} e^{-k\sqrt{s}}(k \ge 0) \qquad \frac{1}{\sqrt{\pi t}} \exp\left(-\frac{k^{2}}{4t}\right) \\ 85  s^{-3/2}e^{-k\sqrt{t}}(k \ge 0) \qquad 2  \sqrt{\frac{t}{\pi}} \exp\left(-\frac{k^{2}}{4t}\right) - k \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right) \\ 86  \frac{ae^{-k\sqrt{t}}}{s(a+\sqrt{s})}(k \ge 0) \qquad -e^{ak}e^{a^{2}t}\operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right) \\ 87  \frac{e^{-k\sqrt{t}}}{\sqrt{s}(a+\sqrt{s})}(k \ge 0) \qquad e^{ak}e^{a^{2}t}\operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right) \\ 88  \frac{e^{-k\sqrt{t}(t+a)}}{\sqrt{s(s+a)}} \qquad \qquad e^{ak}e^{a^{2}t}\operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) \\ 89  \frac{e^{-k\sqrt{t}(t+a)}}{\sqrt{s^{2}+a^{2}}} \qquad \qquad \begin{cases} 0 & \text{when } 0 < t < k \\ d_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} \\ 90  \frac{e^{-k\sqrt{t^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} \qquad \qquad \begin{cases} 0 & \text{when } 0 < t < k \\ d_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} \\ 91  \frac{e^{-k(\sqrt{t^{2}+a^{2}}-t)}}{\sqrt{s^{2}+a^{2}}}(k \ge 0) \qquad \qquad \begin{cases} 0 & \text{when } 0 < t < k \\ d_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} \\ 92  e^{-ks} - e^{-k\sqrt{t^{2}+a^{2}}} \qquad \qquad \begin{cases} 0 & \text{when } 0 < t < k \\ \sqrt{t^{2}-k^{2}}} & \sqrt{t^{2}-k^{2}} & \text{when } t > k \end{cases} \\ 90  \frac{e^{-ks} - e^{-k\sqrt{t^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} \qquad \qquad \begin{cases} 0 & \text{when } 0 < t < k \\ \sqrt{t^{2}-k^{2}}} & \sqrt{t^{2}-k^{2}} & \text{when } t > k \end{cases} \\ 91  \frac{e^{-k(\sqrt{t^{2}+a^{2}}-t)}}{\sqrt{s^{2}+a^{2}}}(k \ge 0) \qquad \qquad \begin{cases} 0 & \text{when } 0 < t < k \\ \sqrt{t^{2}-k^{2}}} & \sqrt{t^{2}-k^{2}} & \text{when } t > k \end{cases} \end{cases}$	82	$e^{-k\sqrt{s}}(k>0)$	$\frac{k}{2\sqrt{\pi t^3}} \exp\left(-\frac{k^2}{4t}\right)$
	83	$\frac{1}{s} e^{-k\sqrt{s}} (k \ge 0)$	$\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
	84	$\frac{1}{\sqrt{s}} e^{-k\sqrt{s}} (k \ge 0)$	$\frac{1}{\sqrt{\pi t}} \exp \left(-\frac{k^2}{4t}\right)$
$ \begin{array}{lll} 87 & \frac{e^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})} (k \geq 0) \\ 88 & \frac{e^{-k\sqrt{s}(s+a)}}{\sqrt{s}(s+a)} \\ 89 & \frac{e^{-k\sqrt{s'(s+a)}}}{\sqrt{s^2+a^2}} \\ 90 & \frac{e^{-k\sqrt{s'+a'}}}{\sqrt{s^2-a^2}} \\ 91 & \frac{e^{-k(\sqrt{s'+a'}-a)}}{\sqrt{s^2+a^2}} (k \geq 0) \\ 92 & e^{-ks} - e^{-k\sqrt{s'+a'}} \end{array} $ $ \begin{array}{lll} e^{ak} e^{a^{it}} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) \\ e^{ak} e^{a^{it}} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) \\ e^{-k(\sqrt{s'+a'}-a)} & \text{when } 0 < t < k \\ e^{-(1/2)(at)} I_0(\frac{1}{2}a\sqrt{t^2-k^2}) & \text{when } t > k \end{array} $ $ \begin{array}{lll} 0 & \text{when } 0 < t < k \\ J_0(a\sqrt{t^2-k^2}) & \text{when } t > k \end{array} $ $ \begin{array}{lll} 0 & \text{when } 0 < t < k \\ I_0(a\sqrt{t^2-k^2}) & \text{when } t > k \end{array} $ $ \begin{array}{lll} 0 & \text{when } 0 < t < k \\ I_0(a\sqrt{t^2-k^2}) & \text{when } t > k \end{array} $	85	$s^{-3/2}e^{-k\sqrt{s}}(k \geq 0)$	$2 \sqrt{\frac{t}{\pi}} \exp\left(-\frac{k^2}{4t}\right) - k \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
88 $\frac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}}$ 89 $\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$ 90 $\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2-a^2}}$ 91 $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$ 92 $e^{-ks} - e^{-k\sqrt{s^2+a^2}}$ 88 $\frac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s^2+a^2}}$ 89 $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$ 90 $\frac{e^{-k\sqrt{s^2+a^2}-s}}{\sqrt{s^2+a^2}} (k \ge 0)$ 91 $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$ 92 $\frac{e^{-ks} - e^{-k\sqrt{s^2+a^2}}}{\sqrt{t^2-k^2}} (k \ge 0)$ 93 $\frac{e^{-ks} - e^{-k\sqrt{s^2+a^2}}}{\sqrt{t^2-k^2}} (k \ge 0)$ 94 $\frac{e^{-ks} - e^{-k\sqrt{s^2+a^2}}}{\sqrt{t^2-k^2}} (k \ge 0)$	86	$\frac{ae^{-k\sqrt{s}}}{s(a+\sqrt{s})} (k \ge 0)$	$-e^{ak}e^{a^{i}t}\operatorname{erfc}\left(a\sqrt{t}+\frac{k}{2\sqrt{t}}\right)+\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
$ \frac{e^{-k\sqrt{s^{2}+a^{2}}}}{\sqrt{s^{2}+a^{2}}} $ 90 $ \frac{e^{-k\sqrt{s^{2}-a^{2}}}}{\sqrt{s^{2}-a^{2}}} $ 91 $ \frac{e^{-k(\sqrt{s^{2}+a^{2}-s})}}{\sqrt{s^{2}+a^{2}}}(k \ge 0) $ 92 $ e^{-ks} - e^{-k\sqrt{s^{2}+a^{2}}} $ $ \begin{cases} 0 & \text{when } 0 < t < k \\ I_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} $ $ \begin{cases} 0 & \text{when } 0 < t < k \\ I_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} $ $ \begin{cases} 0 & \text{when } 0 < t < k \\ I_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} $ $ \begin{cases} 0 & \text{when } 0 < t < k \\ I_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} $ $ \begin{cases} 0 & \text{when } 0 < t < k \\ I_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} $ $ \begin{cases} 0 & \text{when } 0 < t < k \\ I_{0}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases} $	87	$\frac{e^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})} (k \ge 0)$	$e^{ak}e^{a't}\operatorname{erfc}\left(a\sqrt{t}+\frac{k}{2\sqrt{t}}\right)$
89 $\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$ 90 $\frac{e^{-k\sqrt{s^2-a^2}}}{\sqrt{s^2-a^2}}$ 91 $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$ 92 $e^{-ks} - e^{-k\sqrt{s^2+a^2}}$ 89 $\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}} (k \ge 0)$ 90 $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$ 91 $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$ 92 $\frac{ak}{\sqrt{t^2-k^2}} J_1(a\sqrt{t^2-k^2}) \text{ when } t > k$	88	$\frac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ e^{-(1/2)(at)} I_0(\frac{1}{2}a \sqrt{t^2 - k^2}) & \text{when } t > k \end{cases}$
91 $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$ $e^{-ks} - e^{-k\sqrt{s^2+a^2}}$ $\begin{cases} 0 & \text{when } 0 < t < \\ \frac{ak}{\sqrt{t^2-k^2}} J_1(a\sqrt{t^2-k^2}) \text{ when } t > k \end{cases}$	89	$\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$	
91 $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$ $\int_0^{0} (a\sqrt{t^2+2kt}) dt$ $\begin{cases} 0 & \text{when } 0 < t < t < t < t < t < t < t < t < t <$	90	$\frac{e^{-k\sqrt{s^2-a^2}}}{\sqrt{s^2-a^2}}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ I_0(a\sqrt{t^2 - k^2}) & \text{when } t > k \end{cases}$
92 $e^{-ks} - e^{-k\sqrt{s^2 + a^2}}$ $\frac{ak}{\sqrt{t^2 - k^2}} J_1(a\sqrt{t^2 - k^2}) \text{ when } t > k$	91	$\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \ge 0)$	
>	92	$e^{-ks} - e^{-k\sqrt{s^2+a^2}}$	$\begin{cases} 0 & \text{when } 0 < t < k \\ \frac{ak}{\sqrt{t^2 - k^2}} J_1(a\sqrt{t^2 - k^2}) \text{ when } t > k \end{cases}$
94 $\frac{a^{\nu}e^{-k\sqrt{s^{2}-a^{2}}}}{\sqrt{s^{2}+a^{2}}(\sqrt{s^{2}+a^{2}}+s)^{\nu}} \begin{cases} 0 & \text{when } 0 < t < t \\ \left(\frac{t-k}{t+k}\right)^{(1/2)\nu} J_{\nu}(a\sqrt{t^{2}-k^{2}}) & \text{when } t > k \end{cases}$	93	$e^{-k\sqrt{s^2+a^2}}-e^{-ks}$	}
	94	$\frac{a^{\nu}e^{-k\sqrt{s^{2}-a^{2}}}}{\sqrt{s^{2}+a^{2}}(\sqrt{s^{2}+a^{2}}+s)^{\nu}}$ $(\nu > -1)$	$\begin{cases} 0 & \text{when } 0 < t < k \\ \left(\frac{t-k}{t+k}\right)^{(1/2)\nu} J_{\nu}(a\sqrt{t^2-k^2}) \text{ when } t > k \end{cases}$
95 $\frac{1}{s} \log s$ $\Gamma'(1) - \log t \left[\Gamma'(1) = -0.5772\right]$	95	$\frac{1}{s} \log s$	$\Gamma'(1) - \log t \left[\Gamma'(1) = -0.5772\right]$
96 $ \frac{1}{s^k} \log s  (k > 0) $ $ t^{k-1} \left\{ \frac{\Gamma'(k)}{[\Gamma(k)]^2} - \frac{\log t}{\Gamma(k)} \right\} $	96	$\frac{1}{s^k}\log s(k>0)$	$t^{k-1}\left\{\frac{\Gamma'(k)}{\left[\Gamma(k)\right]^2} - \frac{\log t}{\Gamma(k)}\right\}$
97 $\left  \frac{\log s}{s-a} (a>0) \right $ $e^{at} [\log a - \operatorname{Ei}(-at)]$	97	$\frac{\log s}{s-a} (a > 0)$	$e^{at}[\log a - \operatorname{Ei}(-at)]$

	f(s)	F(t)
98	$\frac{\log s}{s^2+1}$	$\cos t \operatorname{Si}(t) - \sin t \operatorname{Ci}(t)$
99	$\frac{s \log s}{s^2 + 1}$	$-\sin t \operatorname{Si}(t) - \cos t \operatorname{Ci}(t)$
100	$\frac{1}{s}\log\left(1+ks\right)(k>0)$	$-\operatorname{Ei}\left(-\frac{t}{k}\right)$
101	$\log \frac{s-a}{s-b}$	$\frac{1}{t}\left(e^{bt}-e^{at}\right)$
102	$\frac{1}{s}\log\left(1+k^2s^2\right)$	$-2\operatorname{Ci}\left(\frac{t}{k}\right)$
103	$\frac{1}{s}\log(s^2+a^2)  (a>0)$	$2 \log a - 2 \operatorname{Ci}(at)$
104	$\frac{1}{s^2} \log (s^2 + a^2)  (a > 0)$	$\frac{2}{a} \left[ at \log a + \sin at - at \operatorname{Ci}(at) \right]$
105	$\log \frac{s^2 + a^2}{s^2}$	$\frac{2}{t}\left(1-\cos at\right)$
106	$\log \frac{s^2 - a^2}{s^2}$	$\frac{2}{t}\left(1-\cosh at\right)$
107	$\frac{k}{s}$	$\frac{1}{t} \sin kt$
108	$\frac{1}{s}$ arctan $\frac{k}{s}$	Si(kt)
109	$e^{k's'}$ erfc $(ks)$ $(k > 0)$	$\frac{1}{k\sqrt{\pi}}\exp\left(-\frac{t^2}{4k^2}\right)$
110	$\frac{1}{s} e^{k's'} \operatorname{erfc}(ks)  (k > 0)$	$\operatorname{erf}\left(\frac{t}{2k}\right)$
111	$e^{ks}\operatorname{erfc}(\sqrt{ks})$ $(k>0)$	$\frac{\sqrt{k}}{\pi\sqrt{t}(t+k)}$
112	$\frac{1}{\sqrt{s}}$ erfc $(\sqrt{ks})$	$\begin{cases} 0 & \text{when } 0 < t < k \\ (\pi t)^{-1/2} & \text{when } t > k \end{cases}$
113	$\frac{1}{\sqrt{s}} e^{ks} \operatorname{erfc} (\sqrt{ks})(k > 0)$	$\frac{1}{\sqrt{\pi(t+k)}}$
114	$\operatorname{erf}\left(\frac{k}{\sqrt{s}}\right)$	$\frac{1}{\pi t} \sin{(2k\sqrt{t})}$
115	$\frac{1}{\sqrt{s}} e^{k^2/s} \operatorname{erfc}\left(\frac{k}{\sqrt{s}}\right)$	$\frac{1}{\sqrt{\pi t}} e^{-2k\sqrt{t}}$

#### LAPLACE TRANSFORMS (Continued)

	f(s)	F(t)
115.1	-e <sup>as</sup> Ei(-as)	$\frac{1}{t+a};(a>0)$
115.2	$\frac{1}{a} + se^{as} \operatorname{Ei}(-as)$	$\frac{1}{(t+a)^2}:(a>0)$
	$\left[\frac{\pi}{2} - \operatorname{Si}(s)\right] \cos s + \operatorname{Ci}(s) \sin s$	$\frac{1}{t^2+1}$
116*	$K_0(ks)$	$\begin{cases} 0 & \text{when } 0 < t < k \\ (t^2 - k^2)^{-1/2} & \text{when } t > k \end{cases}$
117	$K_0(ks)$ $K_0(k\sqrt{s})$ $\frac{1}{s} e^{ks} K_1(ks)$	$\frac{1}{2t} \exp\left(-\frac{k^2}{4t}\right)$
118	$\frac{1}{s} e^{ks} K_1(ks)$	$\frac{1}{k} \sqrt{t(t+2k)}$
119	$\frac{1}{\sqrt{s}} K_1(k \sqrt{s})$ $\frac{1}{\sqrt{s}} e^{k/s} K_0\left(\frac{k}{s}\right)$	$\frac{1}{k} \exp\left(-\frac{k^2}{4t}\right)$
120	$\frac{1}{\sqrt{s}} e^{k/s} K_0 \left(\frac{k}{s}\right)$	$\frac{2}{\sqrt{\pi t}} K_0(2\sqrt{2kt})$
121	$\pi e^{-ks}I_0(ks)$	$\begin{cases} [t(2k-t)]^{-1/2} & \text{when } 0 < t < 2k \\ 0 & \text{when } t > 2k \end{cases}$
122**	$e^{-ks}I_1(ks)$	$\begin{cases} \frac{k-t}{\pi k \sqrt{t(2k-t)}} & \text{when } 0 < t < 2k \\ 0 & \text{when } t > 2k \end{cases}$

 $*K_n(x)$  is Bessel's function of the second kind for the imaginary argument.

<sup>\*\*</sup>Several additional transforms, especially those involving other Bessel functions, can be found in the tables by G. A. Campbell and R. M. Foster, "Fourier Integrals for Practical Applications", or "Vol. 1, Bateman Manuscript Project, Transform Tables, McGraw-Hill, 1955", or N. W. McLachlan and P. Humbert, "Formulaire pour le calcul symbolique". In the tables by Campbell and Foster, only those entries containing the condition 0 < g or k < g, where g is our t, are Laplace transforms.