## LAPLACE OPERATIONS*

| | $F(t)$ | $f(s)$ |
|---|---|---|
| 1 | $F(t)$ | $\int_0^\infty e^{-st}F(t)\,dt$ |
| 2 | $AF(t) + BG(t)$ | $Af(s) + Bg(s)$ |
| 3 | $F'(t)$ | $sf(s) - F(+0)$ |
| 4 | $F^{(n)}(t)$ | $s^n f(s) - s^{n-1}F(+0) - s^{n-2}F'(+0) - \cdots - F^{(n-1)}(+0)$ |
| 5 | $\int_0^t F(\tau)\,d\tau$ | $\frac{1}{s} f(s)$ |
| 6 | $\int_0^t \int_0^\tau F(\lambda)\,d\lambda\,d\tau$ | $\frac{1}{s^2} f(s)$ |
| 7 | $\int_0^t F_1(t-\tau)F_2(\tau)\,d\tau = F_1 * F_2$ | $f_1(s)\,f_2(s)$ |
| 8 | $tF(t)$ | $-f'(s)$ |
| 9 | $t^n F(t)$ | $(-1)^n f^{(n)}(s)$ |
| 10 | $\frac{1}{t} F(t)$ | $\int_s^\infty f(x)\,dx$ |
| 11 | $e^{at}F(t)$ | $f(s-a)$ |
| 12 | $F(t-b)$, where $F(t) = 0$ when $t < 0$ | $e^{-bs}f(s)$ |
| 13 | $\frac{1}{c} F\left(\frac{t}{c}\right)$ | $f(cs)$ |
| 14 | $\frac{1}{c} e^{(bt)/c}\ F\left(\frac{t}{c}\right)$ | $f(cs - b)$ |
| 15 | $F(t+a) = F(t)$ | $\dfrac{\int_0^a e^{-st}F(t)\,dt}{1 - e^{-as}}$ |
| 16 | $F(t+a) = -F(t)$ | $\dfrac{\int_0^a e^{-st}F(t)\,dt}{1 + e^{-as}}$ |
| 17 | $F_1(t)$, the half-wave rectification of $F(t)$ in No. 16 | $\dfrac{f(s)}{1 - e^{-as}}$ |
| 18 | $F_2(t)$, the full-wave rectification of $F(t)$ in No. 16 | $f(s) \coth \frac{as}{2}$ |
| 19 | $\sum_1^m \frac{p(a_n)}{q'(a_n)} e^{a_n t}$ | $\frac{p(s)}{q(s)},\ q(s) = (s-a_1)(s-a_2)\cdots(s-a_m)$ |
| 20 | $e^{at} \sum_{n=1}^{r} \frac{\phi^{(r-n)}(a)}{(r-n)!} \frac{t^{n-1}}{(n-1)!} + \cdots$ | $\frac{p(s)}{q(s)} = \frac{\phi(s)}{(s-a)^r}$ |

*These tables of Laplace Operations, Laplace Transforms, and Finite Fourier sine and cosine transforms were taken from "Modern Operational Mathematics in Engineering", by permission of the author, R. V. Churchill, and the publisher, McGraw-Hill Book Company, Inc.

## LAPLACE TRANSFORMS

| | $f(s)$ | $F(t)$ |
|---|---|---|
| 1 | $\frac{1}{s}$ | $\mu(t)$, unit step function |
| 2 | $\frac{1}{s^2}$ | $t$ |
| 3 | $\frac{1}{s^n}\ (n = 1, 2, \ldots)$ | $\frac{t^{n-1}}{(n-1)!}$ |
| 4 | $\frac{1}{\sqrt{s}}$ | $\frac{1}{\sqrt{\pi t}}$ |
| 5 | $s^{-3/2}$ | $2\sqrt{\frac{t}{\pi}}$ |
| 6 | $s^{-[n+(1/2)]}\ (n = 1, 2, \ldots)$ | $\frac{2^n t^{n-(1/2)}}{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}$ |
| 7 | $\frac{\Gamma(k)}{s^k}\ (k > 0)$ | $t^{k-1}$ |
| 8 | $\frac{1}{s-a}$ | $e^{at}$ |
| 9 | $\frac{1}{(s-a)^2}$ | $te^{at}$ |
| 10 | $\frac{1}{(s-a)^n}\ (n = 1, 2, \ldots)$ | $\frac{1}{(n-1)!} t^{n-1} e^{at}$ |
| 11 | $\frac{\Gamma(k)}{(s-a)^k}\ (k > 0)$ | $t^{k-1} e^{at}$ |
| 12* | $\frac{1}{(s-a)(s-b)}$ | $\frac{1}{a-b}(e^{at} - e^{bt})$ |
| 13* | $\frac{s}{(s-a)(s-b)}$ | $\frac{1}{a-b}(ae^{at} - be^{bt})$ |
| 14* | $\frac{1}{(s-a)(s-b)(s-c)}$ | $-\frac{(b-c)e^{at} + (c-a)e^{bt} + (a-b)e^{ct}}{(a-b)(b-c)(c-a)}$ |
| 15 | $\frac{1}{s^2+a^2}$ | $\frac{1}{a}\sin at$ |
| 16 | $\frac{s}{s^2+a^2}$ | $\cos at$ |
| 17 | $\frac{1}{s^2-a^2}$ | $\frac{1}{a}\sinh at$ |
| 18 | $\frac{s}{s^2-a^2}$ | $\cosh at$ |

*Here $a$, $b$, and (in 14) $c$ represent distinct constants.

| | $f(s)$ | $F(t)$ |
|---|---|---|
| 19 | $\dfrac{1}{s(s^2 + a^2)}$ | $\dfrac{1}{a^2}(1 - \cos at)$ |
| 20 | $\dfrac{1}{s^2(s^2 + a^2)}$ | $\dfrac{1}{a^3}(at - \sin at)$ |
| 21 | $\dfrac{1}{(s^2 + a^2)^2}$ | $\dfrac{1}{2a^3}(\sin at - at \cos at)$ |
| 22 | $\dfrac{s}{(s^2 + a^2)^2}$ | $\dfrac{t}{2a} \sin at$ |
| 23 | $\dfrac{s^2}{(s^2 + a^2)^2}$ | $\dfrac{1}{2a}(\sin at + at \cos at)$ |
| 24 | $\dfrac{s^2 - a^2}{(s^2 + a^2)^2}$ | $t \cos at$ |
| 25 | $\dfrac{s}{(s^2 + a^2)(s^2 + b^2)} \; (a^2 \neq b^2)$ | $\dfrac{\cos at - \cos bt}{b^2 - a^2}$ |
| 26 | $\dfrac{1}{(s - a)^2 + b^2}$ | $\dfrac{1}{b} e^{at} \sin bt$ |
| 27 | $\dfrac{s - a}{(s - a)^2 + b^2}$ | $e^{at} \cos bt$ |
| 27.1 | $\dfrac{1}{[(s + a)^2 + b^2]^n}$ | $\dfrac{-e^{-at}}{4^{n-1} b^{2n}} \sum_{r=1}^{n} \binom{2n - r - 1}{n - 1} (-2t)^{r-1} \dfrac{d^r}{dt^r} [\cos(bt)]$ |
| 27.2 | $\dfrac{s}{[(s + a)^2 + b^2]^n}$ | $\dfrac{e^{-at}}{4^{n-1} b^{2n}} \left\{ \sum_{r=1}^{n} \binom{2n - r - 1}{n - 1} \dfrac{1}{(r-1)!} (-2t)^{r-1} \dfrac{d^r}{dt^r} [a \cos(bt) + b \sin(bt)] - 2b \sum_{r=1}^{n-1} \dfrac{1}{(r-1)!} \binom{2n - r - 2}{n - 1} (-2t)^{r-1} \dfrac{d^r}{dt^r} [\sin bt] \right\}$ |
| 28 | $\dfrac{3a^2}{s^3 + a^3}$ | $e^{-at} - e^{(at)/2} \left( \cos \dfrac{at\sqrt{3}}{2} - \sqrt{3} \sin \dfrac{at\sqrt{3}}{2} \right)$ |
| 29 | $\dfrac{4a^3}{s^4 + 4a^4}$ | $\sin at \cosh at - \cos at \sinh at$ |
| 30 | $\dfrac{s}{s^4 + 4a^4}$ | $\dfrac{1}{2a^2} \sin at \sinh at$ |

## LAPLACE TRANSFORMS (Continued)

| | $f(s)$ | $F(t)$ |
|---|---|---|
| 31 | $\dfrac{1}{s^4 - a^4}$ | $\dfrac{1}{2a^3}(\sinh at - \sin at)$ |
| 32 | $\dfrac{s}{s^4 - a^4}$ | $\dfrac{1}{2a^2}(\cosh at - \cos at)$ |
| 33 | $\dfrac{8a^3s^2}{(s^2 + a^2)^3}$ | $(1 + a^2t^2)\sin at - \cos at$ |
| 34* | $\dfrac{1}{s}\left(\dfrac{s-1}{s}\right)^n$ | $L_n(t) = \dfrac{e^t}{n!}\dfrac{d^n}{dt^n}(t^n e^{-t})$ |
| 35 | $\dfrac{s}{(s - a)^{3/2}}$ | $\dfrac{1}{\sqrt{\pi t}} e^{at}(1 + 2at)$ |
| 36 | $\sqrt{s - a} - \sqrt{s - b}$ | $\dfrac{1}{2\sqrt{\pi t^3}}(e^{bt} - e^{at})$ |
| 37 | $\dfrac{1}{\sqrt{s} + a}$ | $\dfrac{1}{\sqrt{\pi t}} - ae^{a^2t}\operatorname{erfc}(a\sqrt{t})$ |
| 38 | $\dfrac{\sqrt{s}}{s - a^2}$ | $\dfrac{1}{\sqrt{\pi t}} + ae^{a^2t}\operatorname{erf}(a\sqrt{t})$ |
| 39 | $\dfrac{\sqrt{s}}{s + a^2}$ | $\dfrac{1}{\sqrt{\pi t}} - \dfrac{2a}{\sqrt{\pi}} e^{-a^2t}\displaystyle\int_0^{a\sqrt{t}} e^{\lambda^2}\,d\lambda$ |
| 40 | $\dfrac{1}{\sqrt{s}(s - a^2)}$ | $\dfrac{1}{a} e^{a^2t}\operatorname{erf}(a\sqrt{t})$ |
| 41 | $\dfrac{1}{\sqrt{s}(s + a^2)}$ | $\dfrac{2}{a\sqrt{\pi}} e^{-a^2t}\displaystyle\int_0^{a\sqrt{t}} e^{\lambda^2}\,d\lambda$ |
| 42 | $\dfrac{b^2 - a^2}{(s - a^2)(b + \sqrt{s})}$ | $e^{a^2t}[b - a\operatorname{erf}(a\sqrt{t})] - be^{b^2t}\operatorname{erfc}(b\sqrt{t})$ |
| 43 | $\dfrac{1}{\sqrt{s}(\sqrt{s} + a)}$ | $e^{a^2t}\operatorname{erfc}(a\sqrt{t})$ |
| 44 | $\dfrac{1}{(s + a)\sqrt{s + b}}$ | $\dfrac{1}{\sqrt{b - a}} e^{-at}\operatorname{erf}(\sqrt{b - a}\sqrt{t})$ |
| 45 | $\dfrac{b^2 - a^2}{\sqrt{s}(s - a^2)(\sqrt{s} + b)}$ | $e^{a^2t}\left[\dfrac{b}{a}\operatorname{erf}(a\sqrt{t}) - 1\right] + e^{b^2t}\operatorname{erfc}(b\sqrt{t})$ |
| 46† | $\dfrac{(1 - s)^n}{s^{n+(1/2)}}$ | $\dfrac{n!}{(2n)!\sqrt{\pi t}} H_{2n}(\sqrt{t})$ |
| 47 | $\dfrac{(1 - s)^n}{s^{n+(3/2)}}$ | $-\dfrac{n!}{\sqrt{\pi}(2n + 1)!} H_{2n+1}(\sqrt{t})$ |

*$L_n(t)$ is the Laguerre polynomial of degree $n$.

†$H_n(x)$ is the Hermite polynomial, $H_n(x) = e^{x^2}\dfrac{d^n}{dx^n}(e^{-x^2})$.

| | $f(s)$ | $F(t)$ |
|---|---|---|
| 48* | $\dfrac{\sqrt{s+2a}}{\sqrt{s}} - 1$ | $ae^{-at}[I_1(at) + I_0(at)]$ |
| 49 | $\dfrac{1}{\sqrt{s+a}\sqrt{s+b}}$ | $e^{-(1/2)(a+b)t} I_0\left(\dfrac{a-b}{2}t\right)$ |
| 50 | $\dfrac{\Gamma(k)}{(s+a)^k(s+b)^k}\ (k \geq 0)$ | $\sqrt{\pi}\left(\dfrac{t}{a-b}\right)^{k-(1/2)} e^{-(1/2)(a+b)t} I_{k-(1/2)}\left(\dfrac{a-b}{2}t\right)$ |
| 51 | $\dfrac{1}{(s+a)^{1/2}(s+b)^{3/2}}$ | $te^{-(1/2)(a+b)t}\left[I_0\left(\dfrac{a-b}{2}t\right) + I_1\left(\dfrac{a-b}{2}t\right)\right]$ |
| 52 | $\dfrac{\sqrt{s+2a} - \sqrt{s}}{\sqrt{s+2a} + \sqrt{s}}$ | $\dfrac{1}{t} e^{-at} I_1(at)$ |
| 53 | $\dfrac{(a-b)^k}{(\sqrt{s+a} + \sqrt{s+b})^{2k}}\ (k > 0)$ | $\dfrac{k}{t} e^{-(1/2)(a+b)t} I_k\left(\dfrac{a-b}{2}t\right)$ |
| 54 | $\dfrac{(\sqrt{s+a} + \sqrt{s})^{-2\nu}}{\sqrt{s}\sqrt{s+a}}\ (\nu > -1)$ | $\dfrac{1}{a^\nu} e^{-(1/2)(at)} I_\nu\left(\dfrac{1}{2}at\right)$ |
| 55 | $\dfrac{1}{\sqrt{s^2+a^2}}$ | $J_0(at)$ |
| 56 | $\dfrac{(\sqrt{s^2+a^2} - s)^\nu}{\sqrt{s^2+a^2}}\ (\nu > -1)$ | $a^\nu J_\nu(at)$ |
| 57 | $\dfrac{1}{(s^2+a^2)^k}\ (k > 0)$ | $\dfrac{\sqrt{\pi}}{\Gamma(k)}\left(\dfrac{t}{2a}\right)^{k-(1/2)} J_{k-(1/2)}(at)$ |
| 58 | $(\sqrt{s^2+a^2} - s)^k (k > 0)$ | $\dfrac{ka^k}{t} J_k(at)$ |
| 59 | $\dfrac{(s - \sqrt{s^2-a^2})^\nu}{\sqrt{s^2-a^2}}\ (\nu > -1)$ | $a^\nu I_\nu(at)$ |
| 60 | $\dfrac{1}{(s^2-a^2)^k}\ (k > 0)$ | $\dfrac{\sqrt{\pi}}{\Gamma(k)}\left(\dfrac{t}{2a}\right)^{k-(1/2)} I_{k-(1/2)}(at)$ |
| 61 | $\dfrac{e^{-ks}}{s}$ | $S_k(t) = \begin{cases} 0 \text{ when } 0 < t < k \\ 1 \text{ when } t > k \end{cases}$ |
| 62 | $\dfrac{e^{-ks}}{s^2}$ | $\begin{cases} 0 \text{ when } 0 < t < k \\ t - k \text{ when } t > k \end{cases}$ |
| 63 | $\dfrac{e^{-ks}}{s^\mu}\ (\mu > 0)$ | $\begin{cases} 0 & \text{when } 0 < t < k \\ \dfrac{(t-k)^{\mu-1}}{\Gamma(\mu)} & \text{when } t > k \end{cases}$ |
| 64 | $\dfrac{1 - e^{-ks}}{s}$ | $\begin{cases} 1 \text{ when } 0 < t < k \\ 0 \text{ when } t > k \end{cases}$ |

*$I_n(x) = i^{-n} J_n(ix)$, where $J_n$ is Bessel's function of the first kind.

## LAPLACE TRANSFORMS (Continued)

| | $f(s)$ | $F(t)$ |
|---|---|---|
| 65 | $\dfrac{1}{s(1-e^{-ks})} = \dfrac{1+\coth \frac{1}{2}ks}{2s}$ | $S(k,t) = n$ when $(n-1)k < t < nk \ (n = 1, 2, \ldots)$ |
| 66 | $\dfrac{1}{s(e^{ks}-a)}$ | $\begin{cases} 0 \text{ when } 0 < t < k \\ 1 + a + a^2 + \cdots + a^{n-1} \\ \quad \text{when } nk < t < (n+1)k \ (n = 1, 2, \ldots) \end{cases}$ |
| 67 | $\dfrac{1}{s} \tanh ks$ | $M(2k,t) = (-1)^{n-1}$ when $2k(n-1) < t < 2kn$ $(n = 1, 2, \ldots)$ |
| 68 | $\dfrac{1}{s(1+e^{-ks})}$ | $\dfrac{1}{2} M(k,t) + \dfrac{1}{2} = \dfrac{1-(-1)^n}{2}$ when $(n-1)k < t < nk$ |
| 69* | $\dfrac{1}{s^2} \tanh ks$ | $H(2k,t)$ |
| 70 | $\dfrac{1}{s \sinh ks}$ | $2S(2k, t+k) - 2 = 2(n-1)$ when $(2n-3)k < t < (2n-1)k \quad (t > 0)$ |
| 71 | $\dfrac{1}{s \cosh ks}$ | $M(2k, t+3k) + 1 = 1 + (-1)^n$ when $(2n-3)k < t < (2n-1)k \quad (t > 0)$ |
| 72 | $\dfrac{1}{s} \coth ks$ | $2S(2k,t) - 1 = 2n - 1$ when $2k(n-1) < t < 2kn$ |
| 73 | $\dfrac{k}{s^2+k^2} \coth \dfrac{\pi s}{2k}$ | $\lvert \sin kt \rvert$ |
| 74 | $\dfrac{1}{(s^2+1)(1-e^{-\pi s})}$ | $\begin{cases} \sin t \text{ when } (2n-2)\pi < t < (2n-1)\pi \\ 0 \quad \text{when } (2n-1)\pi < t < 2n\pi \end{cases}$ |
| 75 | $\dfrac{1}{s} e^{-k/s}$ | $J_0(2\sqrt{kt})$ |
| 76 | $\dfrac{1}{\sqrt{s}} e^{-k/s}$ | $\dfrac{1}{\sqrt{\pi t}} \cos 2\sqrt{kt}$ |
| 77 | $\dfrac{1}{\sqrt{s}} e^{k/s}$ | $\dfrac{1}{\sqrt{\pi t}} \cosh 2\sqrt{kt}$ |
| 78 | $\dfrac{1}{s^{3/2}} e^{-k/s}$ | $\dfrac{1}{\sqrt{\pi k}} \sin 2\sqrt{kt}$ |
| 79 | $\dfrac{1}{s^{3/2}} e^{k/s}$ | $\dfrac{1}{\sqrt{\pi k}} \sinh 2\sqrt{kt}$ |
| 80 | $\dfrac{1}{s^\mu} e^{-k/s} (\mu > 0)$ | $\left(\dfrac{t}{k}\right)^{(\mu-1)/2} J_{\mu-1}(2\sqrt{kt})$ |

*$H(2k,t) = k + (r-k)(-1)^n$ where $t = 2kn + r; 0 \leq r < 2k; n = 0, 1, 2, \ldots.$

**LAPLACE TRANSFORMS (Continued)**

| | $f(s)$ | $F(t)$ |
|---|---|---|
| 81 | $\frac{1}{s^{\mu}} e^{k/s} (\mu > 0)$ | $\left(\frac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}(2\sqrt{kt})$ |
| 82 | $e^{-k\sqrt{s}} (k > 0)$ | $\frac{k}{2\sqrt{\pi t^3}} \exp\left(-\frac{k^2}{4t}\right)$ |
| 83 | $\frac{1}{s} e^{-k\sqrt{s}} (k \geqq 0)$ | $\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$ |
| 84 | $\frac{1}{\sqrt{s}} e^{-k\sqrt{s}} (k \geqq 0)$ | $\frac{1}{\sqrt{\pi t}} \exp\left(-\frac{k^2}{4t}\right)$ |
| 85 | $s^{-3/2} e^{-k\sqrt{s}} (k \geqq 0)$ | $2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{k^2}{4t}\right) - k \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$ |
| 86 | $\frac{ae^{-k\sqrt{s}}}{s(a+\sqrt{s})} (k \geqq 0)$ | $-e^{ak} e^{a^2 t} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$ |
| 87 | $\frac{e^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})} (k \geqq 0)$ | $e^{ak} e^{a^2 t} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right)$ |
| 88 | $\frac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}}$ | $\begin{cases} 0 & \text{when } 0 < t < k \\ e^{-(1/2)(at)} I_0(\frac{1}{2}a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$ |
| 89 | $\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$ | $\begin{cases} 0 & \text{when } 0 < t < k \\ J_0(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$ |
| 90 | $\frac{e^{-k\sqrt{s^2-a^2}}}{\sqrt{s^2-a^2}}$ | $\begin{cases} 0 & \text{when } 0 < t < k \\ I_0(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$ |
| 91 | $\frac{e^{-k(\sqrt{s^2+a^2}-s)}}{\sqrt{s^2+a^2}} (k \geqq 0)$ | $J_0(a\sqrt{t^2+2kt})$ |
| 92 | $e^{-ks} - e^{-k\sqrt{s^2+a^2}}$ | $\begin{cases} 0 & \text{when } 0 < t < k \\ \frac{ak}{\sqrt{t^2-k^2}} J_1(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$ |
| 93 | $e^{-k\sqrt{s^2+a^2}} - e^{-ks}$ | $\begin{cases} 0 & \text{when } 0 < t < k \\ \frac{ak}{\sqrt{t^2-k^2}} I_1(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$ |
| 94 | $\frac{a^{\nu} e^{-k\sqrt{s^2-a^2}}}{\sqrt{s^2+a^2}(\sqrt{s^2+a^2}+s)^{\nu}}$ $(\nu > -1)$ | $\begin{cases} 0 & \text{when } 0 < t < k \\ \left(\frac{t-k}{t+k}\right)^{(1/2)\nu} J_{\nu}(a\sqrt{t^2-k^2}) & \text{when } t > k \end{cases}$ |
| 95 | $\frac{1}{s} \log s$ | $\Gamma'(1) - \log t \, [\Gamma'(1) = -0.5772]$ |
| 96 | $\frac{1}{s^k} \log s \, (k > 0)$ | $t^{k-1} \left\{ \frac{\Gamma'(k)}{[\Gamma(k)]^2} - \frac{\log t}{\Gamma(k)} \right\}$ |
| 97 | $\frac{\log s}{s-a} (a > 0)$ | $e^{at}[\log a - \operatorname{Ei}(-at)]$ |

## LAPLACE TRANSFORMS (Continued)

| | $f(s)$ | $F(t)$ |
|---|---|---|
| 98 | $\dfrac{\log s}{s^2+1}$ | $\cos t\,\mathrm{Si}(t) - \sin t\,\mathrm{Ci}(t)$ |
| 99 | $\dfrac{s\log s}{s^2+1}$ | $-\sin t\,\mathrm{Si}(t) - \cos t\,\mathrm{Ci}(t)$ |
| 100 | $\dfrac{1}{s}\log(1+ks)(k>0)$ | $-\mathrm{Ei}\left(-\dfrac{t}{k}\right)$ |
| 101 | $\log\dfrac{s-a}{s-b}$ | $\dfrac{1}{t}(e^{bt}-e^{at})$ |
| 102 | $\dfrac{1}{s}\log(1+k^2s^2)$ | $-2\mathrm{Ci}\left(\dfrac{t}{k}\right)$ |
| 103 | $\dfrac{1}{s}\log(s^2+a^2)\quad(a>0)$ | $2\log a - 2\mathrm{Ci}(at)$ |
| 104 | $\dfrac{1}{s^2}\log(s^2+a^2)\quad(a>0)$ | $\dfrac{2}{a}[at\log a + \sin at - at\,\mathrm{Ci}(at)]$ |
| 105 | $\log\dfrac{s^2+a^2}{s^2}$ | $\dfrac{2}{t}(1-\cos at)$ |
| 106 | $\log\dfrac{s^2-a^2}{s^2}$ | $\dfrac{2}{t}(1-\cosh at)$ |
| 107 | $\arctan\dfrac{k}{s}$ | $\dfrac{1}{t}\sin kt$ |
| 108 | $\dfrac{1}{s}\arctan\dfrac{k}{s}$ | $\mathrm{Si}(kt)$ |
| 109 | $e^{k^2s^2}\operatorname{erfc}(ks)\quad(k>0)$ | $\dfrac{1}{k\sqrt{\pi}}\exp\left(-\dfrac{t^2}{4k^2}\right)$ |
| 110 | $\dfrac{1}{s}e^{k^2s^2}\operatorname{erfc}(ks)\quad(k>0)$ | $\operatorname{erf}\left(\dfrac{t}{2k}\right)$ |
| 111 | $e^{ks}\operatorname{erfc}(\sqrt{ks})\quad(k>0)$ | $\dfrac{\sqrt{k}}{\pi\sqrt{t}(t+k)}$ |
| 112 | $\dfrac{1}{\sqrt{s}}\operatorname{erfc}(\sqrt{ks})$ | $\begin{cases}0 & \text{when } 0<t<k\\ (\pi t)^{-1/2} & \text{when } t>k\end{cases}$ |
| 113 | $\dfrac{1}{\sqrt{s}}e^{ks}\operatorname{erfc}(\sqrt{ks})(k>0)$ | $\dfrac{1}{\sqrt{\pi(t+k)}}$ |
| 114 | $\operatorname{erf}\left(\dfrac{k}{\sqrt{s}}\right)$ | $\dfrac{1}{\pi t}\sin(2k\sqrt{t})$ |
| 115 | $\dfrac{1}{\sqrt{s}}e^{k^2/s}\operatorname{erfc}\left(\dfrac{k}{\sqrt{s}}\right)$ | $\dfrac{1}{\sqrt{\pi t}}e^{-2k\sqrt{t}}$ |

## LAPLACE TRANSFORMS (Continued)

| | $f(s)$ | $F(t)$ |
|---|---|---|
| 115.1 | $-e^{as}\,\mathrm{Ei}(-as)$ | $\dfrac{1}{t+a}$; $(a>0)$ |
| 115.2 | $\dfrac{1}{a} + se^{as}\,\mathrm{Ei}(-as)$ | $\dfrac{1}{(t+a)^2}$; $(a>0)$ |
| 115.3 | $\left[\dfrac{\pi}{2} - \mathrm{Si}(s)\right]\cos s + \mathrm{Ci}(s)\sin s$ | $\dfrac{1}{t^2+1}$ |
| 116* | $K_0(ks)$ | $\begin{cases} 0 & \text{when } 0<t<k \\ (t^2-k^2)^{-1/2} & \text{when } t>k \end{cases}$ |
| 117 | $K_0(k\sqrt{s})$ | $\dfrac{1}{2t}\exp\left(-\dfrac{k^2}{4t}\right)$ |
| 118 | $\dfrac{1}{s}e^{ks}K_1(ks)$ | $\dfrac{1}{k}\sqrt{t(t+2k)}$ |
| 119 | $\dfrac{1}{\sqrt{s}}K_1(k\sqrt{s})$ | $\dfrac{1}{k}\exp\left(-\dfrac{k^2}{4t}\right)$ |
| 120 | $\dfrac{1}{\sqrt{s}}e^{k/s}K_0\left(\dfrac{k}{s}\right)$ | $\dfrac{2}{\sqrt{\pi t}}K_0(2\sqrt{2kt})$ |
| 121 | $\pi e^{-ks}I_0(ks)$ | $\begin{cases} [t(2k-t)]^{-1/2} & \text{when } 0<t<2k \\ 0 & \text{when } t>2k \end{cases}$ |
| 122** | $e^{-ks}I_1(ks)$ | $\begin{cases} \dfrac{k-t}{\pi k\sqrt{t(2k-t)}} & \text{when } 0<t<2k \\ 0 & \text{when } t>2k \end{cases}$ |

*$K_n(x)$ is Bessel's function of the second kind for the imaginary argument.

**Several additional transforms, especially those involving other Bessel functions, can be found in the tables by G. A. Campbell and R. M. Foster, "Fourier Integrals for Practical Applications", or "Vol. 1, Bateman Manuscript Project, Transform Tables, McGraw-Hill, 1955", or N. W. McLachlan and P. Humbert, "Formulaire pour le calcul symbolique". In the tables by Campbell and Foster, only those entries containing the condition $0 < g$ or $k < g$, where $g$ is our $t$, are Laplace transforms.