True Analog Fractional Order Control

Small Business Innovation Research Final Report

Principal Investigator: Gary W. Bohannan, PhD

This small business innovation research (SBIR) Phase I project was completed under a 2006, National Science Foundation Grant, OII-0538866, True Analog Fractional Order Control. Previously, improved response in a temperature control application had been demonstrated by simply replacing an integrating capacitor with a fractor, the fractional order impedance device, in an otherwise conventional analog controller. In this demonstration project, the fractor enabled wide bandwidth fractional order control in a mixed analog/digital signal servomotor control environment. The primary goal was to determine if further development of the fractor devices was justified, i.e. was it straightforward to design a fractional order control system, did the available fractors support the design, and did the resulting controller produce significantly improved response? These questions were all unambiguously answered: Yes.

Unfortunately, there was inadequate investment interest to support a Phase II effort. Additional work will need to be done improve the production methods, and there remains a significant hurdle in getting general acceptance of the concepts of Fractional Order Control. These two efforts will probably depend on research and educational institutions that can look at the long term payoff rather than investment based industries that require rapid return on investment.

Introduction and objectives

The objective of this Small Business Innovation Research project was to design and build a controller for a flexible robotic joint using a new class of electronic circuit element that allows implementation of true fractional order control (FOC) – the fractor. The feasibility demonstration focused on the control of robotic devices requiring human-compatible response in order to augment human activity. Applications include prosthetics, powered wheelchairs, remote robotics, and human exoskeletons. The technology also readily applies to hard disc drives and many other light and flexible systems, offering a broad commercial market for the emerging technology. By introducing a new control paradigm with an analog fractional order controller, human-compatible, and human-like, motion behavior becomes possible – implemented with simple, intuitive circuit elements.

The fractor, shown in Figure 1, is a simple, two-lead electronic component in the category of resistor, capacitor, or inductor. Currently made by hand, the typical unit is 3.5 cm on a side and 1.0 cm thick. Using more sophisticated manufacturing techniques, the devices can be reduced in size to that of comparable tantalum capacitors.

Fractors exhibit fractional order impedance, or "fractance," given by

$$Z_F(\omega) \cong K \big/ (j\tau\omega)^{\alpha}\,, \tag{1}$$

where K is the impedance magnitude at a calibration frequency $\omega_0 = 1/\tau$, $j = \sqrt{-1}$ the imaginary radix, and α is non-integer value $0 < \alpha < 1$. The impedance phase shift, ϕ, is related to α by $\phi = -90° * \alpha$. The term "Constant Phase Element" (or "CPE") has also been used.

Figure 2 shows how the fractor can be used to create a fractional order integrator. This allows for the creation of otherwise complex control circuits by simply replacing conventional resistors and capacitors with fractors. The "fractance" symbol was designed to give the impression of a mixture of resistive and capacitive attributes with the underlying impression of the generalized Warburg impedance.

The primary question addressed in this feasibility project was whether it was worth proceeding with further development of the fractor devices. The devices were adequate for purposes of addressing the engineering issues of whether it was possible to design systems using the device and whether the systems performed better and less expensively than with conventional components.

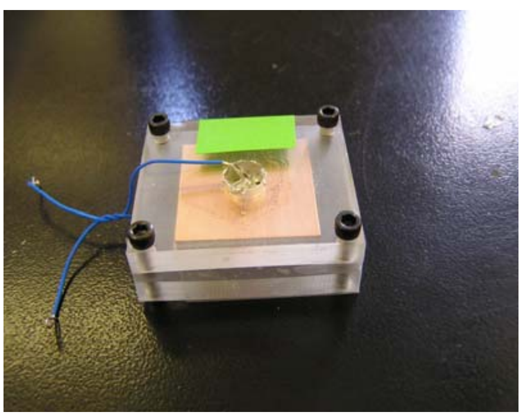


Figure 1. A sample hand-built prototype fractor.

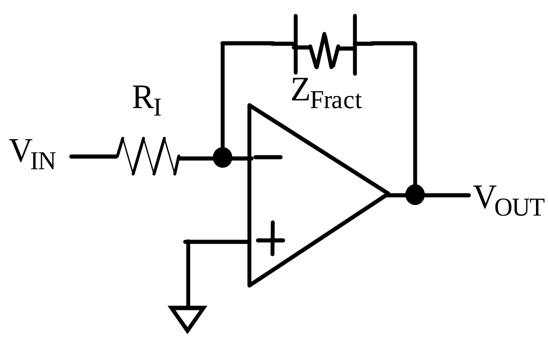


Figure 2. A fractional order integrator circuit using a fractor in the feedback path.

Technical Objectives

The key technical objectives of this research were to extend control theory to incorporate the more powerful and complete calculus of arbitrary order, the fractional calculus, and then apply that theory to the design and construction of a breadboard demonstration of a true fractional order controller for a flexible joint robot arm. The target plant presented the major challenges required to evaluate the technical and commercial feasibility of constructing a full-scale prototype of a true fractional order controller for the targeted industries.

1. Show that design and construction of a hybrid analog/digital fractional order controller is feasible.

- *Are adequate characterization techniques available to make use of the additional flexibility of the FOC? Can any plant be characterized for control via true FOC systems?*
- *Does the design process parallel or augment the current state-space design techniques?*
- *Are educational curricular changes evolutionary or revolutionary?*
- *How do the size, power requirements, bandwidth, and relative complexity of the FOC compare with state-of-the-art?*
- *How readily does it interface with sensors and actuators to affect a complete control loop?*
- *How readily does the fractor integrate into an otherwise digital system?*

2. Determine whether the available fractors support FOC for robotics.

- *Estimate the cost of laboratory fabrication of fractors. What production rate could be sustained in laboratory fabrication before external facilities are required?*
- *Evaluate the fractor characteristics. What are the phase and impedance magnitude required for ideal systems? What variation in parameters, e.g. phase ripple over frequency, can be tolerated within the control scheme?*
- *Will a single fractor adapt to the full range of environment required for a prosthetic? Will it replace the multiple control variable sets in hard disc drives? Will it adapt to changing drive requirements for wheelchair motors? Will it reduce the complexity and cost as promised?*

3. Determine if FOC actually produces the human-like motion behavior as promised.

- *Quantitatively, what improvements to a flexible joint or drive system are possible with true FOC? Is the maximum squared error in reaching a setpoint reduced? Compare to approximated FOC, neural networks, and adaptive-predictive control techniques.*
- *Evaluate the robustness of true FOC. Without changing control parameter settings, what is the range of arm length, weight, flexibility and speed that can still be controlled within the maximum squared error of a state-of-the-art control system?*
- *Quantify improvements in terms of over-shoot, mean-squared-error, time-to-settle, cost, power-savings, ease of design, and other criteria.*

Summary of results

The overall goal of the project was met. The fractance devices created for this project exhibited impedance phase and magnitude nearly perfectly suited to the design requirements for robotics applications. The question of whether additional investment in fractional order impedance devices is justified was clearly answered in the affirmative.

The most surprising aspect of this experience was that the construction of the instrument was that the incorporation of the fractor into the instrument was very straightforward. It was an evolutionary step of adding one more component to the resistor/capacitor/inductor catalog, placed in operational amplifier circuits in very much the same manner as with conventional components. What was substantially different was the thinking involved in deciding how to set the gain and phase. Conventional integer order design requires a delicate balance between the integral, proportional, derivative terms. This often leads to inefficient and "fragile" control systems with large gain values. With FOC, it became clear that this fragile balance is neither necessary nor appropriate. We were able to use "just enough" integral order and gain to get the steady state error to go to zero without causing overshoot or other instability while drastically reducing the gain values – in some cases to zero. The design approach, the thinking about the system, needed to change to accept a more general and encompassing view of the system.

The demonstration target plant

The target pilot plant for this feasibility study was the Rotary Flexible Joint (RFJ) module shown in Figure 3, provided by Utah State University, as part of the sub-award. The module, built by Quanser consists of a free arm (or "load") attached to the base (or "hub") with two identical springs. The base is mounted on a stand, which is clamped to a test bench. The base rotation angle is driven by the SRV02 load motor and gears.

Encoder 1 measures the relative angle of the arm with respect to the base. Encoder 2 measures the angle of the base with respect to the stand. The total arm rotation angle is the sum of the two angles.

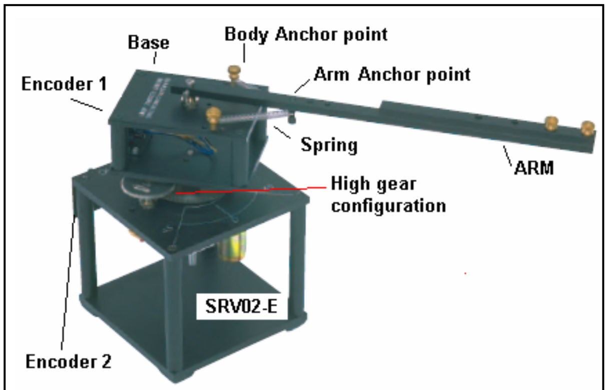


Figure 3. The rotary flexible joint test plant.

The plant allows variation of parameters of arm length and weight and strength of coupling at the upper joint via changeable springs with selectable mounting points. The plant is used extensively for control system education and is well characterized from a conventional point of view. The Quanser video clip of the response using their state space feedback control is fairly impressive and is available on their website.

The flexible joint roughly simulates the flexibility in tendons and muscles. The variable length and mass loading simulate the effects of lifting and moving objects with a prosthetic or augmentation device.

Summary of objective 1 results

The design and construction of a fractional order control was proven feasible; two "Fractroller" instruments were built. Early on, it was realized that the characterization and design rules for fractional order control were not as far along as the literature led us to believe at the time of writing the proposal for this project. For this reason, we decided to create a very general and flexible control instrument, the Fractroller, which would allow for adjustment of any and all control parameters. The instruments include plug-in option boards that allow for configurations to be changed rapidly, in a repeatable way, among conventional integer and fractional order schemes.

A major objective was to test the system in a human augmentation mode. To support this objective, we included a human interface option using a low-cost joystick as the control input. A control and display panel was implemented on the PC to allow for setting system gains as well as echoing the control and actual angle data back to the user. The block diagram for this setup is shown in Figure 4.

To allow analysis of the complete system and to aid in design decisions concerning maximum gain values and such, a numerical simulation package was created to extend state space theory using fractional order operators as opposed to just first order systems. This code turned out to be a major design tool, assisting with the selection of appropriate gain and phase terms for use in the final instrument. It can also be used as a training aid in introducing fractional order control.

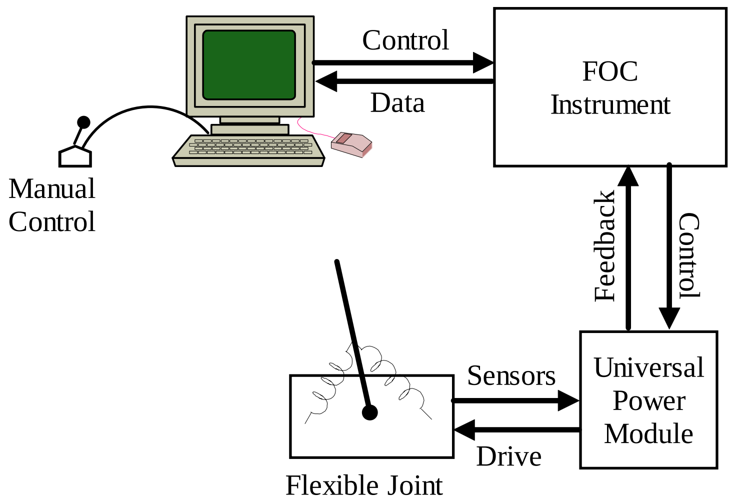


Figure 4. Block diagram for the Phase I demonstration.

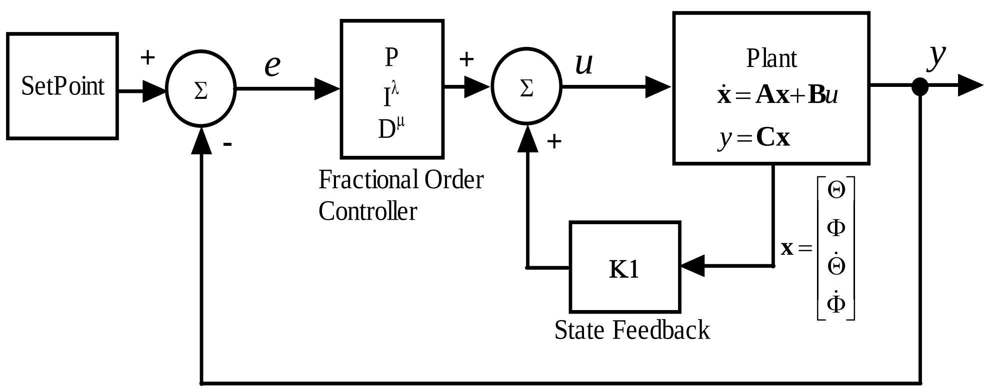


Figure 5. Inner and outer control loops. Θ is the base (hub) angle and Φ is the arm (load) angle with respect to the base. The total angle $y = \Theta + \Phi$. The exponents λ and μ are set by the fractors used in the circuit. Maximum gains are established by the analog components, with gain adjustments controlled by a digital microprocessor (mcu).

The Fractroller instrument implements the state space feedback inner loop configuration recommended by Quanser and adds an outer loop for setpoint control as shown in Figure 5. It extends conventional PID (Proportional plus Integral plus Derivative) to $PI^{\lambda}D^{\mu}$ by allowing non-integer λ and μ using fractors.

A fractional order operator is implemented as in Figure 2 using the standard gain calculation for operational amplifiers,

$$G(s) = \frac{Z_{Feedback}(s)}{Z_{input}(s)}, \tag{2}$$

where $s = j\omega$ is the Laplace transform variable. In the example of Figure 2,

$Z_{Feedback}(s) = Z_F(s) = K/(s\tau)^{\alpha}$ and $Z_{Input}(s) = R_I$, giving:

$$G(s) = \frac{K}{R_I}\frac{1}{(s\tau)^{\alpha}}. \tag{3}$$

Fractors are definitely not "leaky capacitors." Trying to use standard techniques for determining the impedance of fractors in series and parallel using the simple rules for capacitors does not work. The generalized rules for arbitrary impedances must be used:

$$\begin{aligned} Z_{PARALLEL}(s) &= Z_1(s)Z_2(s) \big/ \left(Z_1(s) + Z_2(s)\right) \\ Z_{SERIES}(s) &= Z_1(s) + Z_2(s) \end{aligned} \tag{4}$$

where the individual impedances are given by equation (1).

In fact, resistors and capacitors are only special cases of fractance, with exponents of 0 and 1 respectively, just as integer order derivatives and integrals are special cases of the fractional order operators.

The form of equation (3) is that of the Laplace transform of the α order integral. While the integral is often written in the form $1/s^{\alpha}$, this leads to inconsistency in physical units. The form of Equations (1) and (3) retains the physical units, including the time-scaling, while (3) reduces to a dimensionless magnitude and phase as required.

This basic building block is used to construct the fractional order filter (FOF) shown in Figure 6. The Multiplying Digital to Analog Converter (MDAC) allows for computer control of the gain. It is general enough to allow for integration or differentiation to any order by rearrangement of the placement of resistors, capacitors, and fractors. By mixing fractors with conventional elements, much more complex functions can be implemented.

It must be emphasized that the time scale parameter $\boldsymbol{\tau}$ is associated with the description of the fractor itself. The time scales associated with $R{\cdot}C$ products are due to interactions between elements. As can be seen, there is plenty of design room for adjusting the overall gain by the choice of the resistance values. The time scales can be adjusted by the choice of the capacitors. Note that the time units are in seconds and the gain is dimensionless, as mentioned above. The fractional order integrator, I^{λ}, and differentiator, D^{μ}, are easily implemented as special cases of the general fractional order filter. By summing FOF units, it is possible to smoothly shift between different filters and therefore to shift between orders of operators. This allows a degree of auto-tuning of both gain and phase of any control element if desired.

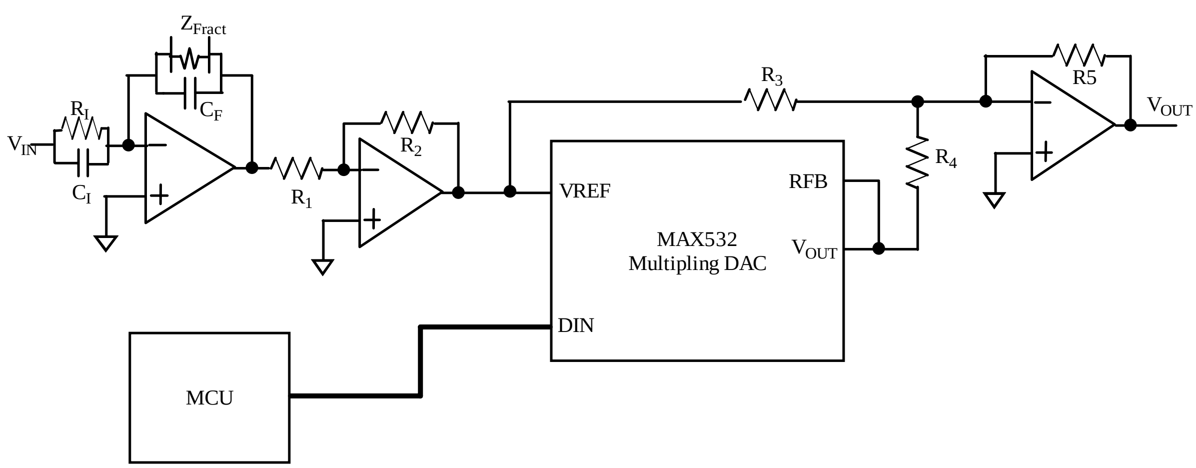
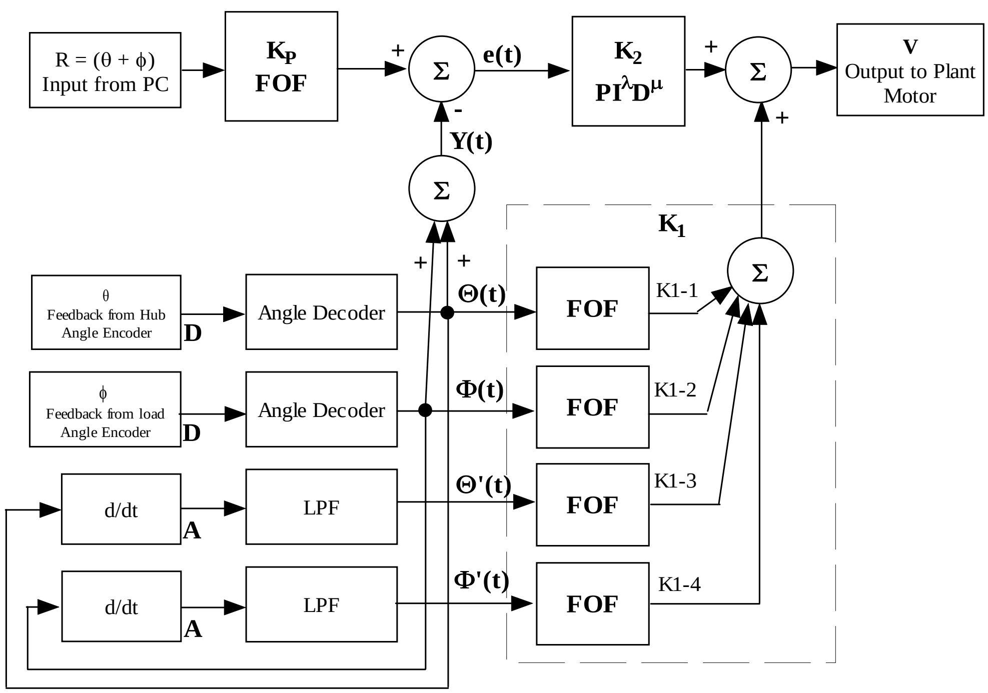


Figure 6. A generalized fractional order filter with gain controlled by a micro-processor unit (mcu). In this example, the circuit acts as a fractional order lead-lag compensator.

The final instrument configuration is shown in Figure 7. For initial testing, the FOF circuits in the $\mathbf{K_1}$ state feedback section were set to proportional amplifiers, implementing $\mathbf{K_1}$ as a set of constant multipliers. Due to the low resolution of the joystick (8 bit) sampled at a 60 ms cadence, derivatives of the error term were exceptionally noisy. Also, it was determined in the simulation analysis that any D^{μ} term was unnecessary. A conventional integrator term was implemented in this slot instead to allow easier shift between I and I^{λ} response by just setting the appropriate gains on the PC control panel.

As can be seen in Figures 6 and 7, the fractor has been blended into a completely mixed signal environment (analog/digital). They currently occupy little more space than a 3.3 µF metal film plate-through capacitor. They optimally operate at voltages below 2 V and dissipate no more than a few tens of micro-watts of power – on the order of the dissipation of a 10 kΩ resistor. The power-law characteristics of the devices used in this project covered the band from <10 mHz to >10 kHz, fully supporting all relevant time scales of robotics. By eliminating the need for anti-windup circuitry, the fractional order integrator actually proved to be less complex than conventional integer order. By off-loading the fractional order "computation" to the fractor, the demands on performance of the mcu were substantially reduced.



Note: for feedback signals, "**D**" represents a digitally encoded signal, "**A**" represents an analog (voltage) encoded signal.
"**FOF**" represents a Fractional Order Filter.
$\mathbf{PI^{\lambda}D^{\mu}}$ implemented as a set of three parallel Fractional Order Filters.
$\mathbf{K_P}$ is a possible input pre-filter.
V may also include amplifier to re-scale voltage for compatibility with the UPM.

Figure 7. A generalized state feedback controller with two angle and two rate measurements as states. Kp acts as an input filter to smooth out the D/A signal from the joystick. K_1 implements the state feedback. K_2 responds to the error in the total angle.

Simulation of the proposed complete system, using the state matrices provided by Quanser, and a fractional order PI$^{\lambda}$ controller operating on the total angle error signal, produced the prediction shown in Figure 8. It is interesting to note that the phase angle needed to be less than about 35° for optimal results. The low phase angle is consistent with the results of testing by the Utah State University team using a digital approximation of a fractional order controller with the test plant. The numerical stability of the digital approximation remains suspect, but the trend toward low exponent value was clear. Note that the simulation predicts an overshoot of approximately 1%. This did not show up in the actual system, as discussed below in the performance section.

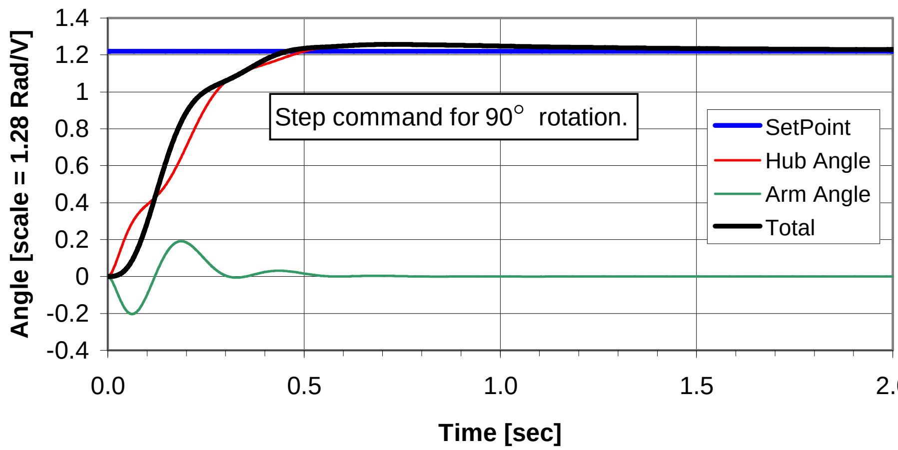


Figure 8. Predicted performance of the robotic arm using a fractional order integrator in the controller. The order of the integration was 0.27.

Once the gain values were estimated using the simulation, it was a straightforward task to determine the appropriate matching impedances for the fractance elements by graphical means using the impedance spectra scaled in radians per second. On a log-log scale, the point at which the impedance magnitude line crosses the $\omega = 1.0$ gridline is marked. From this point the desired nominal gain is determined by the ratio of that number to the resistance value. This is just a vertical distance on a log-log chart.

Designing with the fractor involved an intuitive evolutionary step beyond conventional methods. The simulation and analysis techniques kept track of actual physical units throughout the process, making selection of component values much easier at the end.

Summary of objective 2 results

The second objective of Phase I was to characterize the fractor and to determine the applicability of the existing devices for robotics applications. Construction of the fractors used in this demonstration project was done in the MSU Dept. of Chemistry and Biochemistry under a subaward.

The device characteristics for the units made so far have fallen roughly into one of two classes; those with phase shifts of approximately 45°- 65° and those with phase shifts of approximately 25-35°. These phase shifts are consistent (±10°) over the spectral range of 0.01 Hz to >10 kHz.

Impedance magnitude can be influenced somewhat by variation in the concentration of the components in the recipe. The greatest contribution to the impedance magnitude is due to the geometry of the construction, just as with capacitors and resistors; i.e. the size of the electrode plates and the distance between the electrode plates. The pressure applied to the plates also affects the impedance magnitude and, possibly, the phase. The pressure is currently adjusted by the torque on the four corner screws shown in

Figure 1. The electrolyte is sensitive to exposure to humidity, so after initial testing the edges of the devices are sealed.

Figure 9 shows an impedance spectroscopic scan of a fractor that appears nearly optimal for the frequency range of robotic systems. Variation among the fractors used in performance testing did not cause notable change in the system behavior. The phase "ripple" over frequency is less than that achieved by any discrete approximation so far demonstrated.

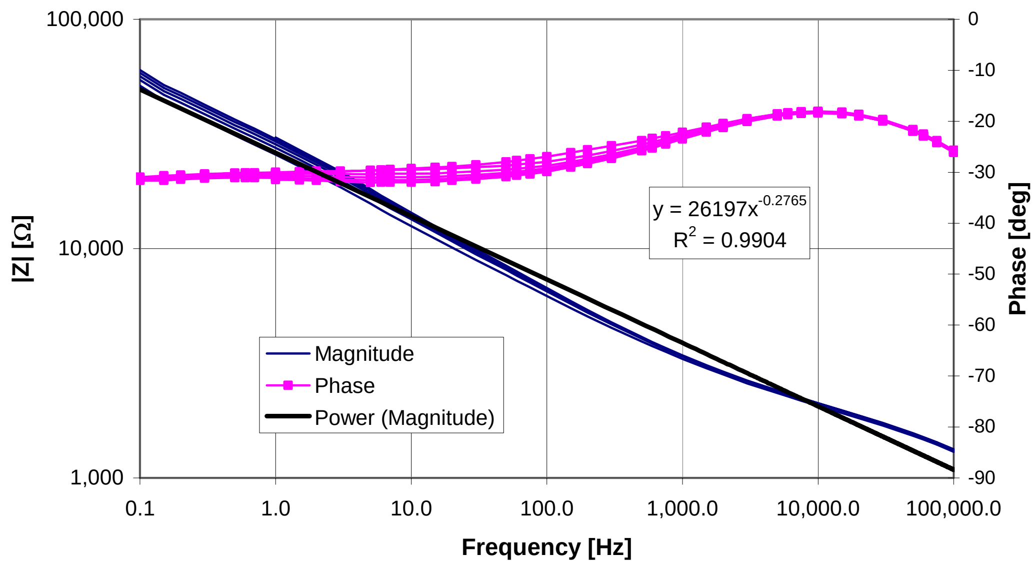


Figure 9. Impedance spectrum for a fractor with near optimal parameters for use with robotic systems.

The yield for the hand crafting of the devices was 100%, even on the first try by novices with little chemistry background. Actual material costs for each fractor are on the order of a few cents, although the manual preparation method is currently wasteful and tedious. The materials and processes comply with European Restriction on Hazardous Substances (RoHS) and all other material safety standards at the time of this writing.

Over fifty devices were made during the course of this project. What we have learned:

- The device parameters are repeatable with a consistency of medium grade capacitors: approximately -20% to +80%. Variation within a batch and between batches is about the same.
- The greatest variation in impedance characteristics is due to exposure of the electrolyte to humidity. This variation can be alleviated by sealing the unit. The impedance magnitude is affected by temperature, with the impedance being reduced at elevated temperature, i.e. increased carrier mobility, and possibly concentration, at higher temperatures.
- Each of the recipe constituents is necessary to the overall performance, although the performance is less sensitive to parameter variations than expected.
- Different phase angles can be produced. The complete set of steps, from electrode preparation to the timing of the electrolyte plating cycles, determines the final characteristics.
- Shelf-life is at least two years.

Conversion of the recipe to automated methods was investigated by a firm specializing in solid state device manufacturing. Their conclusion was that it would require "significant" effort, such as was required for production of Li-ion cells. Manual production of several dozen per month could be sustained pending conversion to more automated methods.

Phase II was to focus on more of the electrochemistry “what” and “why” questions left over in order to produce specific devices with specific characteristics for various applications. As with all solid state devices, the fractor will need to go through several evolutionary, and possibly revolutionary, stages before it is a fully marketable product. What has been shown is they can be produced with some consistency and that the impedance characteristics can be produced in the range of utility to robotic control applications.

Summary of objective 3 results

The third objective of Phase I was to determine if the use of the fractor and fractional order control actually offered measurable improvement in system response. The feasibility testing was to go beyond the scope of the manufacturer’s pedagogical demonstration, which was limited to the application of a step function change in arm angle. The intent here was to look at how it responded to human input. For example, could a human learn to set the arm angle at a certain point without inducing oscillations? What would the system do if the human operator’s hand shook, simulating a palsied condition of a wheelchair user? Figure 10 shows the graduate intern for the project with the Fractroller instrument integrated into the configuration of Figure 4. Figure 11 provides a top view of the Fractroller with the top cover removed showing the fractor mounted on the plug-in configuration board.

The system allowed recording of analog signals within the Fractroller via a multi-channel A/D converter in bench-top PC or notebook computer. Figure 12 is a recording of the setpoint signal along with the voltage representations of the hub (base), arm, and total angles. Figures 13 and 14 show the response of the test plant with the Fractroller configured as conventional PI and P-only configured control, respectively. Since the P gain was most responsible for fast initial response, this value was kept constant for all tests.

As can be seen in Figures 12-14 and summarized in Table 1, the PI^{λ} fractional order controller outperformed either of the two conventional control schemes of similar complexity. Considering that the Fractroller was built with ±1% resistors and there was no attempt to remove OpAmp offsets as would be done in a product instrument, the FOC settling offset is statistically indistinguishable from zero.

Table 1. Comparison of results

Type of controller	**Integrated squared error over 2 sec**	**% over/undershoot in first 2 sec**	**Time to stability < ± 10%**
PI^{λ} (FOC)	0.245	-1.7%	0.36 sec
PI	0.333	+10%	> 2 sec
P	0.339	-9.3%	1.56 sec

Figure 10. Graduate intern Calvin Coopmans with the Fractroller instrument.

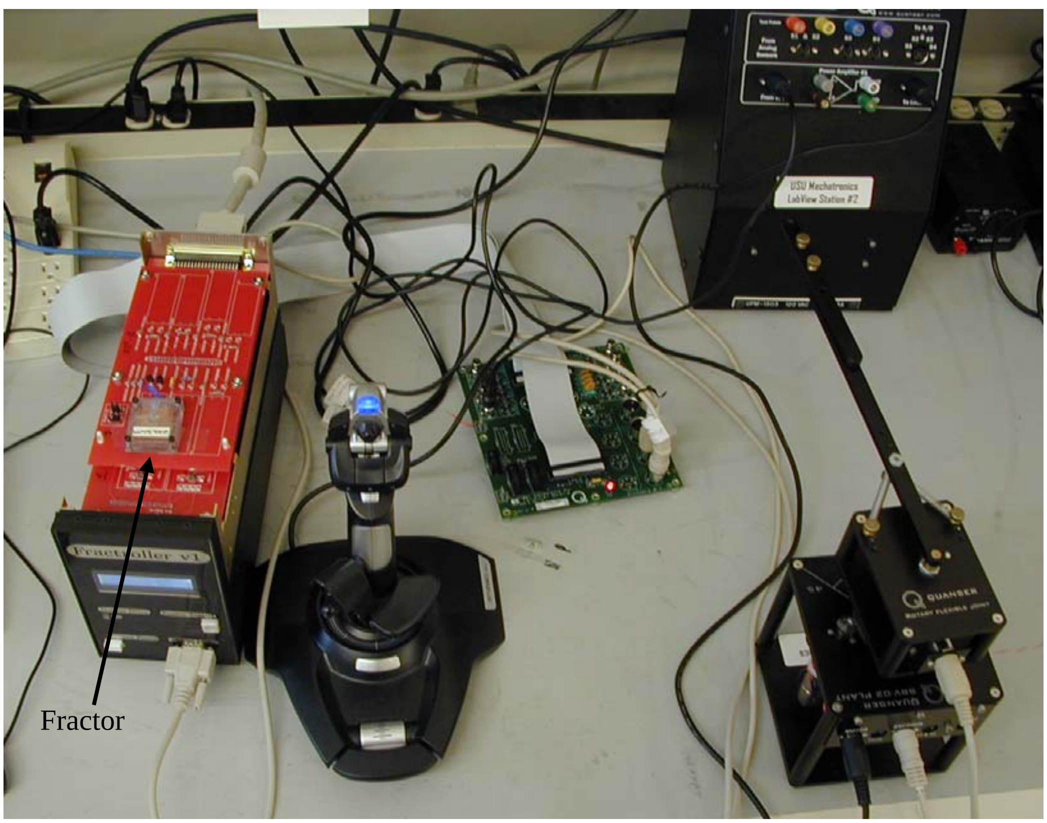


Figure 11. Top view of the Fractroller instrument and the test layout.

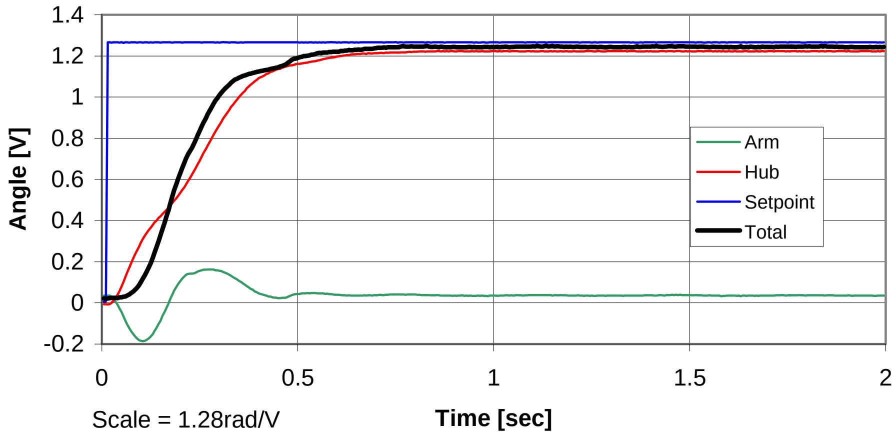


Figure 12. Recorded operation of the test plant with the Fractroller using the fractor with characteristics shown in figure 9.

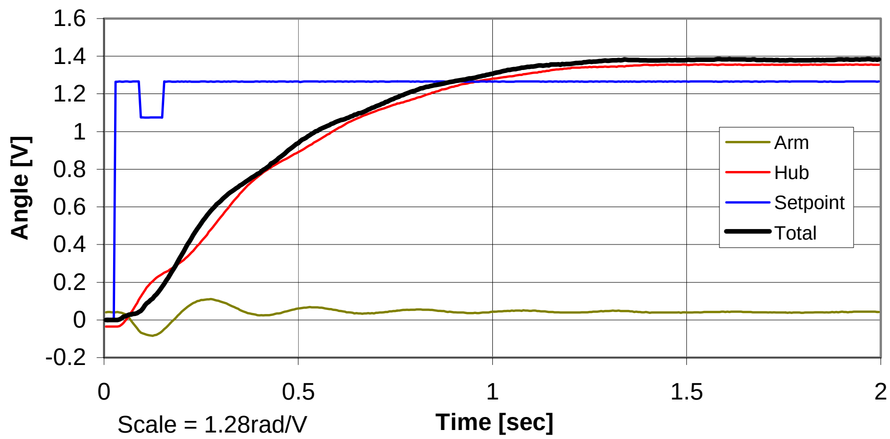


Figure 13. Recorded operation of the test plant with the Fractroller configured as a conventional PI circuit. Note the overshoot. The system recovers toward the set point after about 5 sec.

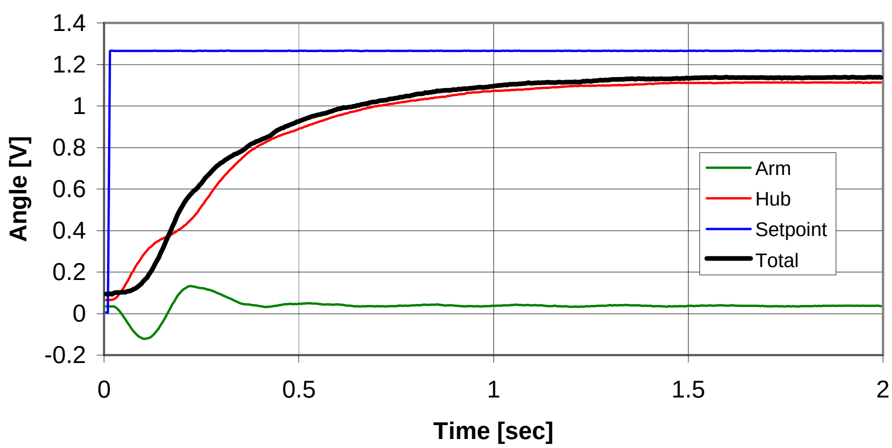


Figure 14. Recorded operation of the test plant with the Fractroller configured as a conventional P-only circuit. Note the undershoot.

As mentioned earlier, due to numerical stability problems, direct meaningful comparison with the digital approximation of FOC was not possible. In comparison with more advanced and complex control schemes, such as neural networks and model based predictive control, all of the others predict an initial overshoot and are limited by the physics of the arm/spring attachments for ultimate speed. As noted in recent texts on control theory, virtually all of the advanced control schemes reduce to PID at the core, wrapped with sophisticated automated gain adjustment algorithms, e.g. Fuzzy Logic Gain Scheduling. (See e.g. Karl J. Åström and Tore Hägglund, *Advanced PID Control*, ISA Press, 2006.) FOC provides a more robust core to any of these approaches, making the automated gain adjustment task less critical.

Figure 15 shows the plant tracking the joystick under fractional order control. As can be seen, the joystick used here was a low cost, low resolution (8 bit) device, sampled with a 60 ms cadence. Nonetheless, the plant tracked the joystick setpoint action with a few hundred millisecond lag. Additionally, an "open house" was held to allow students engaged in the FOC research group to try out the controller. A couple of parents also showed up (they hadn't had the youngsters' experience with using gaming joysticks). They all tried to get the arm to go unstable by rapidly moving the joystick, but none could. They changed the length and weight of the arm by varying the position of the extension. They even mounted the extension loosely with a single loose bolt, allowing the extension to spin freely. In each case, the arm settled at the setpoint. This demonstrates the robustness or the controller. The overwhelming and consistent response from all who tried the controller was: "Smooth!"

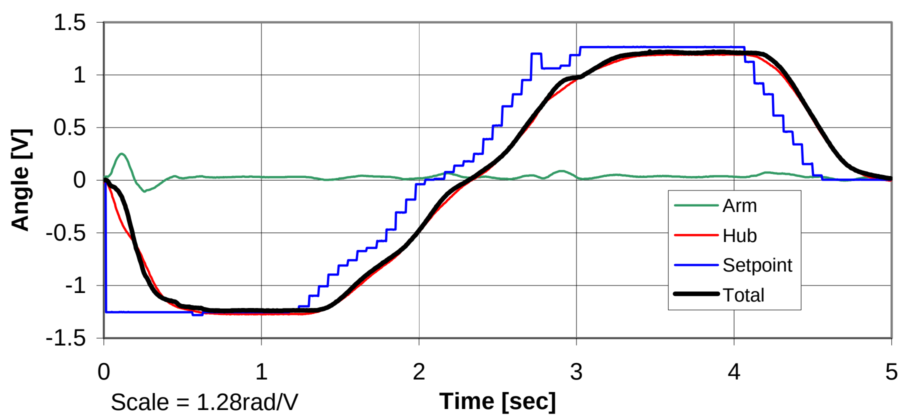


Figure 15. Recorded operation of the test plant with the Fractroller configured for FOC showing tracking of the joystick input.

In summary, the system was robust to all of the variation we could apply in the course of the two hour open house period.

As claimed by Quanser, the use of the state feedback did significantly reduce the oscillations in the arm at the end of the rotation travel. This was true with or without the fractional order control. The difference is in the time to settling and the steady state error. These were not readily apparent in the graphics and video clip provided by Quanser. What is clear is that FOC produces more accurate results without over- or undershooting the setpoint.

This ability of FOC to deal with nonlinear loads has been predicted repeatedly over the last half century and was proven here. Part of the explanation for the lack of any overshoot with the PI^{λ} FOC controller is due to what Quanser refers to as "sticky friction." This was traced to an apparent dead-band in the motor and drive gears. A minimum voltage/current is required to actually cause the motor to turn. The linear plant model predicts that a P-only system should reach setpoint exponentially due to the integrative nature of the load. In reality, once the error signal gets small, the P term is no longer large enough to drive the motor and the system gets “stuck.” Likewise, the PI controller sits in the overshoot condition until the integrator builds up enough integrated error to turn the motor on again to return toward setpoint. The I^{λ} term provides just enough added boost signal to the P term to push the system to very near the setpoint without the need to overshoot and “unwind.” In essence, by operating the arm at maximum speed in step response, Quanser’s demonstrations brush over the nonlinear effects that show up in direct tracking.

Given what we have learned here, it is likely that the low value exponent fractor will be applicable to a broad range of electric motor control challenges.

A mode of operation with no conventional equivalent involves setting the P term gain to zero and using only the I^{λ} term in the error controller. The result is a smooth, high efficiency “transport” motion. This is shown in Figure 16. How many other new modes with no conventional equivalent exist is an open question.

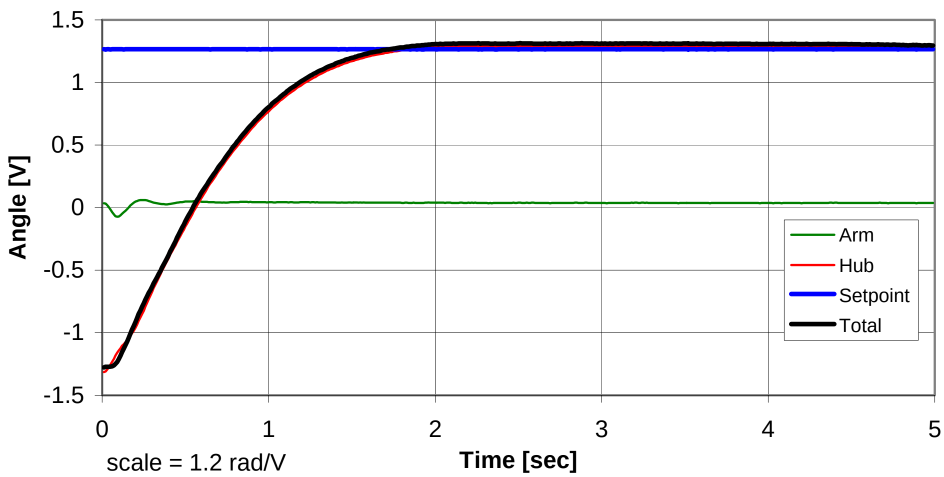


Figure 16. Recorded operation of the test plant with the Fractroller configured for FOC with the I^{λ} term only.

Several researchers in the area of Fractional Derivatives and their Applications (FDA) have commented that the extremely deep memory exhibited by the fractors may be a detriment, as the devices cannot be easily discharged or pre-charged as a capacitor might be. Actual experimental evidence indicates that the memory of previous control values does not act as a detrimental disturbance to future operation of control systems based on the fractor. In fact, the conventional integrator term in digital and analog systems needs to be reset prior to operation. This was not necessary with the I^{λ} term; the initial reset disturbance was barely noticeable with FOC, but very evident with an integer order integral.

The Utah State University group acted as the “alpha level” customer in this project. They were provided a sample set of fractors to see if they could use them in their own breadboard circuits and/or integrate them with a GP-6 tabletop analog computer to create their own prototype controller(s). This was done in parallel with design and construction of the feasibility demonstration instrument at WEI. The USU team dubbed their fractional order integrator as the “Fractogrator.” Their experience highlighted a number of misconceptions with fractance behavior that will need to be addressed in commercialization of the fractor. In particular, it was their early characterization of fractance as “leaky capacitance” that brought into question whether the devices obey the rules of mixing impedances. They do, in a general way, but not with the simplified rules of series and parallel resistances and capacitances. A simple, two page tutorial was developed to deal with this. In most cases, simple explanations with a few graphics were all it took to overcome any paradigm problems.

The USU group also discovered what appeared to be a non-linear saturation response in some of the early fractor units they worked with. This led to using a longer burn-in period after construction to ensure stability of the devices.

Conclusions of the Phase I Findings

Fractors offer a non-integer phase shift over five to seven (or more) decades of frequency with a single, two-lead circuit element. With the phase shift selectable by adjustment of process variables in manufacturing, a wide range of non-integer phase shifts become available. The term "fractance" now takes its place beside resistance, capacitance, and inductance. Fractance refers to a class of electronic devices characterized by phase angle, impedance magnitude, and frequency band. They have voltage and temperature limitations just as exhibited by capacitors, inductors, and other circuit elements.

The need for circuit elements with phase shifts other than integer multiples of 90° dates back at least to H. Bode's text of 1945, in which he described a 17 element R-L-C circuit to obtain the desired near constant phase over some frequency band. The fractor now fulfills that need; ironically, using ideas that were published in the 1940s. By providing very accurate power-law impedance over very wide bandwidth, it makes possible the construction of fractional order controllers with the robustness to fulfill the promise of these long dreamed of systems.

The fractional order derivatives and integrals of the mathematics underlying fractional order control are linear operators. Although the fractors show some nonlinearity in impedance magnitude due to changes in applied voltage, they are quite linear in their characteristics and are well described by fractional order operators. In this sense, they "carry out" fractional order mathematics. The impedance is time dependent, just as it is with capacitors. The difference between fractance and capacitance is that the dynamic relaxation follows a power-law relation rather than exponential. What distinguishes fractional order control from integer order conventional techniques is the natural ability of a linear process to deal with nonlinear effects such as backlash, actuator saturation, and "sticky friction" (as demonstrated here).

Fractional order control achieves robustness to system parameter variation due to the inherent improvement in phase margin over a very broad frequency range. This results in elimination of the need for competing integral and derivative terms normally found in phase compensation networks. The result is lower gain values and greater efficiency in the total system. This robustness was proven here by modifying gain values by factors of two with no discernable difference in response. However, when the gains were set to over three times that required for optimal response, the system did become unstable, to the point of destroying a set of springs.

In all, the design and component selection method followed conventional methods with evolutionary steps toward a more general approach. The difference is in the way one thinks about gains and time scales. It replaces mixing gains and "time constants" with a single procedure capable of covering any order of derivative or integral operator. The analysis and design process was helped along immensely by the generalized state space modeling software developed as part of this project.

The Fractroller units along with the simulation package were sent to Utah State University to support further research into factional order control. The general impression so far is that students grasp the concepts of FOC much faster than people with extensive experience.

The demonstrations achieved in this Phase I project exceeded many of the ambitious objectives set forth in the initial proposal. The research has highlighted specific issues with the fractor devices that need to be addressed in future research but it showed conclusively that efforts to improve the environmental response and manufacturability of the devices are justified.

The current paradigm seems to be that control theory is moving from one digital paradigm to the next digital paradigm. There is a general trend toward treating analog processing as passé, even if it is simpler and more efficient. “As long as the system is cheap and reliable, who cares if it’s complex?” Unfortunately, there are many underserved markets where there isn’t the volume to achieve return on investment for the design and construction of complex controllers. Fractional order control can produce simple, robust systems to serve these market areas.

To get around the “world is digital” mental block, we are looking into encapsulating the fractance element into an otherwise digital processor, such as a DSP or FPGA, with D/A and A/D interface stages and gain controls much like those use in this demonstration. This will dovetail with the other issues associated with improving the packaging as discussed above.

The controller demonstrated here has direct application to control of wheelchairs and similar electric drive systems. In particular, the need is to achieve stable control of the vehicle on uneven, slippery, or rough terrain. For higher power efficiency, the controller would have a pulse width modulated (PWM) output stage instead of a linear system such as in Quanser’s Output Power Module (OPM), but the technology for such power stages is well developed, see, e.g. T. Wildi, *Electrical Machines, Drives, and Power Systems* (6^{th} ed.), Prentice Hall, 2006.

As an example in powered prosthetics, Motion Control, of Salt Lake City, Utah, maker of the Utah Arm™, implemented proportional control using myoelectric implants in residual muscle tissue as the source of the control signal. From what we have seen here, adding an integer order integral tern to this controller would not be advisable, but adding fractional order control would make the arm move more smoothly and naturally for a wide range of body sizes and physical demands. Given the modularity of the human interface developed for testing the Fractroller, it would be straightforward to integrate the myoelectric signal into the Fractroller to investigate this more direct interaction between human and machine and to complete the integration of FOC into powered prosthetics. As a possible adjunct to the prosthetic control, fractor based circuits may be able to generate the appropriately shaped low voltage stimulus signal that will help deaden phantom limb pain for amputees. This was suggested in limited testing done a decade ago.

In order to support entry into other markets, an additional test instrument will be developed. The student participants dubbed the proposed analytical device the “Fractolyzer.” This will be a plant test instrument to probe for basic system parameters and detect nonlinear properties. It is an extension of the autotuning devices now being used for conventional control applications. Coupled with a version of the system modeling software already developed here, it will speed up the process of analyzing the plant under consideration and designing an appropriate controller.