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Analog Fractional Order Controller in Temperature and Motor Control Applications

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Abstract: An analog fractional order PI$^{\lambda}$ controller, using a circuit element with fractional order impedance, a Fractor (patent pending), is demonstrated in both a simple temperature control application and a more complex motor controller. The performance improvement over a standard PI controller was notable in both reduction of overshoot and decreased time to stable temperature, while retaining long term stability. In the motor controller, set point accuracy was considerably improved over conventional control. The modification of the standard controller to a fractional order controller was as simple as replacing the integrator capacitor with a Fractor. Mixing (i.e., hybridization) of digital and analog control was demonstrated.

Keywords: Fractional order control, temperature control, robotic control, fractional order impedance

1. INTRODUCTION

Significant work has been focused on the fact that dynamic properties of materials, particularly electrical impedance, can often be described with high accuracy using non-integer order impedance descriptions (Macdonald, 1987). The Warburg impedance, with the impedance varying as the square-root of frequency, was originally described at the end of the 19^{th} century (Warburg, 1899). The non-integer order impedance spectroscopic description has been tied to fractional order calculus time domain dynamics by a number of researchers (see, e.g., Westerlund and Ekstam, 1994).

By reversing the point of view, from the fractional calculus describing the dynamic behavior of a material to viewing the material's dynamics as carrying out the fractional calculus operation, we can create broadband analog fractional order operators with a high degree of accuracy. The output of a fractional order integrator can be summed with the output of a standard proportional amplifier to implement the PI$^{\lambda}$ fractional order controller (FOC) described by Podlubny (1999) and others. The idea of using a non-integer order impedance to create a fractional order mathematical operator dates back at least as far as 1961 (Carlson and Halijak, 1961). Figure 1 is configured as a fractional order integrator.

Since thermal loads involve thermal diffusion, a half-order controller, $\lambda \approx 0.5$, represents a better match than an integer order control to the physics of the plant to be controlled.

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Figures 2–6, 8–9 appear in color online: http://jvc.sagepub.com

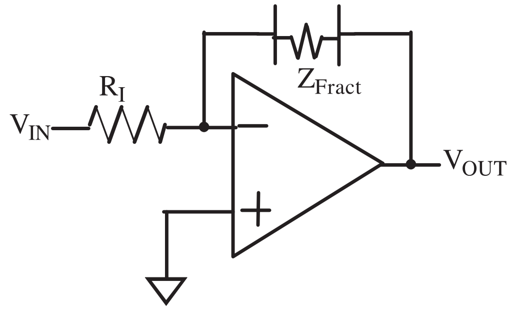


Figure 1. Schematic for a fractional order integrator. Z_{Fract} represents the Fractor element. The schematic symbol for the Fractor was designed to give the impression of a generalized Warburg impedance; a mixture of resistive and capacitive characteristics.

Earlier work (Petras and Vinagre, 2002) showed that a digital half-order controller was effective for temperature control, and suggested that the test should be conducted with an analog circuit. This project completed that demonstration.

For the robotics servomotor control application, with nonlinear "sticky friction" in the motor, a lower exponent value, $\lambda \approx 0.3$, was found to be near optimal to work through the nonlinearity and achieve a high accuracy in set-point tracking.

2. THE FRACTOR CIRCUIT ELEMENT

A Fractor is a two lead passive electronic circuit element similar to a resistor or capacitor, but exhibiting a non-integer order power-law impedance versus frequency. The term "fractance" has been used to describe fractional order impedance, consistent with the terms "resistance" and "capacitance". A fractance has a nearly constant loss ratio ("tan δ") over a large effective bandwidth.

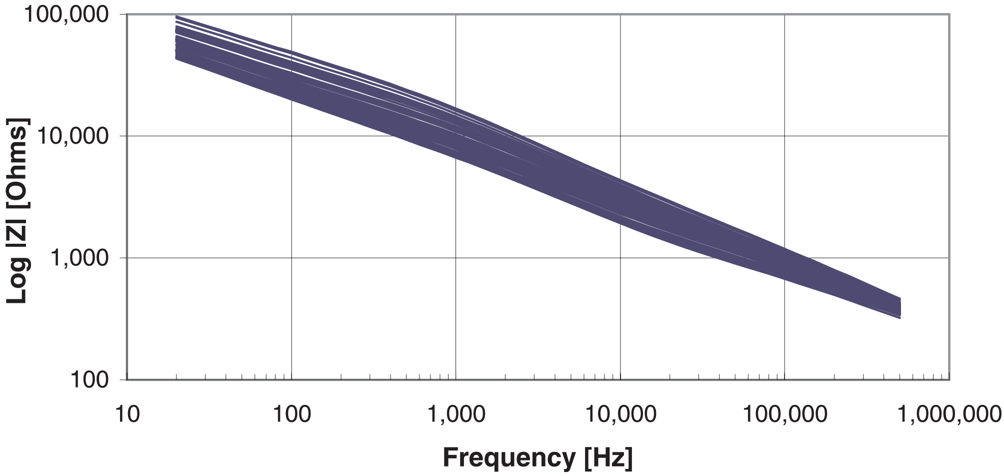
The impedance behavior is accurately modelled by the power-law form

$$Z_{Frac}(\omega) = \frac{K}{(j\omega\tau)^{\lambda}}, \tag{1}$$

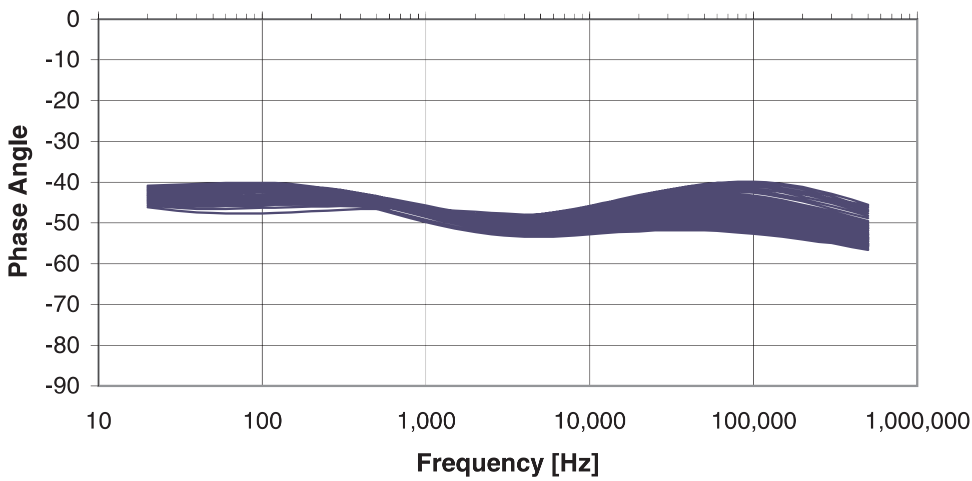
where K is the impedance magnitude (in Ohms, at a calibration frequency $\omega_C = 1/\tau$), and λ is the non-integer exponent, $0 < \lambda < 1$. The phase shift is related to the exponent by $\phi = -90^{\circ} \times \lambda$. The fractance description in equation 1 also includes ideal resistance and capacitance at the limits as $\lambda \rightarrow 0$ and $\lambda \rightarrow 1$, respectively.

Note that the impedance definition in equation (1) uses only real, integer order, units of Ohms, seconds, and Hertz. For actual circuit performance specification and design, this provides for a direct link between the impedance spectroscopic measurements and the ultimate device performance, without trying to determine the meaning of such descriptions as Farads to a fractional power (Westerlund and Ekstam, 1994).

Figure 2 shows the frequency response for the Fractor used in the temperature control demonstration. Additional measurements have shown that the fractance behavior with $\lambda \approx$



(a)



(b)

Figure 2. Spectral response of the Fractor used in the temperature control demonstration; (a) impedance magnitude and (b) impedance phase. The multiple lines show the variation over 26 impedance measurement scans.

0.5 adheres to the milli-Hertz regime, covering over seven decades of frequency. A Fractor with $\lambda \approx 0.3$ was used in the motor control demonstration.

No digital implementation has yet achieved this effective bandwidth. The phase ripple over frequency is no worse than that achieved through approximations. The prototype Fractor elements are currently made by hand, but are still not much larger than typical through-hole capacitors, 3.5 cm × 3.5 cm × 0.6 cm. It is thus feasible to implement fractional order control without the excessive space requirements of a network of integer-order analog elements, or the computing power requirements of a digital approximation. Tenriero Machado (1997) provided a useful description of some of these approximation techniques.

Haba et al. (2005) demonstrated that it is possible to create fractional order impedances in the 100 kHz to 10 GHz range by fabrication of a fractal structure on silicon. Fractance behavior has now been demonstrated over the frequency range $< 10^{-2}$ to $> 10^9$ Hz.

As with any circuit element, the Fractor has voltage and power dissipation limitations, as well as some other restrictions on operating environment. These are being actively investigated. Fractors developed to date do not contain lead or other hazardous substances currently covered under European RoHS standards.

3. THE FRACTIONAL ORDER INTEGRATOR

Analog fractional order operators can be designed using standard rules for operational amplifier circuits. The gain of an amplifier circuit, such as that shown in Figure 1, is represented in the frequency domain as the ratio of the feedback to input impedances:

$$G(\omega) = \frac{V_{OUT}}{V_{IN}} = -\frac{Z_{FB}(\omega)}{Z_{IN}(\omega)}. \tag{2}$$

In this instance, the feedback impedance is given by equation (1), and the input impedance is a resistance, $\mathrm{R_I}$, resulting in an overall gain of

$$G(\omega) = -\frac{\mathrm{K}}{\mathrm{R_I}} \frac{1}{(j\omega\tau)^{\lambda}}. \tag{3}$$

Rewriting this equation in terms of the Laplace variable, $s = j\omega$, we get

$$G(s) = -\frac{\mathrm{K}}{\mathrm{R_I}} \frac{1}{(s\tau)^{\lambda}}. \tag{4}$$

It is apparent that equation (4) has the form of the Laplace transform of a fractional order integrator of order λ (Oldham and Spanier, 1974). When summed with the output of a proportional amplifier, the result is a PI^{λ} controller. Since the summing amplifier is also typically inverting, positive polarity is restored without further circuitry.

While the impedance spectroscopy data is most useful in characterizing a Fractor, a more convincing demonstration of the power-law nature of the fractional order integrator is shown in the time domain. Figure 3 shows the response of an analog fractional order integrator with $\lambda \approx 0.5$ to a square wave input. The circuit shown in Figure 1 was followed by a unity gain inverter to restore positive polarity. The $\sqrt{t}$ shape is clearly evident in the output.

4. TEMPERATURE CONTROL

For the temperature control demonstration, a standard 5 Amp analog temperature controller (Wavelength Electronics MPT-5000) was modified by removing the 1 μF integrator capacitor and installing the Fractor characterized by Figure 2. No other changes were required.

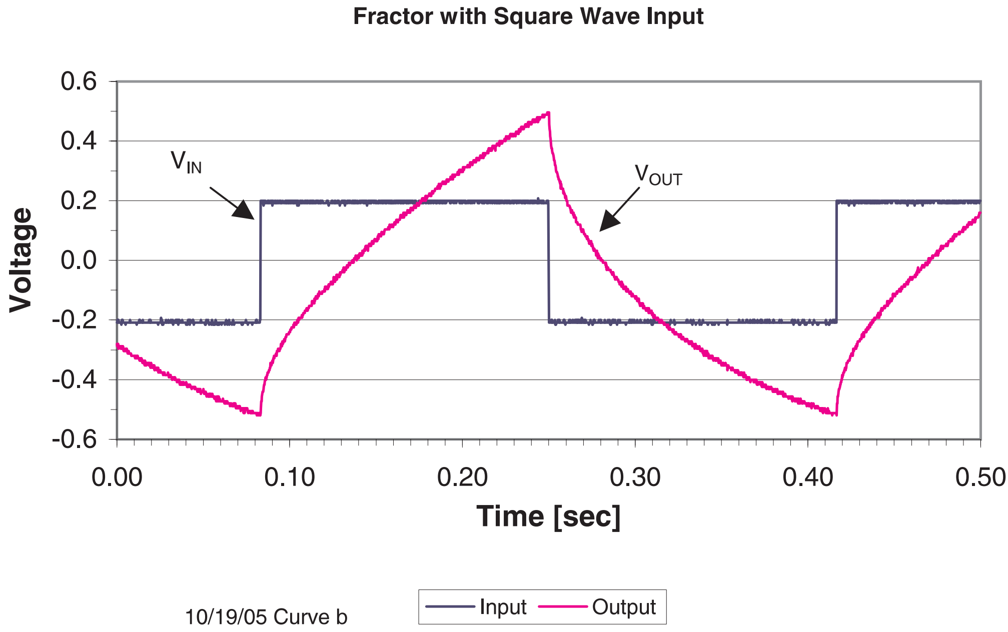


Figure 3. Time domain response of a half-order integrator using a Fractor with properties similar to those shown in Figure 2.

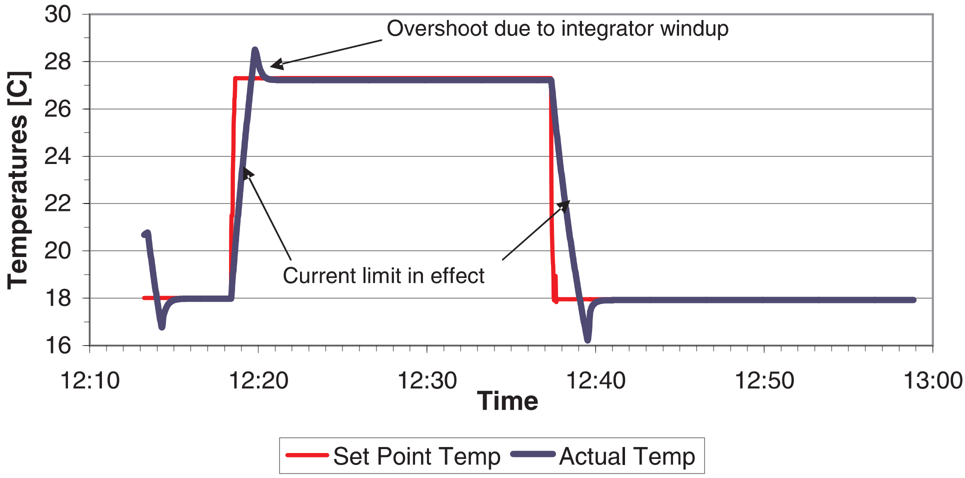
The thermal load (the test "plant") for this demonstration was a thermoelectric cooler (TEC) mounted on a 14 × 7.5 × 0.5 cm aluminum block. The system was driven to current limit, to demonstrate the nonlinear effect of actuator saturation.

A 10 kΩ negative temperature coefficient (NTC) thermistor was used as the temperature sensor. The MPT Series controllers provide a constant 100 μA excitation current for probing the thermistor resistance. Since the impedance of the sensor varies nearly exponentially with temperature, there is an inherent non-linearity in the feedback gain. The Steinhart-Hart relation was used to convert resistance to temperature.

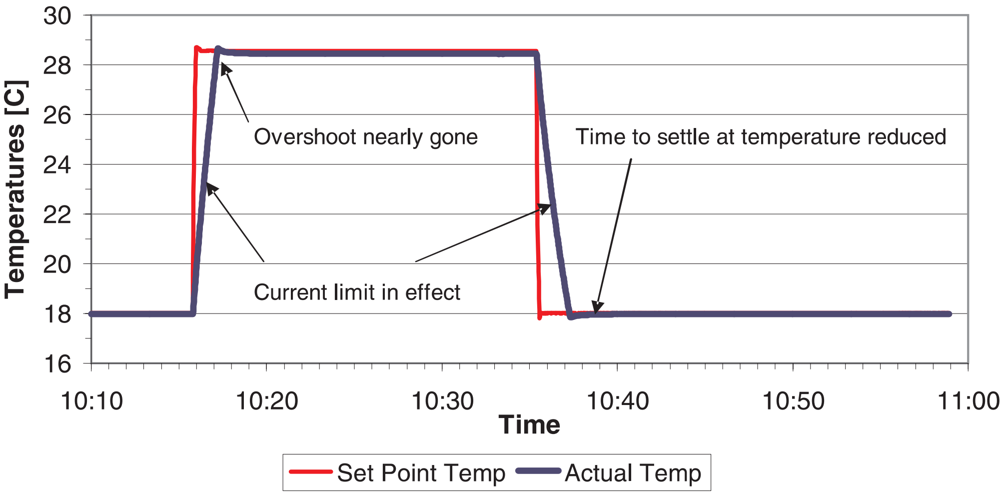
A comparison of the effectiveness of a conventional PI controller with that of a PI^λ controller, with $\lambda \approx 0.5$, is shown in Figures 4(a) and (b). Note that the overshoot is virtually eliminated. Time to stable temperature is reduced by a factor of almost three. The integrator windup effect due to the time spent in actuator saturation was virtually eliminated, without the extra complexity or expense of anti-windup circuitry.

5. ROBOTIC MOTION CONTROL

The rotary flexible joint (RFJ) "challenge" shown in Figure 5, made by Quanser^{TM}, was selected to demonstrate fractional order control in a servomotor control application. This more complex control project was aimed at demonstrating more "natural", or human-like, motion, such as would be required in prosthetic devices or other human augmentation systems. The



(a) Conventional PI.



(b) PI^{λ} control ($\lambda \approx 0.5$).

Figure 4. Temperature control results (a) with standard PI (b) with PI^{λ} control (b). x-axis is time of day in [HH:MM].

RFJ includes a servomotor which rotates the hub (base). On top of the hub is a passively hinged arm, coupled to the hub by two springs.

The overall experimental setup is shown in Figure 6. This configuration allowed for both real-time joystick tracking of human operator input, and step (or other programmed) response under PC control.

The control block diagram shown in Figure 7 was implemented in a "Fractroller" FOC instrument. Simulation showed that the D^{μ} term was unnecessary, so the test set was configured to allow for an integer order integral term in that part of the hardware. The state variables

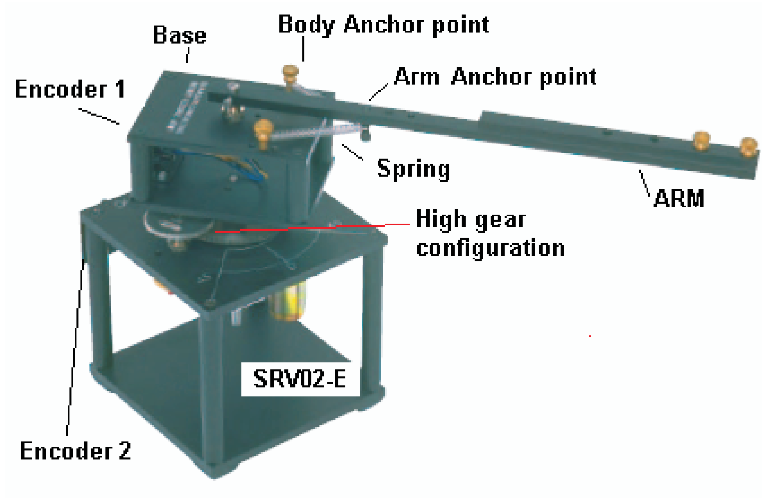


Figure 5. The rotary flexible joint used.

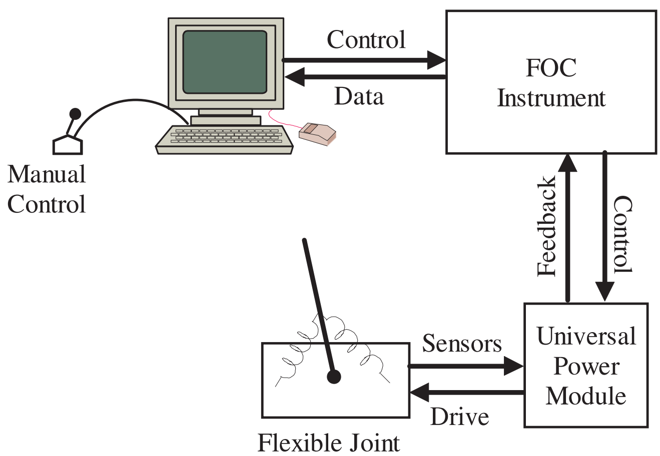


Figure 6. The overall experimental setup.

are the hub-to-stand angle Θ, the arm-to-hub relative angle Φ, and their time derivatives. The angles were measured with optical encoders and the time derivatives were computed with analog differentiators.

The maximum control gains were set by ratios of analog components. For example, the input impedance R_I was chosen using equation (4) to obtain the desired gain once the desired Fractor had been characterized. The gain values could then be adjusted by a factor in the range -1 to $+1$, using a computer interface to multiplying D/A converters. This allowed direct switching between fractional order and integer order control, simply by changing values in a dialog box on a PC monitor.

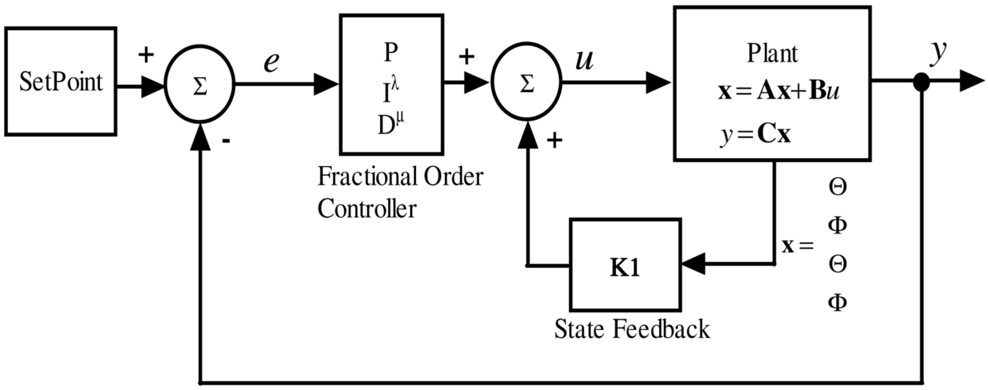


Figure 7. Control block diagram for the fractional order control of the rotary flexible joint using the "Fractroller" instrument. The total angle $y = \Theta + \Phi$.

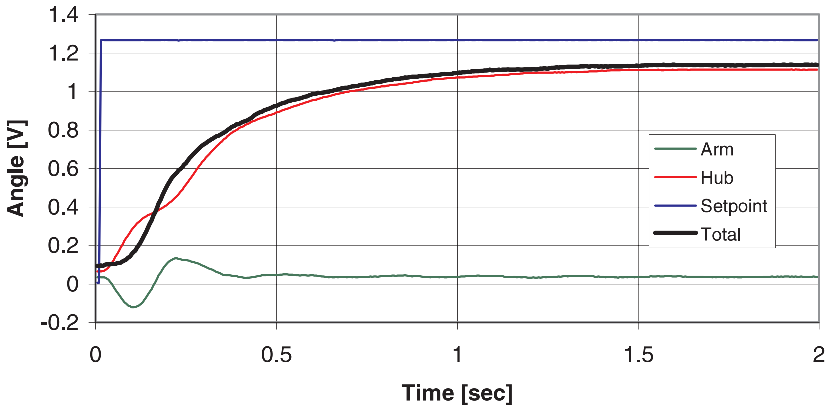
The state feedback **K1** block was implemented as proportional amplifiers. The **K1** values were set consistent with the manufacturer's suggestions, except that $\mathbf{K1}_1$ was set to zero, to avoid conflict with the set-point control.

For the robotic system, a Fractor with an exponent value of $\lambda \approx 0.3$ was used. Modelling indicated that if the servomotors had been purely linear, no integral term would have been required. However, the motors showed nonlinear "sticky" friction, with no torque output for voltages/currents below a certain threshold. This resulted in undershoot in purely proportional control, and an "overshoot and stick" behavior with integer order integral control. The I^{λ} term, with small exponent and relatively low gain, was just enough to "push" the arm through the nonlinearity, but also had rapid enough memory decay to avoid any need to overshoot in order to get the opposite polarity of error signal to "unwind" the integral effect.

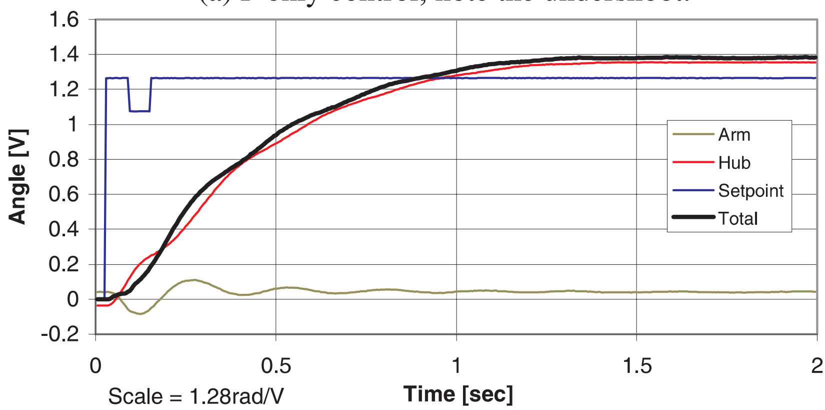
The results of P only, integer order PI, and PI^{λ} controls for the rotary flexible joint demonstration are shown in Figure 8. The curve labelled "Total" represents the angle of the arm tip with respect to the bench-top. Note that the arm lags behind the hub as the springs are stretched. With integer order integral control, the arm would begin to swing back towards the set-point after about 5 seconds. The I^{λ} term augments the P term earlier in the process, giving a faster response.

Increases in the proportional gain were limited by the inertia of the arm and the stretch of the springs. With integer order integral PI control, increasing the integer order gain resulted in oscillations around the set-point value, while decreasing the integral gain resulted in interesting sticking in the P-only undershoot, followed by slow drift toward the set-point after several seconds.

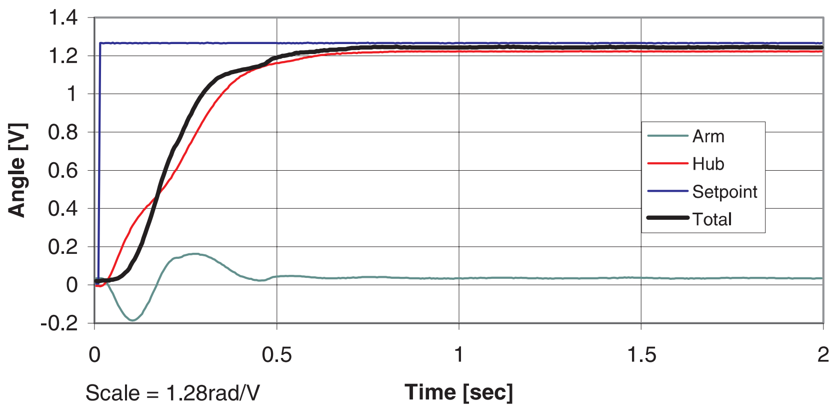
Attempts were made to cause the I^{λ} term to go into an "integrator windup" condition by holding the arm and base at various angles to cause large error signals. This presented a large load disturbance to the system. The arm consistently returned to set-point without overshoot or any evidence of resonant oscillation. The low order integrator appeared to be immune to this effect.



(a) P only control; note the undershoot.



(b) Conventional integer order PI; note the overshoot.



(c) Fractional order PI$^{\lambda}$ ($\lambda \approx 0.3$).

Figure 8. Comparison of results for (a) P only, (b) conventional PI, and (c) PI$^{\lambda}$ controllers for the rotary flexible joint.

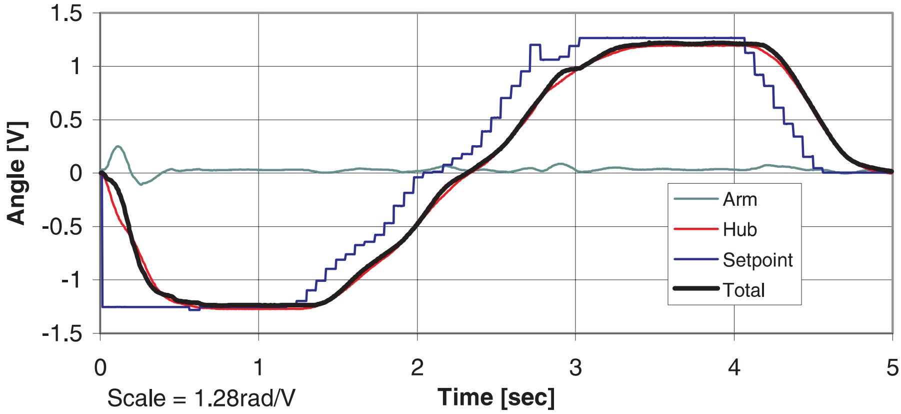


Figure 9. Joystick tracking performance of the robot arm.

Tracking of the joystick is shown in Figure 9. Note the smooth and faithful tracking in spite of the noisy (8 bit encoding with 60 msec sampling interval) input signal from the joystick. The tracking lag was less than 300 msec, and was only barely noticeable to experienced "gamers", used to high-speed on-line video games.

6. ONGOING RESEARCH AND FUTURE PROSPECTS

One of the Fractroller units has been transferred to Utah State University, for further experimentation; to develop auto-tuning mechanisms for selection of gain magnitude and phase (see, e.g., Chen et al., 2004), for example. Other uses for the Fractor suggest themselves, such as creating a hybrid analog/digital mathematical co-processor, offering the potential to increase computational throughput over digital only systems by many orders of magnitude while dramatically reducing power requirements. This idea has been demonstrated for conventional integer order mathematics (see, e.g., Cowen et al., 2005).

The memory disturbance effects outlined by Hartley and Lorenzo (2002) did not affect the performance of the temperature or motor controller. In actual practice, the startup transient was far less noticeable than with the integer order integrator. It seems paradoxical that the long-term memory implied in the fractional order operator is essential to its ability to deal so effectively with short term transients, as well as long term effects.

Much work remains to be done to improve the thermal, aging, and other characteristics of the Fractor devices. Additional assembly and packaging options for shrinking the size of the devices also need to be investigated.

As shown in the demonstration examples discussed here, different circumstances will dictate a variety of impedance magnitude, phase, and frequency band requirements. Requirements for additional fractance properties will evolve, guiding additional work into the electrochemistry of the devices. Compared with the similar solid-state electrochemical systems, such as batteries and capacitors, achieving the fractance properties of the prototype Fractors on first attempt is quite remarkable.

7. CONCLUSION

The two demonstrations described above show that fractional (non-integer) order control provides a degree of robustness against resonance and other instabilities that is difficult or even impossible to achieve with conventional control. Disturbance rejection was achieved without sacrificing set-point control. The ability of FOC to deal with nonlinearities, which had been predicted almost a half century ago (see, e.g., Manabe, 1960), was demonstrated here.

What is most remarkable about fractional order control using the fractance circuit element is how easy it was to design and construct the controllers. Appropriate phase margin was achieved over very broad frequency and time scales, without resorting to complex lead-lag compensation circuitry. The fractional order "operators" were implemented using otherwise standard components such as operational amplifiers, and A/D and D/A converters. For the most part, analog gain calculations were done graphically using impedance spectra like that shown in Figure 2.

The Fractor adds a very simple but useful device to the catalog of electronic elements available to the system designer, allowing for intuitive and straightforward design of fractional order controllers. Circuit configurations that would be impractical or impossible to implement with conventional devices may become commonplace.

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