Time evolution of mode-summed radial self-force of a scalar field on an eliptical orbit in a Schwarszchild background using extrapolations in both the mode-sum and the Discontinuous Galerkin error

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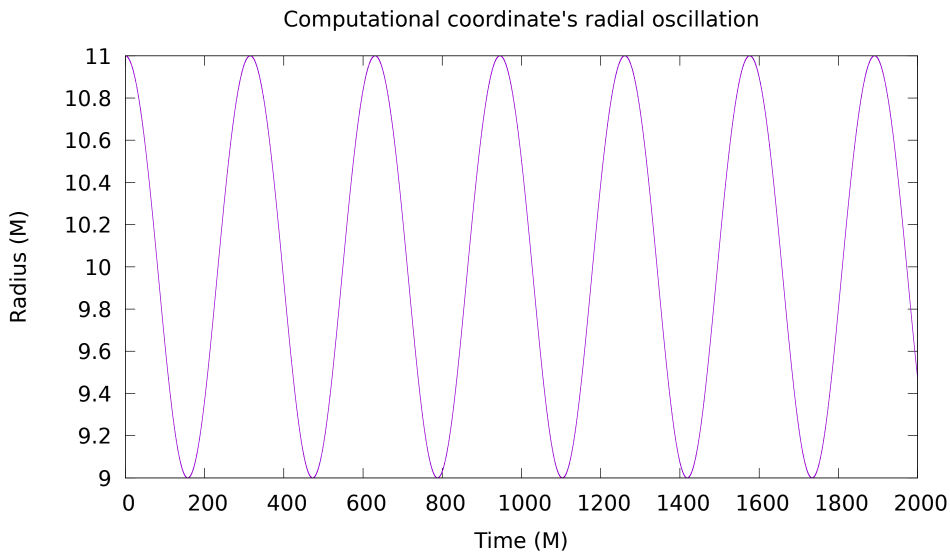
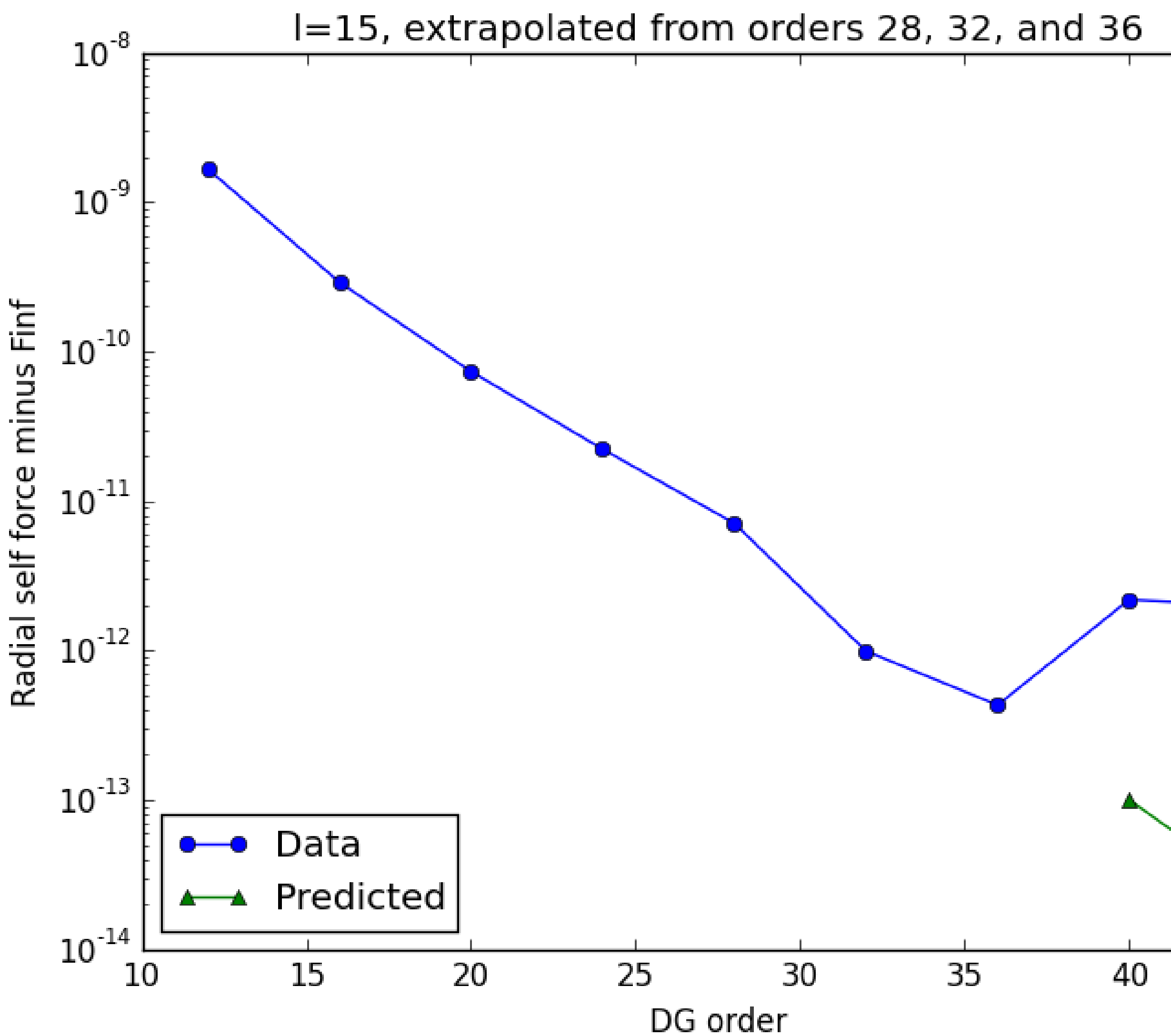
1 l=2

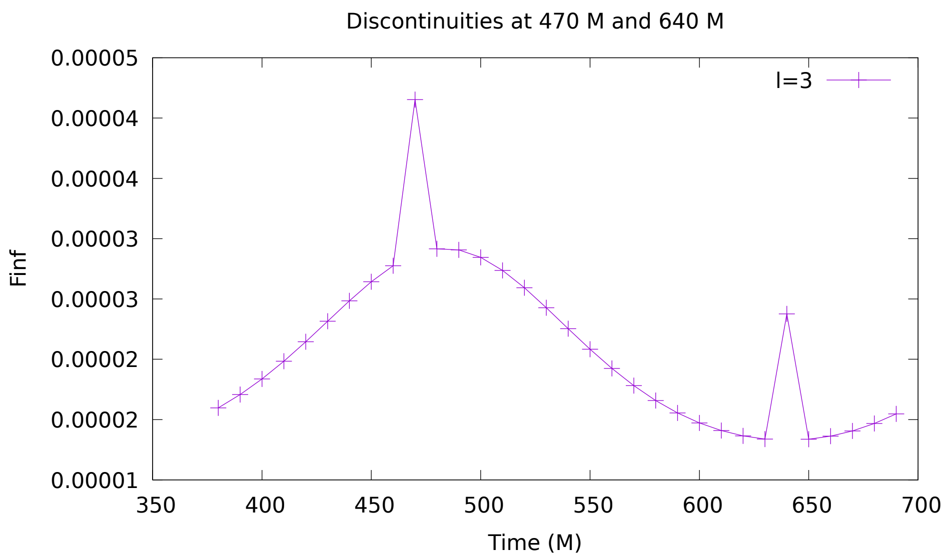
Starting index	finf
2	4.18128309016e-05
3	mode failed
4	4.18128307505e-05
5	4.18128308245e-05
6	4.1812830828e-05



time	starting order	finf
632	0	mode failed
632	1	2.40975299617e-05
632	2	2.40975300465e-05
632	3	2.40975300114e-05
632	4	mode failed
632	5	2.40975299291e-05
632	6	2.40975299148e-05
634	0	mode failed (however, 6 selected)
634	1	2.39990698129e-05
634	2	2.39990699318e-05
634	3	2.39990698774e-05
634	4	mode failed
634	5	2.39990697065e-05
634	6	2.39990696758e-05
636	0	mode failed (however, 6 selected)
636	1	2.391047416e-05
636	2	2.39104742806e-05
636	3	2.39104742249e-05
636	4	2.39104737911e-05
636	5	2.39104739924e-05
636	6	2.39104739079e-05



l=15, extrapolated from orders 28, 32, and 36
Radial self force minus Finf
DG order
Data
Predicted



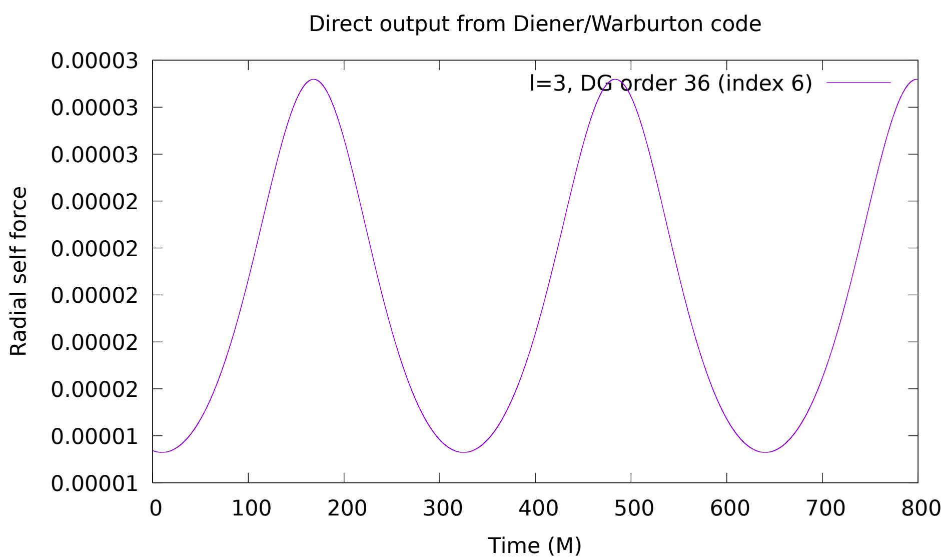
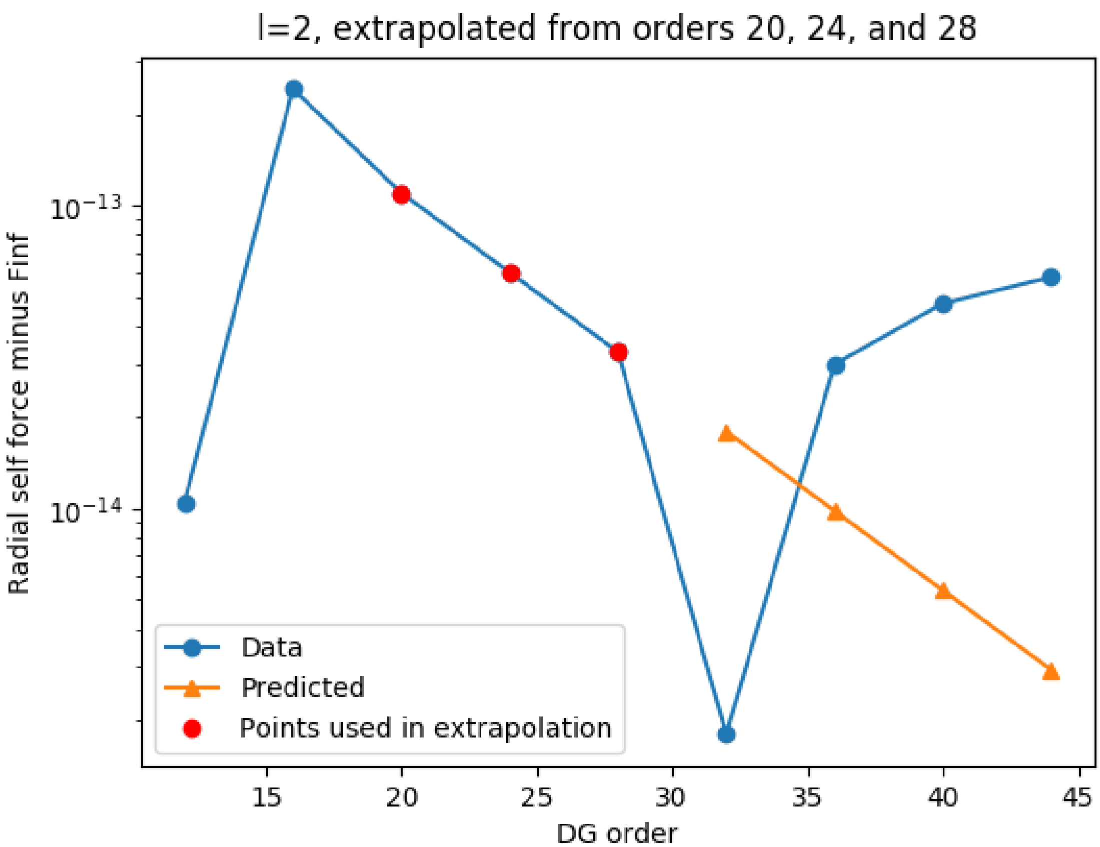


Figure 1: 470 M near perihelion, 640 M at aphelion



l=2, extrapolated from orders 20, 24, and 28
Radial self force minus Finf
10^{-13}
10^{-14}
DG order
15
20
25
30
35
40
45
Data
Predicted
Points used in extrapolation

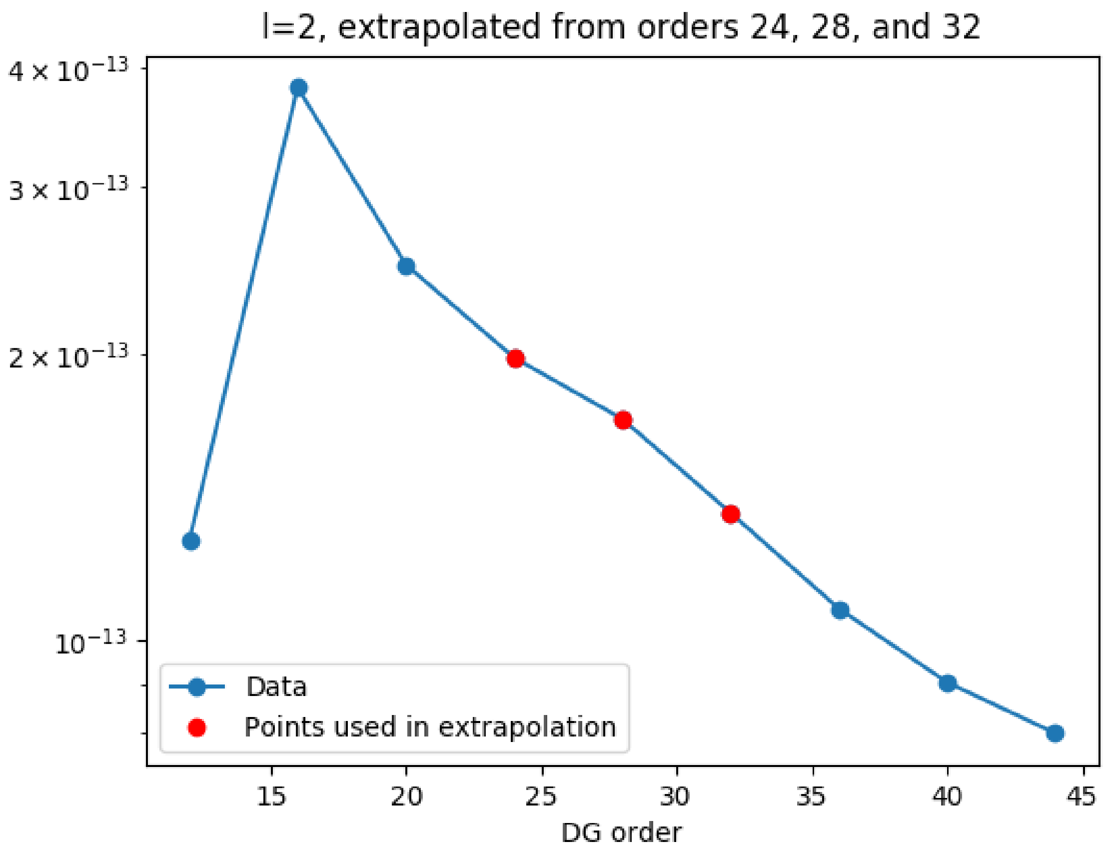
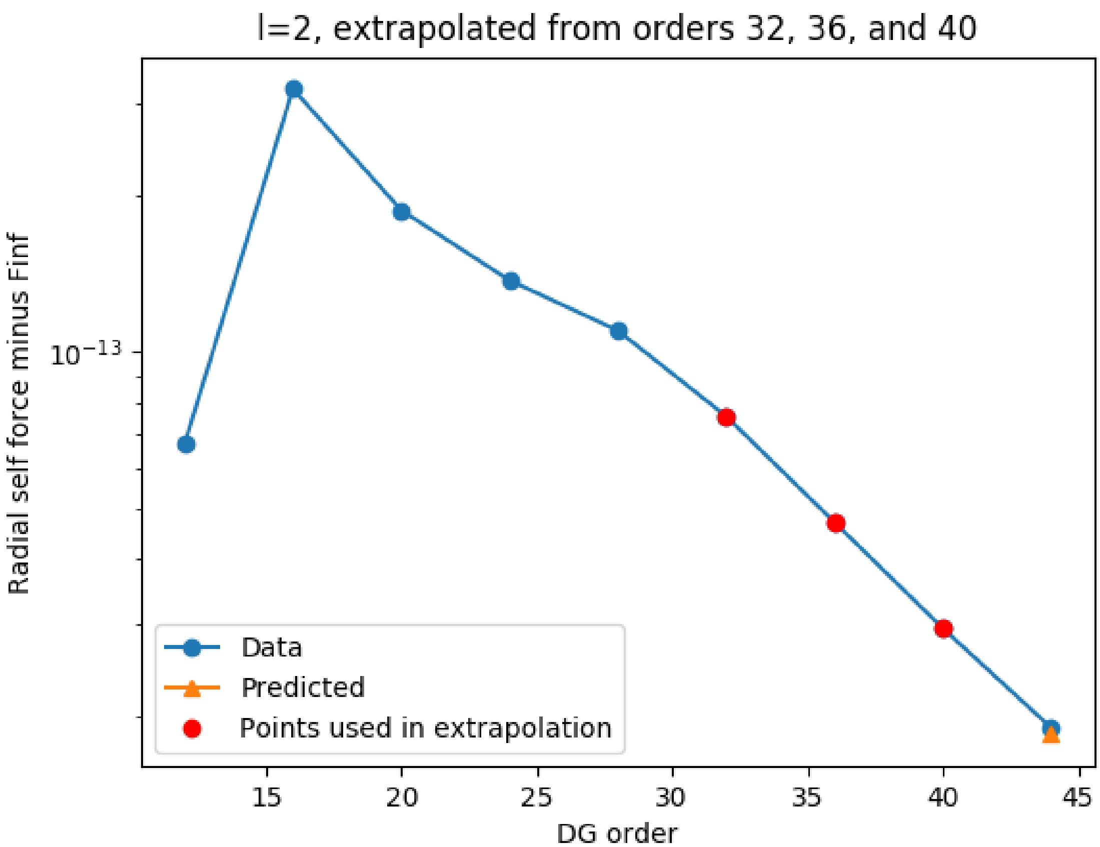
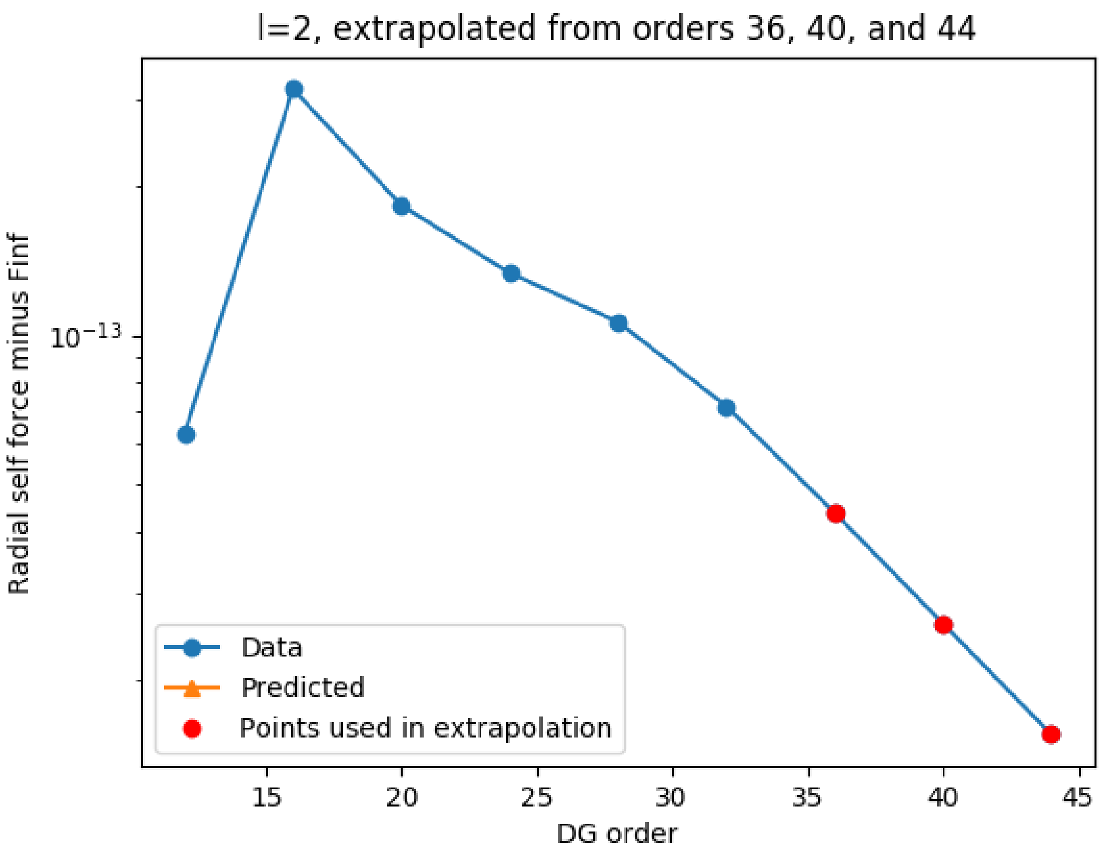
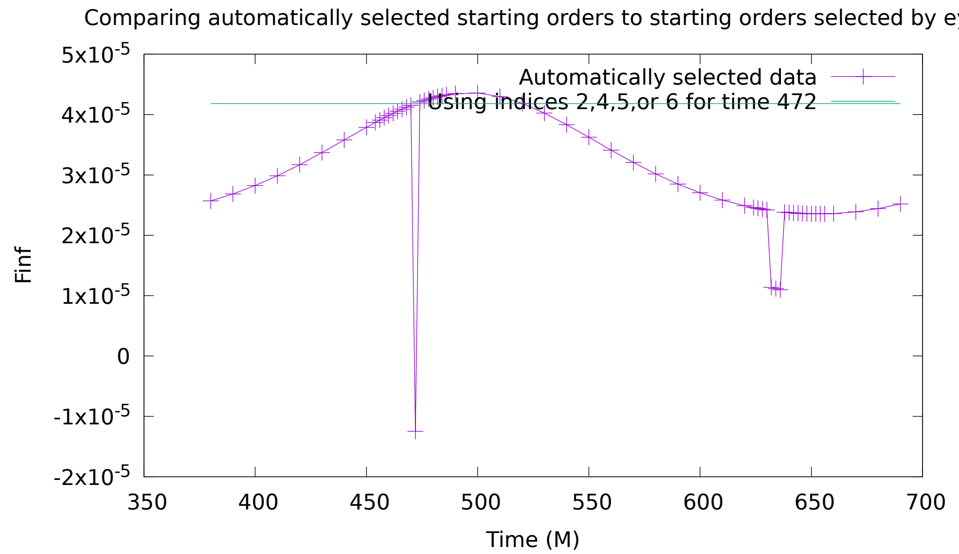


Figure 2: Note that the three points used in the extrapolation are not on a line on a semilog scale– it is not possible to fit an exponential through them. That is why this mode failed.



l=2, extrapolated from orders 32, 36, and 40
Radial self force minus Finf
10^{-13}
DG order
15
20
25
30
35
40
45
Data
Predicted
Points used in extrapolation


l=2, extrapolated from orders 36, 40, and 44
Radial self force minus Finf
10^{-13}
15
20
25
30
35
40
45
DG order
Data
Predicted
Points used in extrapolation



1.1 Checking for discontinuities in F_{inf} for each each l-mode

There are no discontinuities in F_{inf} for any of the l-modes when the median approach is used. See attached figures zero through thirty.

1.2 Determining F_{inf} using maximum likelihood fits to subsegments of lines in semilog space

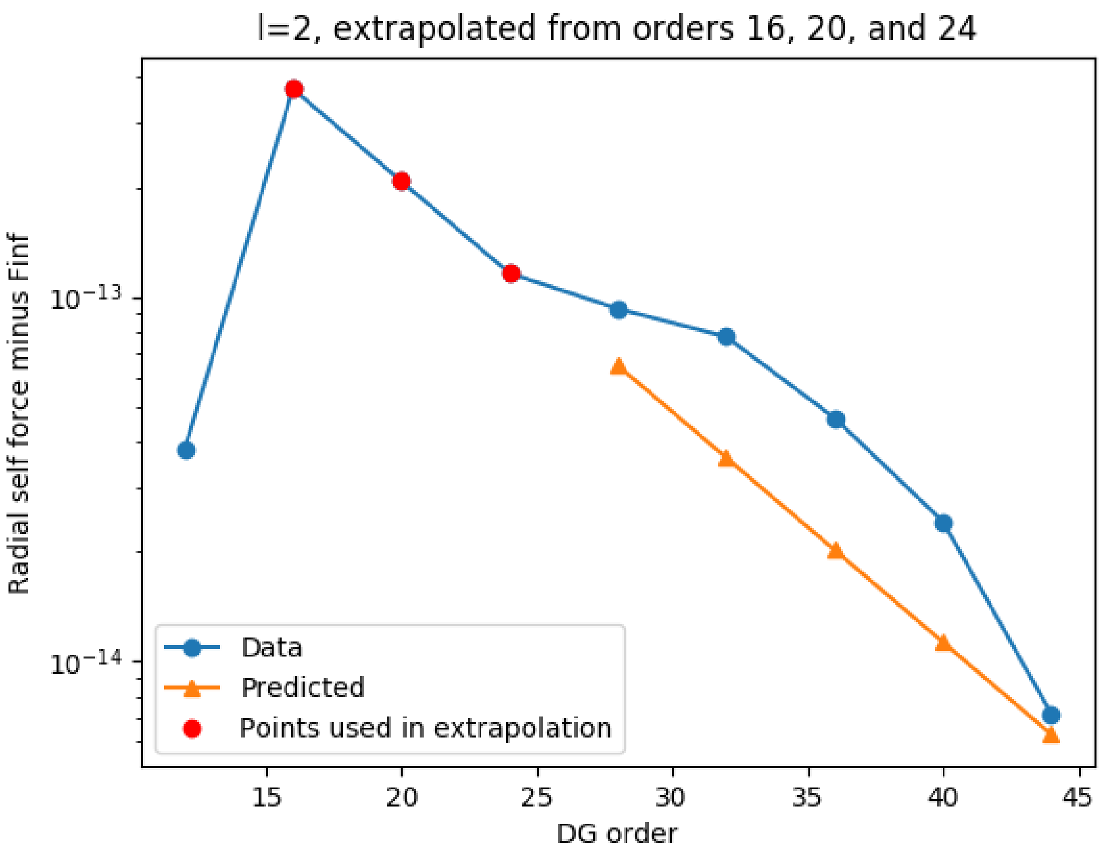
Fit subsegments of lines in semilog space on DG order convergence plot after subtracting Finf for each possible starting order. pick starting order and starting and ending index of line segment with best possible chi-sq per dof (closest to one). use that finf. veto modes and starting indices that fail the alpha ratio test.

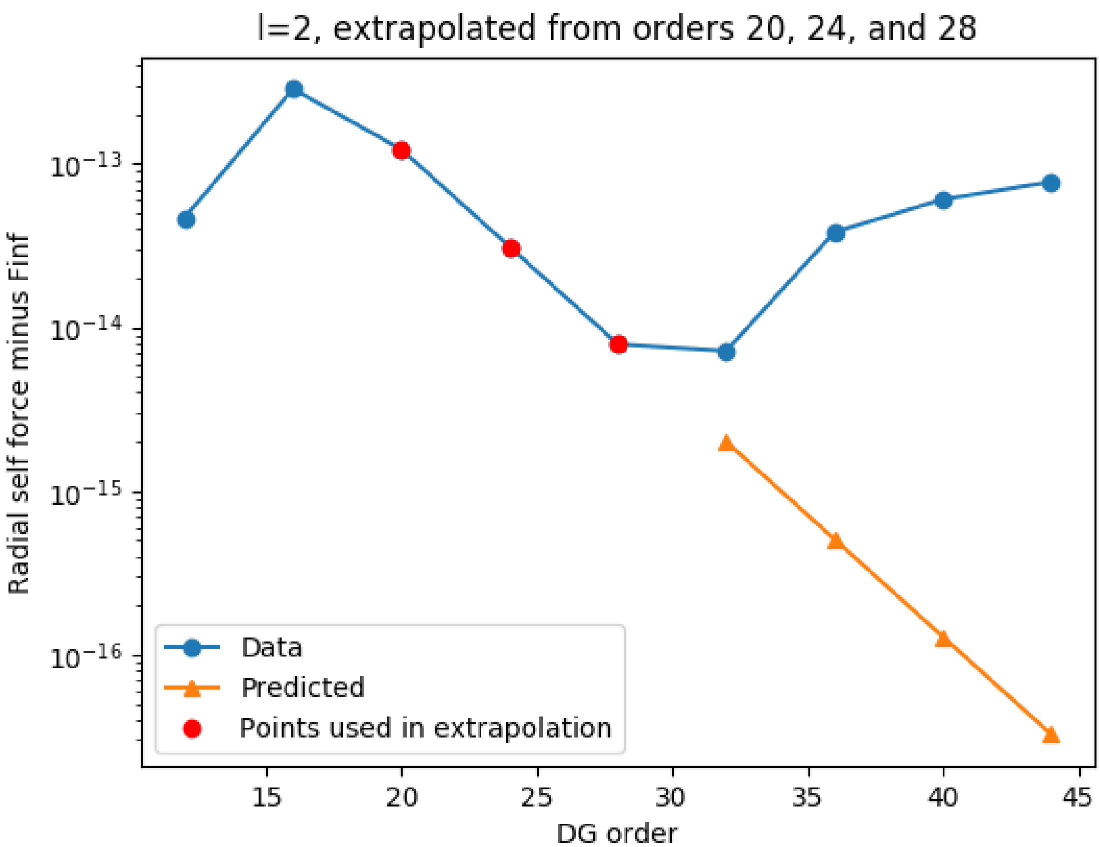
take standard deviation of surface plot as well as average.

1.3 Fractional errors

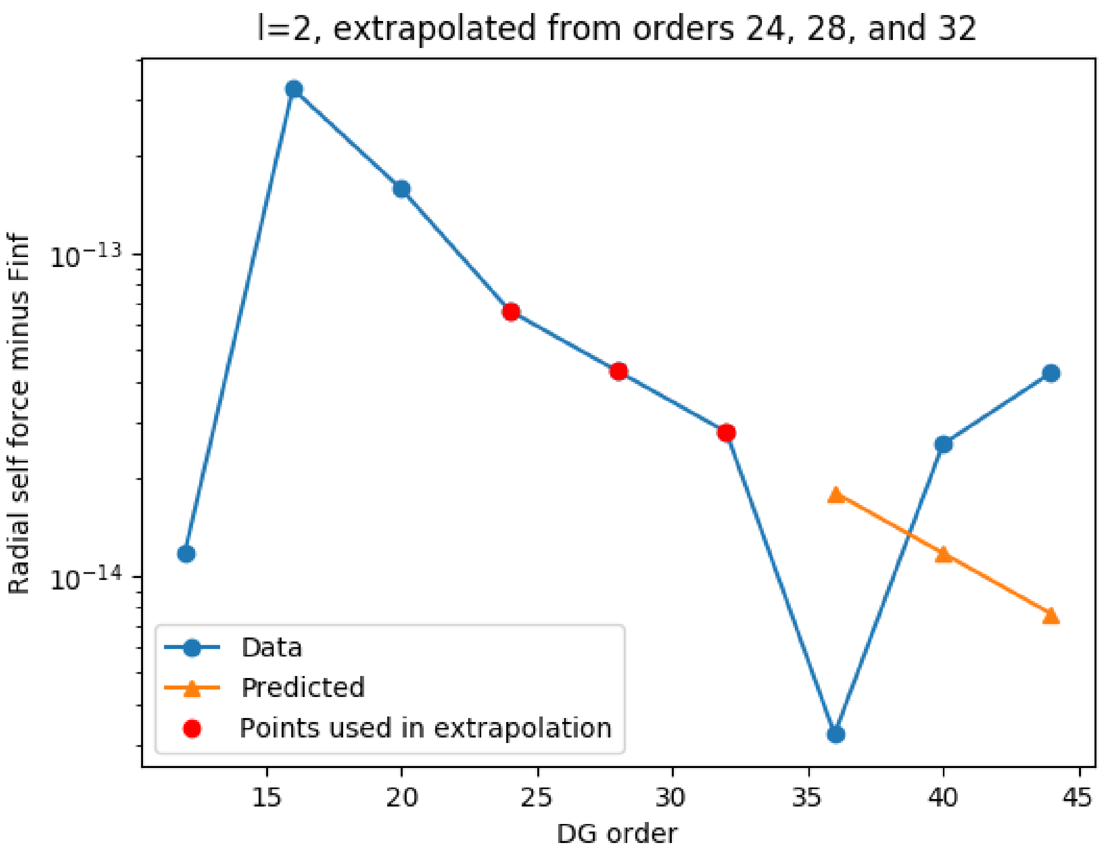
1.4 Relative error as a function of mode

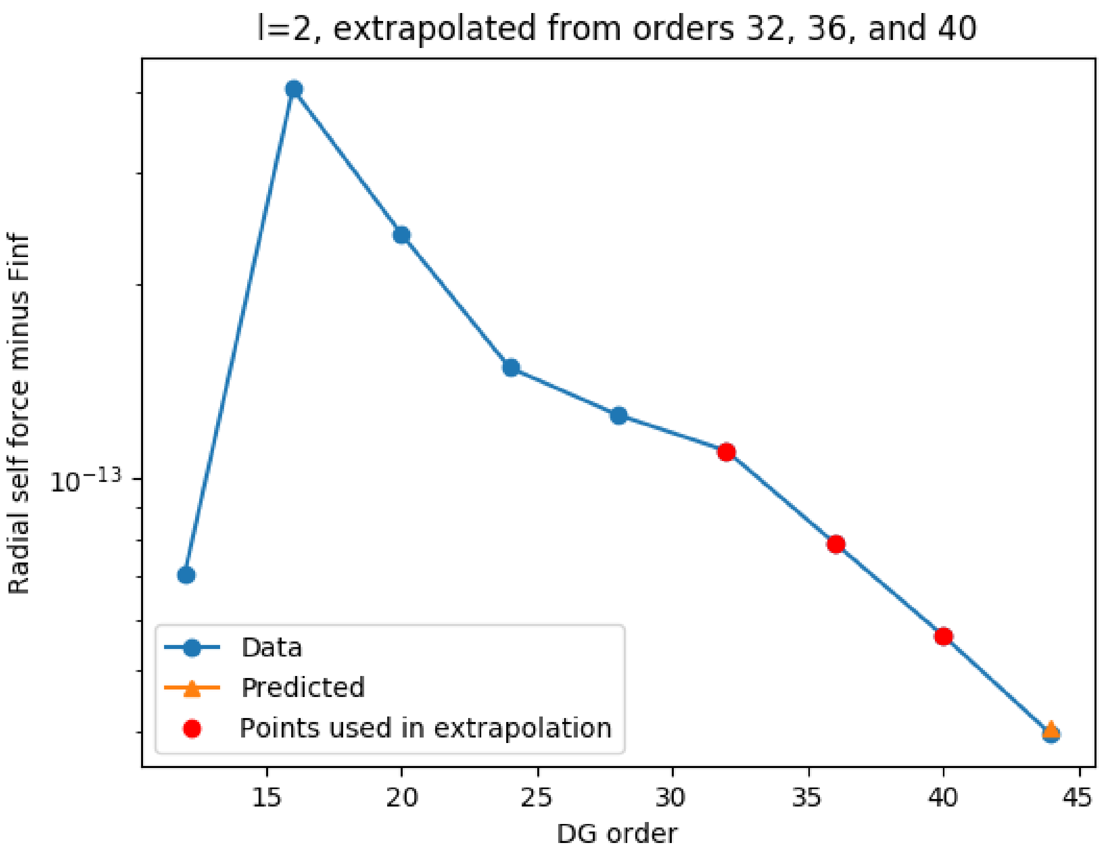
We can understand why it is so hard to produce good fits by examining the relative error between different fitting techniques as a function of mode. Look at the relative error between the fit method and the median method. Both the relative and absolute error grow with l, explaining why the sigma-suppresion technique does not produce good results.

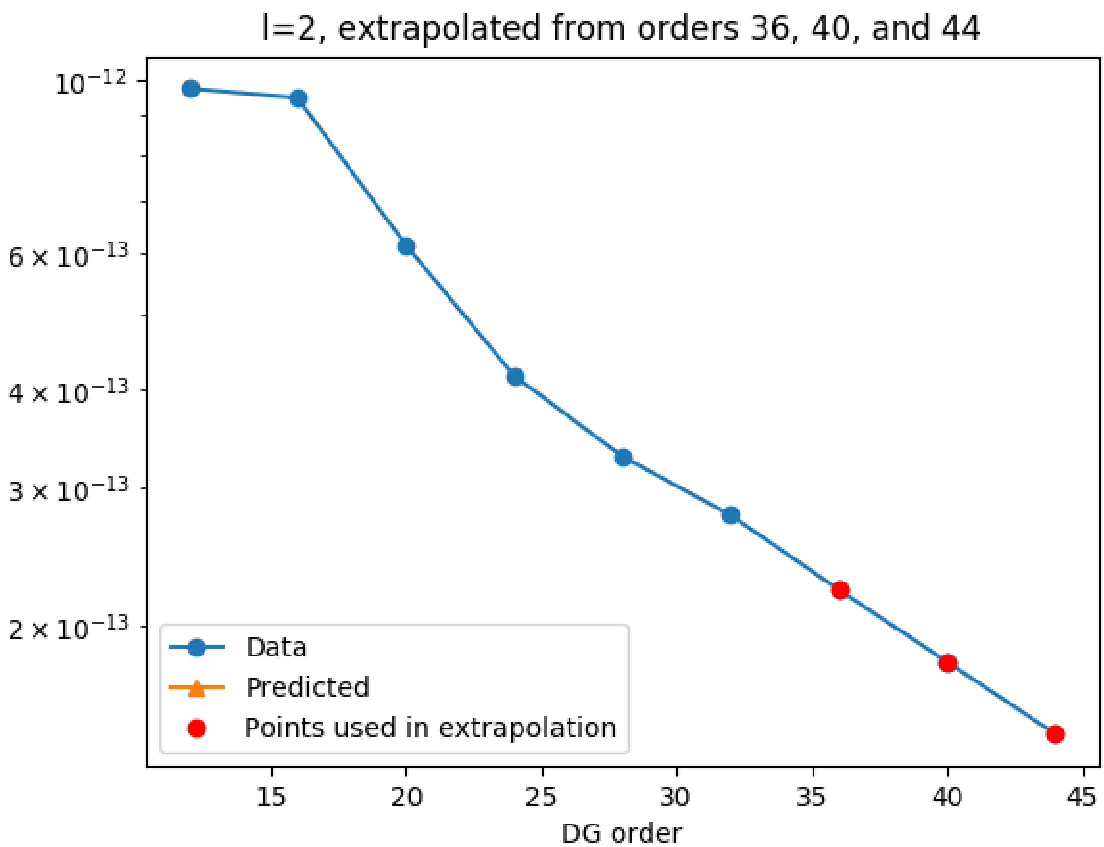

l=2, extrapolated from orders 16, 20, and 24
Radial self force minus Finf
10^{-13}
10^{-14}
15
20
25
30
35
40
45
DG order
Data
Predicted
Points used in extrapolation



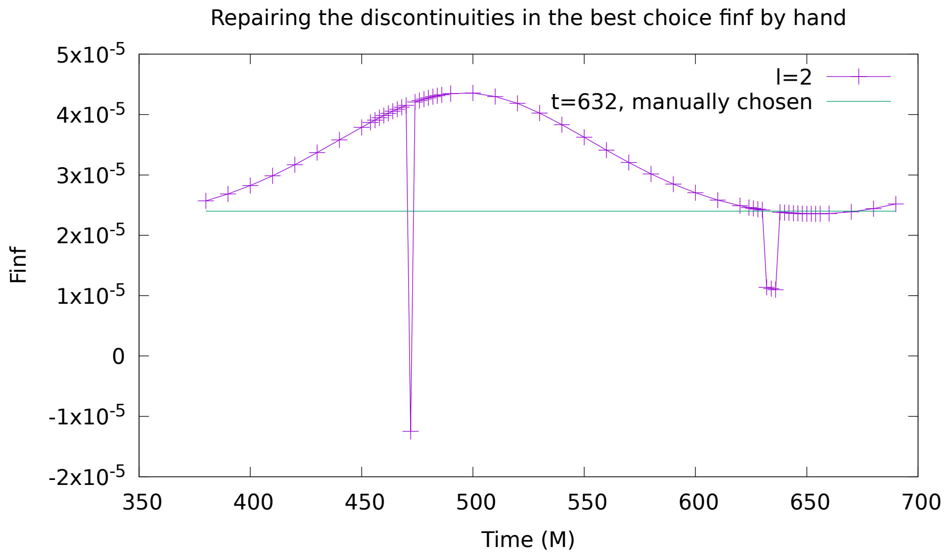
l=2, extrapolated from orders 20, 24, and 28
Radial self force minus Finf
DG order
Data
Predicted
Points used in extrapolation


l=2, extrapolated from orders 24, 28, and 32
Radial self force minus Finf
10^{-13}
10^{-14}
Data
Predicted
Points used in extrapolation
15
20
25
30
35
40
45
DG order


l=2, extrapolated from orders 32, 36, and 40
Radial self force minus Finf
10^{-13}
DG order
15
20
25
30
35
40
45
Data
Predicted
Points used in extrapolation



l=2, extrapolated from orders 36, 40, and 44
10^{-12}
6×10^{-13}
4×10^{-13}
3×10^{-13}
2×10^{-13}
Data
Predicted
Points used in extrapolation
15
20
25
30
35
40
45
DG order



1.5 Structure of the error compared to the evolution in time

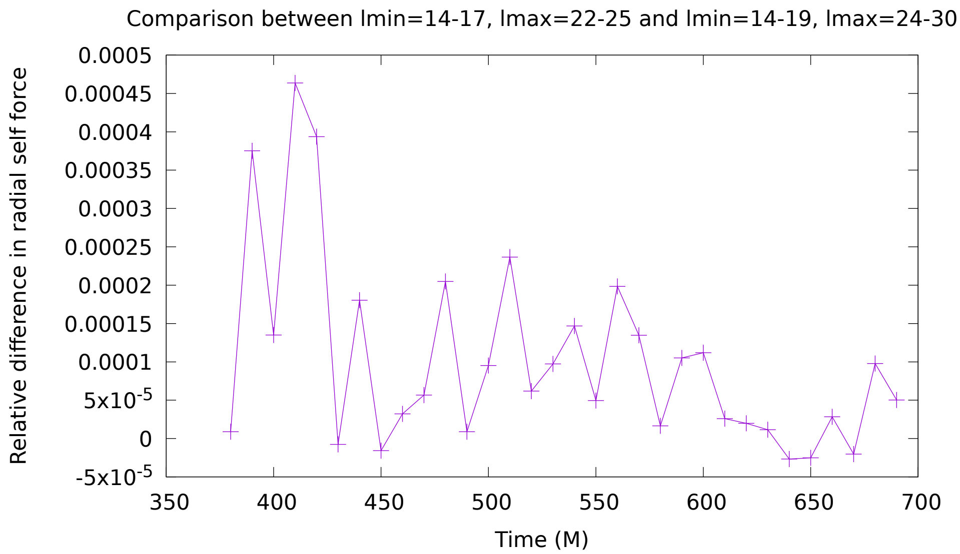


Figure 3: This is the relative difference between the total radial self force measured in two different ways. In both cases, the self force was extrapolated to infinite order at every l-mode at every possible DG starting order. The infinite DG order self forces over the various starting orders were sorted, eliminating NaNs. The median was chosen for each l-mode. Then the self force as a function of l-mode was fit to its three term form, and the sum was summed from zero to lmax, then extrapolated from $lmax + 1$ to infinity using an analytic form determined using Mathematica. All possible choices with lmin between 14 and 17 and lmax between 22 and 25 were averaged to obtain the total radial self force as a function of time. Similarly, all possible choices with lmin between 14 and 19 and lmax between 24 and 30 were averaged to obtain the total radial self force as a function of time. This plot shows the relative difference. I believe the smaller range is in the denominator.

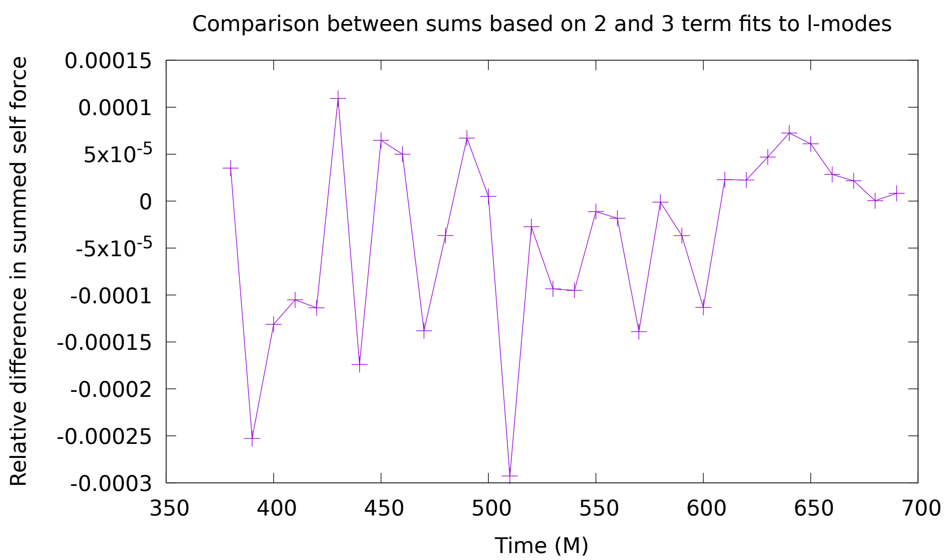


Figure 4: This figure was produced in the same manner as the previous figure, averaging over the smaller range, only it is a comparison between including either two or three terms in the l-mode fit. I believe the three term fit is in the denominator of the relative difference.

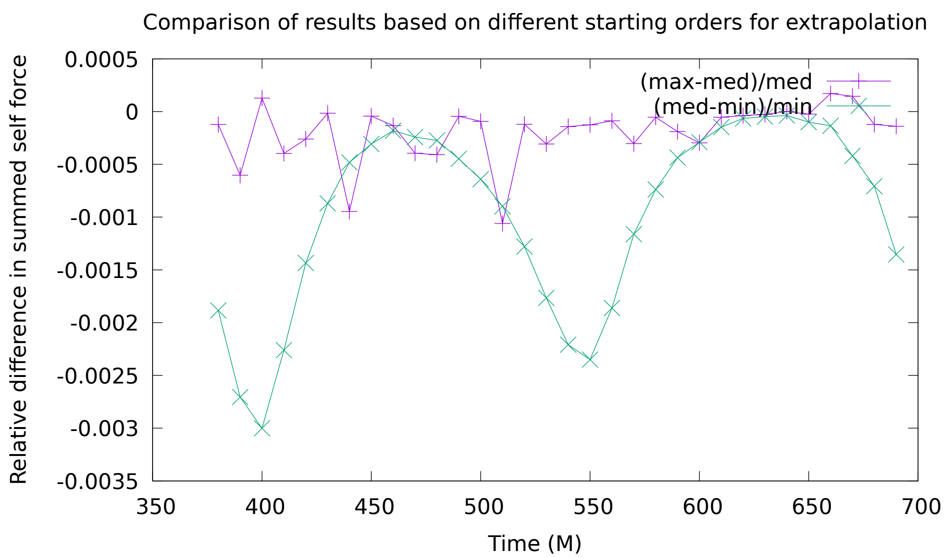


Figure 5: This figure was produced in a similar manner to the first figure, only instead of using the median, it is a comparison between using the median, the maximum, and the minimum. The purple line is the relative difference between the maximum and the median, which is subject to roundoff error due to the potential for the maximum to contian roundoff error. The green line is the relative difference between the median and the minimum, which is subject to effects due to failure to converge. I suspect the median is the best compromise between these two effects, rejecting outliers in both directions, though it is a simplistic approach to doing so, and does not guarantee success. It is possible to have a starting order that has not converged and is also in the roundoff regime, for example. A better guarantee of success, though not a certain one, would be to do a fit over part of the error convergence plot to determine exponentiality, by fitting a line in semilog scale. However, this seems unnecessarily complex at this time.

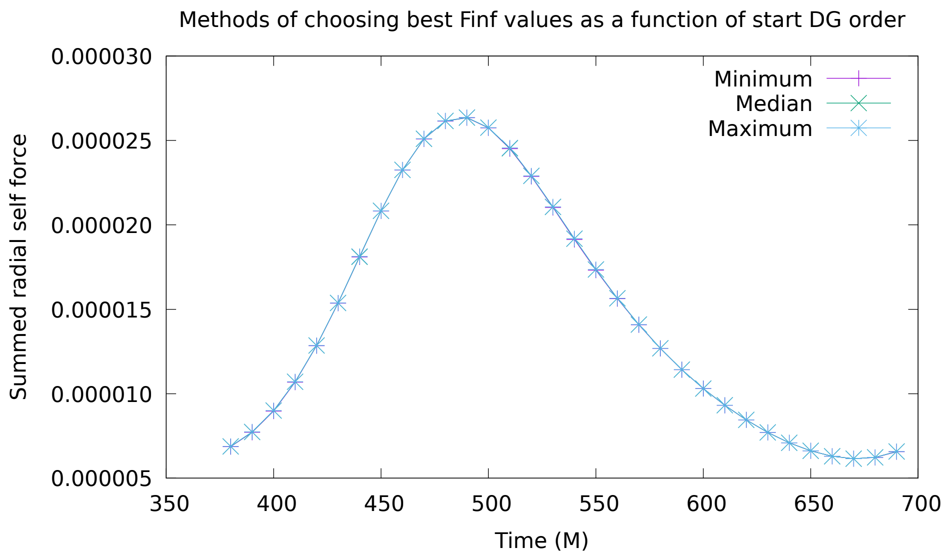
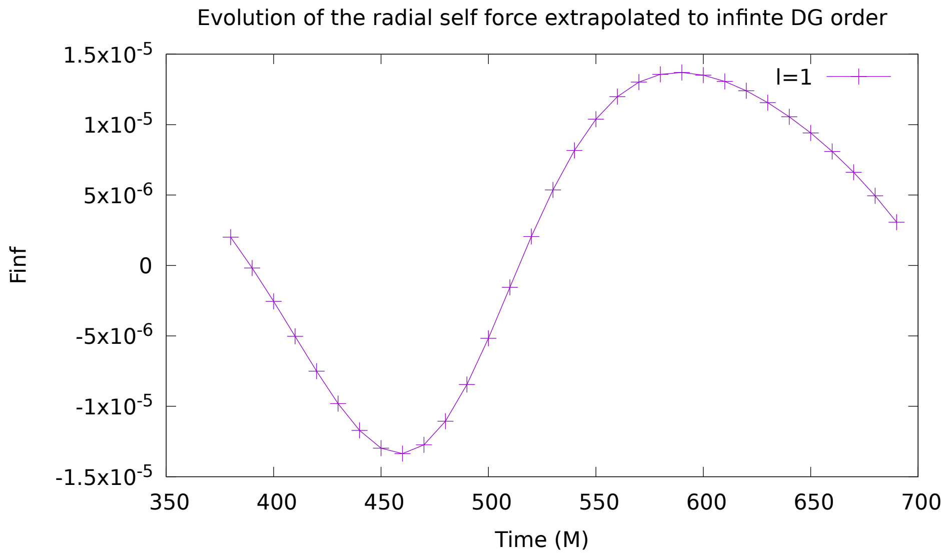
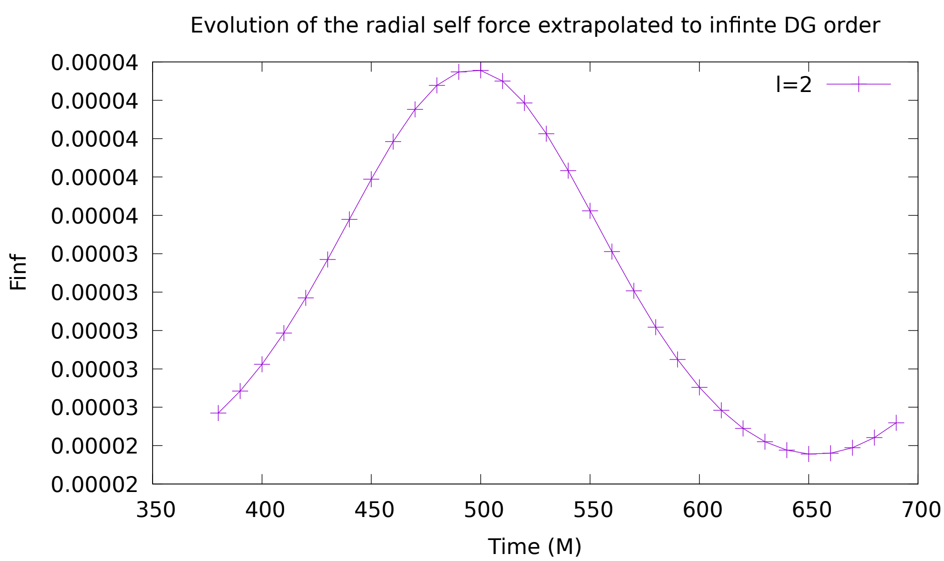
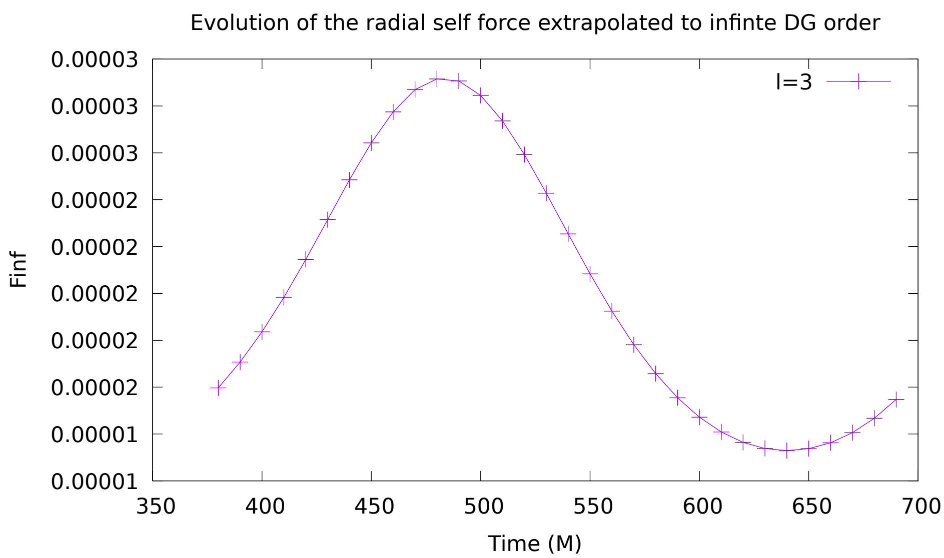
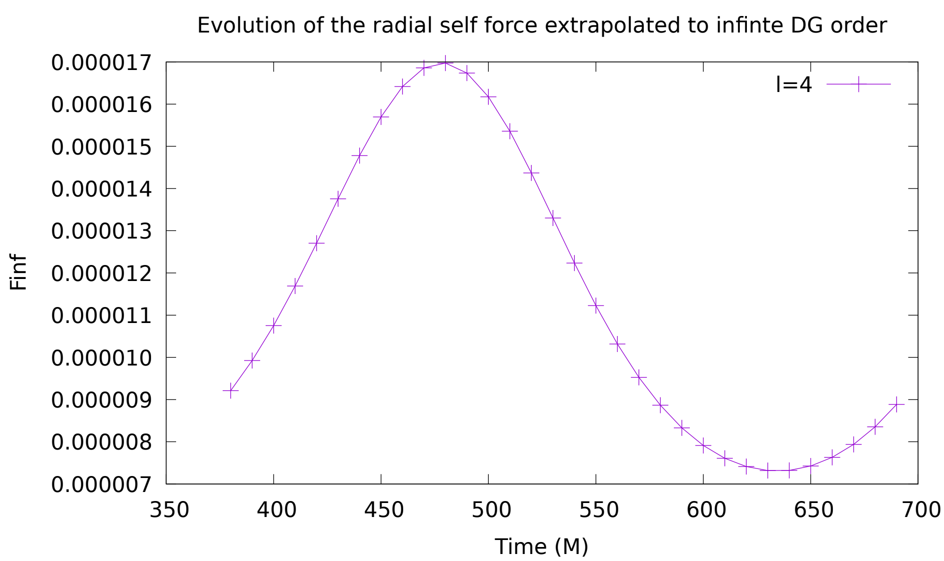


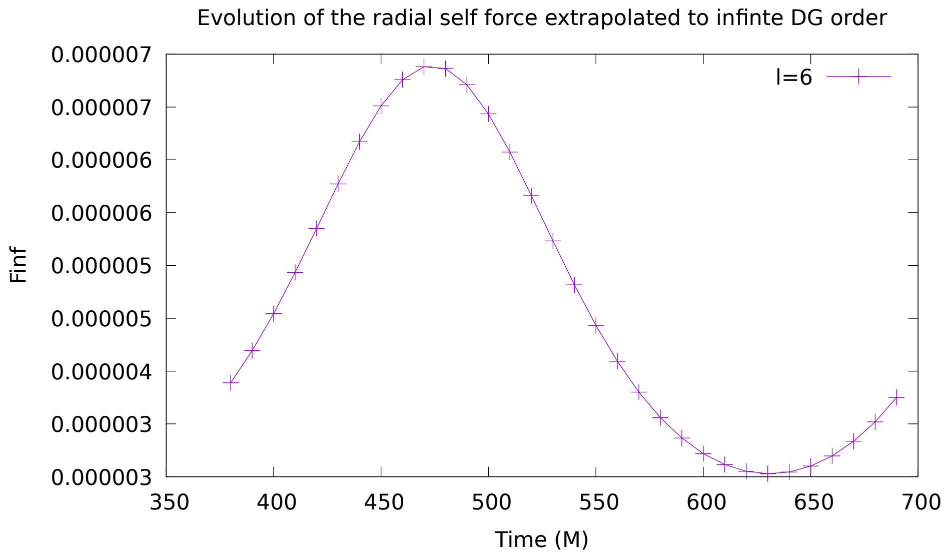
Figure 6: This is the actual summed, doubly extrapolated, radial self force, measured in three different ways as described in the three figures above.

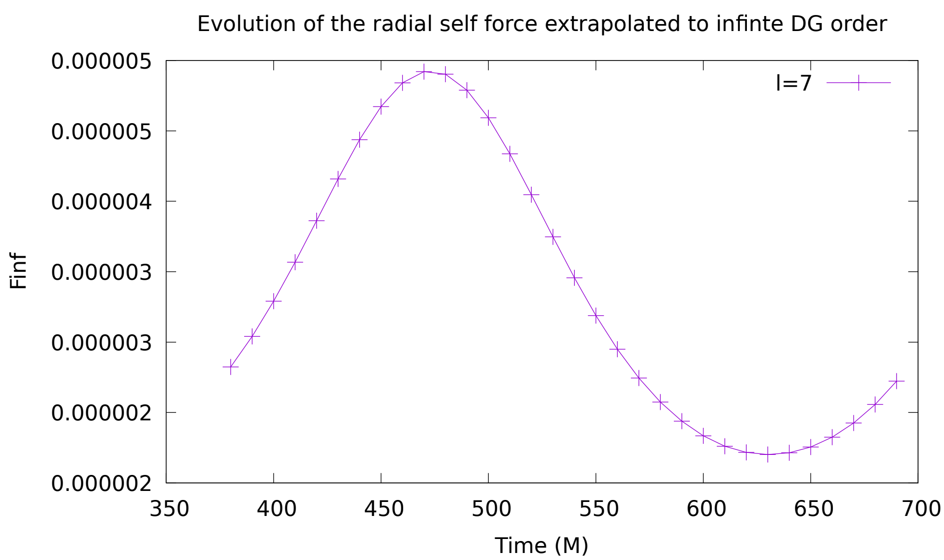

Evolution of the radial self force extrapolated to infinte DG order
1.5x10-5
1x10-5
5x10-6
0
-5x10-6
-1x10-5
-1.5x10-5
Finf
350
400
450
500
550
600
650
700
Time (M)
l=1

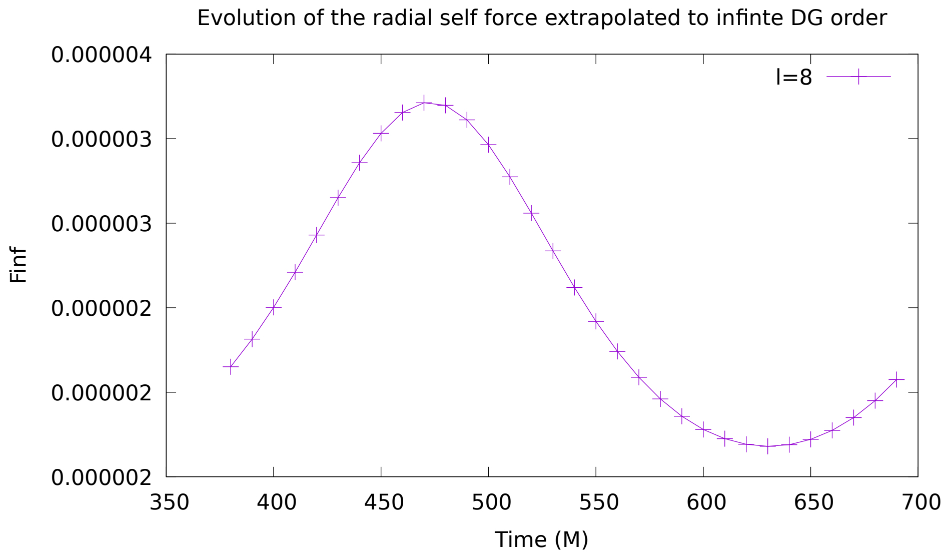

Evolution of the radial self force extrapolated to infinte DG order
0.00004
0.00004
0.00004
0.00004
0.00004
0.00003
0.00003
0.00003
0.00003
0.00003
0.00002
0.00002
Finf
350
400
450
500
550
600
650
700
Time (M)
l=2

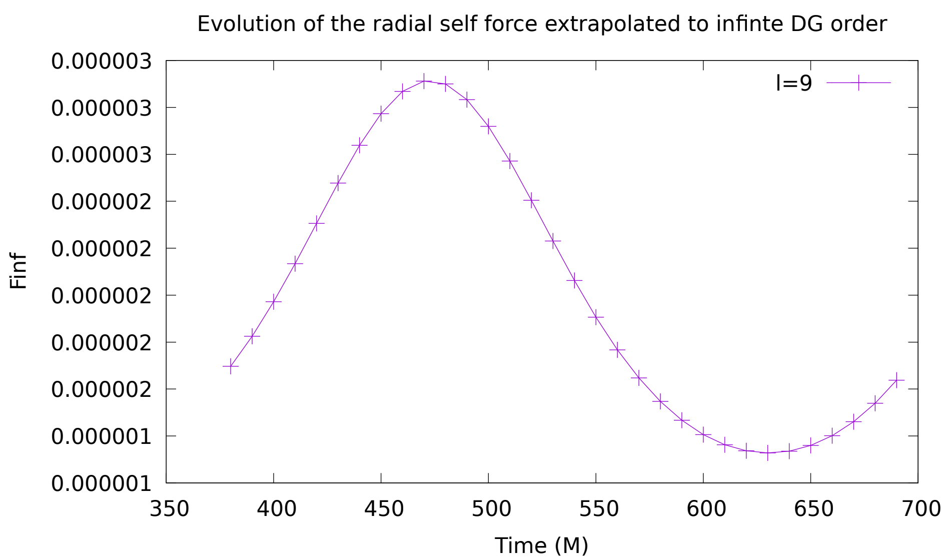

Evolution of the radial self force extrapolated to infinte DG order
l=3
Finf
Time (M)
0.00003
0.00003
0.00003
0.00002
0.00002
0.00002
0.00002
0.00002
0.00001
0.00001
350
400
450
500
550
600
650
700

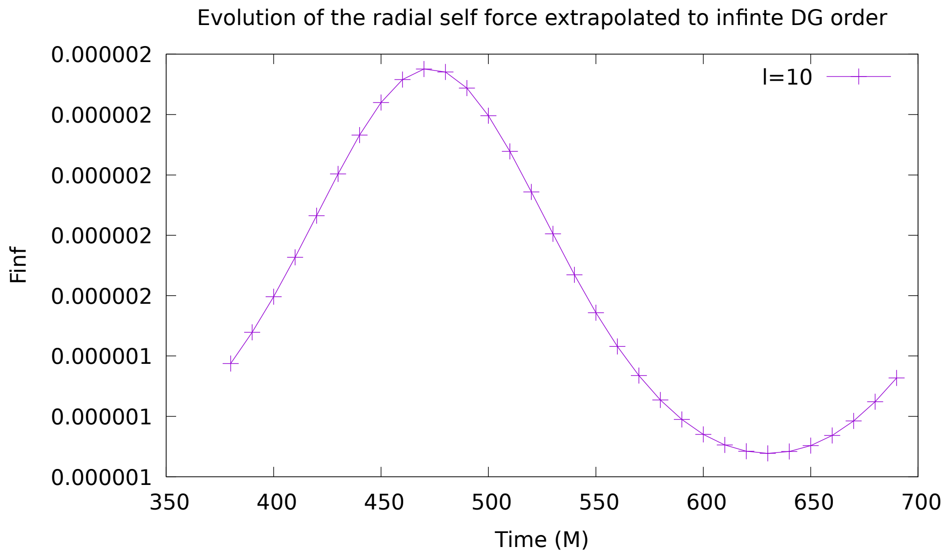

Evolution of the radial self force extrapolated to infinte DG order
l=4
Finf
Time (M)
0.000017
0.000016
0.000015
0.000014
0.000013
0.000012
0.000011
0.000010
0.000009
0.000008
0.000007
350
400
450
500
550
600
650
700

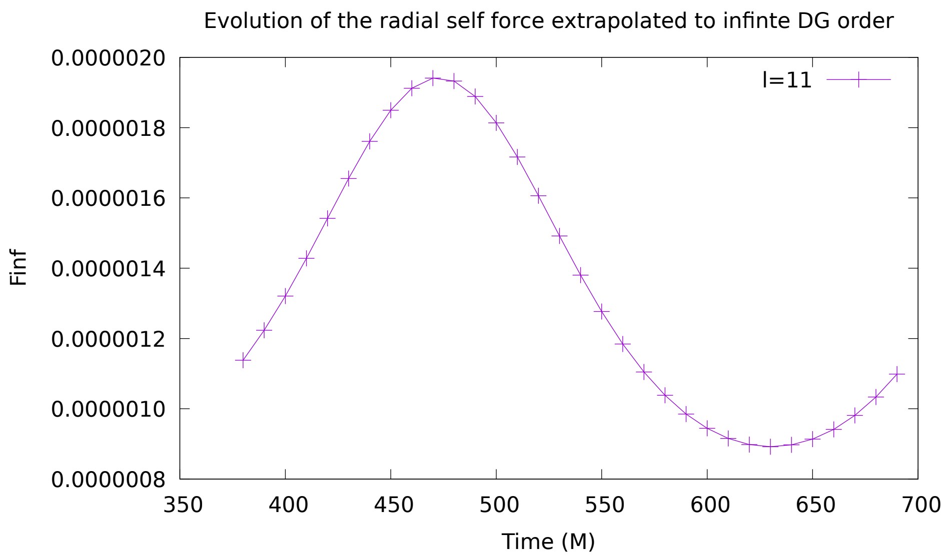

Evolution of the radial self force extrapolated to infinte DG order
l=6
Finf
Time (M)
0.000007
0.000007
0.000006
0.000006
0.000005
0.000005
0.000004
0.000003
0.000003
350
400
450
500
550
600
650
700

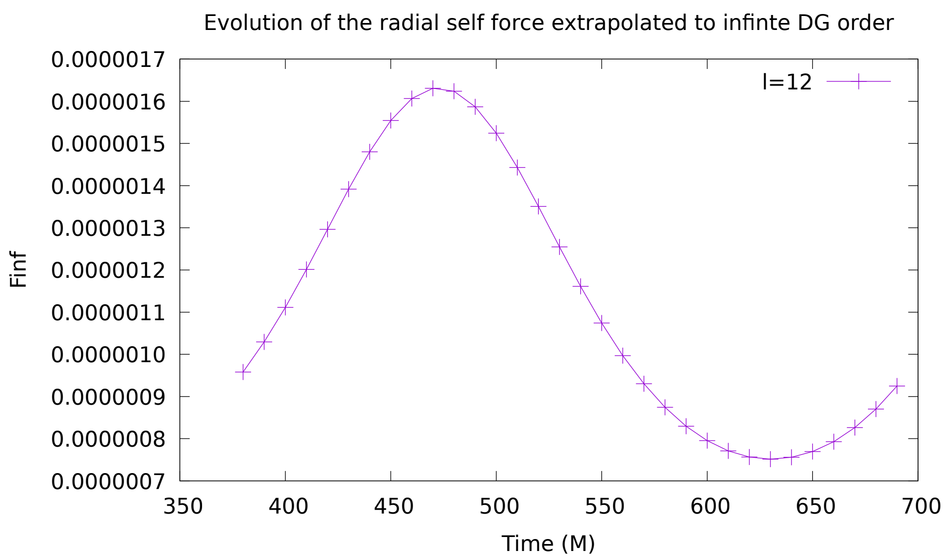

Evolution of the radial self force extrapolated to infinte DG order
l=7
Finf
Time (M)
0.000005
0.000005
0.000004
0.000003
0.000003
0.000002
0.000002
350
400
450
500
550
600
650
700

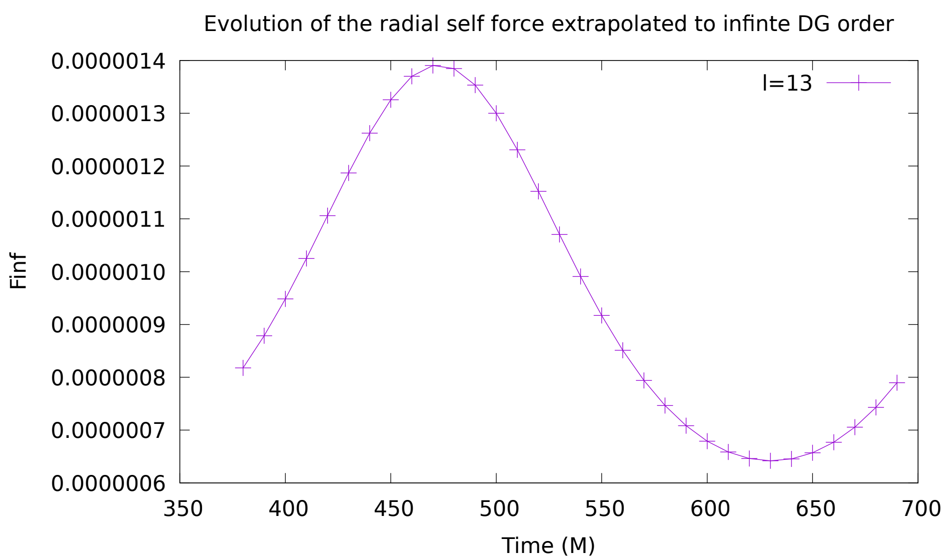

Evolution of the radial self force extrapolated to infinte DG order
0.000004
0.000003
0.000003
0.000002
0.000002
0.000002
Finf
350
400
450
500
550
600
650
700
Time (M)
l=8

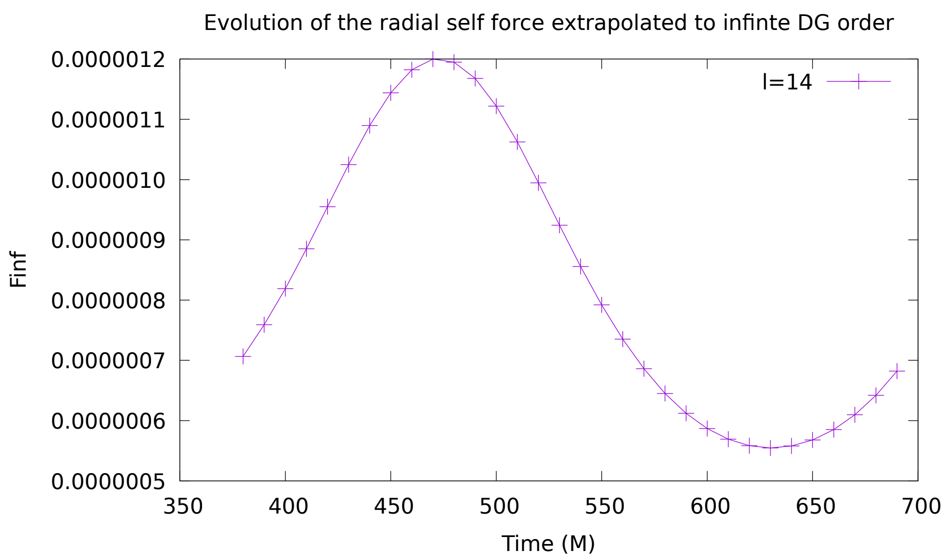

Evolution of the radial self force extrapolated to infinte DG order
0.000003
0.000003
0.000003
0.000002
0.000002
0.000002
0.000002
0.000002
0.000001
0.000001
Finf
350
400
450
500
550
600
650
700
Time (M)
l=9

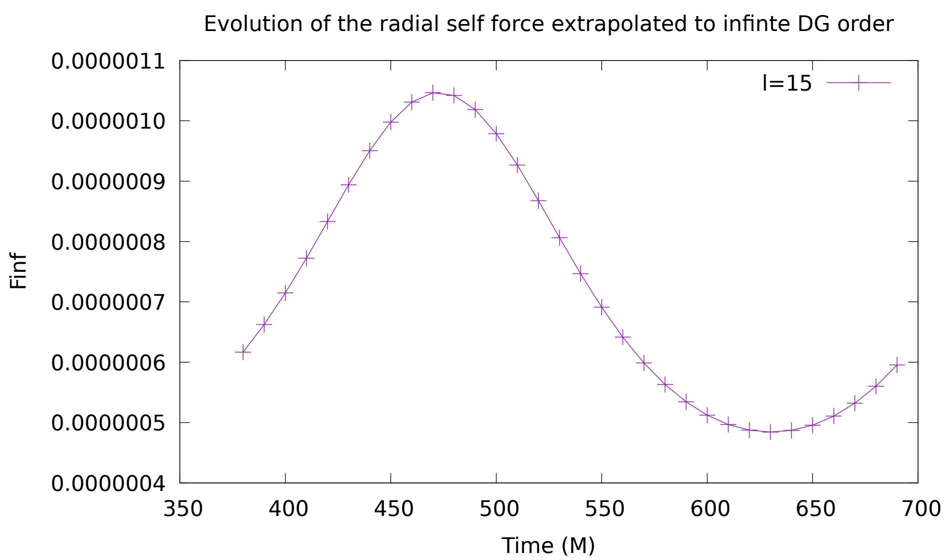

Evolution of the radial self force extrapolated to infinte DG order
l=10
Finf
Time (M)
0.000002
0.000002
0.000002
0.000002
0.000002
0.000001
0.000001
0.000001
350
400
450
500
550
600
650
700

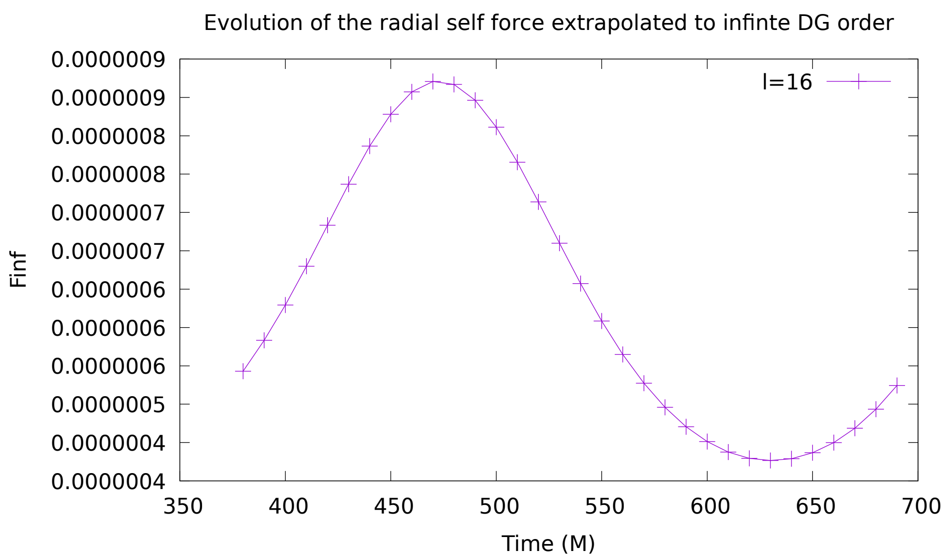

Evolution of the radial self force extrapolated to infinte DG order
l=11
Finf
Time (M)
0.0000020
0.0000018
0.0000016
0.0000014
0.0000012
0.0000010
0.0000008
350
400
450
500
550
600
650
700

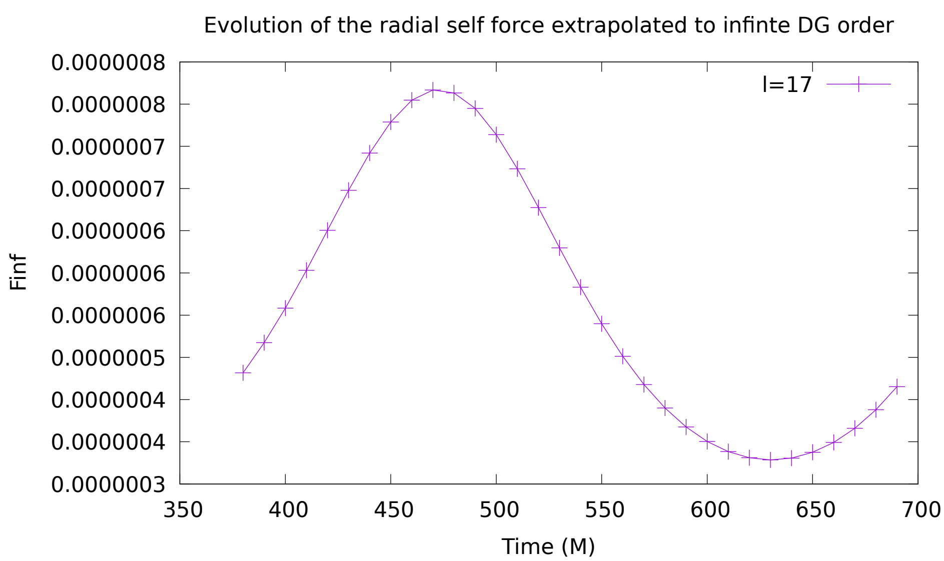

Evolution of the radial self force extrapolated to infinte DG order
l=12
Finf
0.0000017
0.0000016
0.0000015
0.0000014
0.0000013
0.0000012
0.0000011
0.0000010
0.0000009
0.0000008
0.0000007
350
400
450
500
550
600
650
700
Time (M)

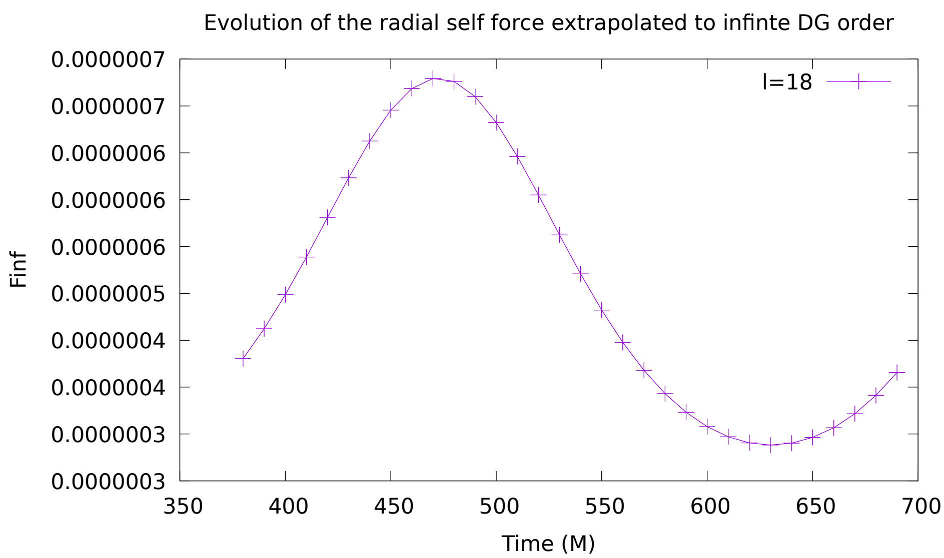

Evolution of the radial self force extrapolated to infinte DG order
l=13
Finf
0.0000014
0.0000013
0.0000012
0.0000011
0.0000010
0.0000009
0.0000008
0.0000007
0.0000006
350
400
450
500
550
600
650
700
Time (M)


Evolution of the radial self force extrapolated to infinte DG order
l=14
Finf
Time (M)
0.0000012
0.0000011
0.0000010
0.0000009
0.0000008
0.0000007
0.0000006
0.0000005
350
400
450
500
550
600
650
700

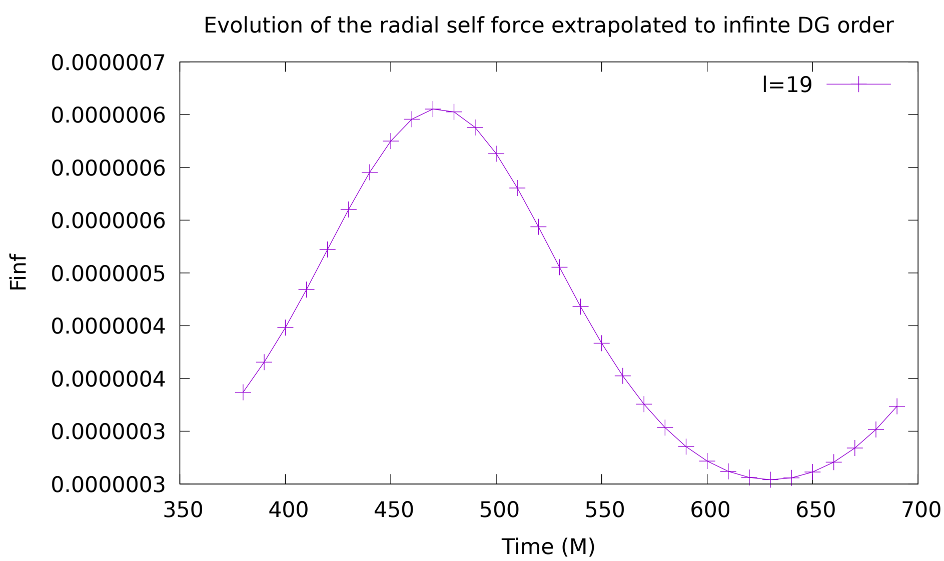

Evolution of the radial self force extrapolated to infinte DG order
l=15
Finf
Time (M)
0.0000011
0.0000010
0.0000009
0.0000008
0.0000007
0.0000006
0.0000005
0.0000004
350
400
450
500
550
600
650
700


Evolution of the radial self force extrapolated to infinte DG order
l=16
Finf
Time (M)
0.0000009
0.0000009
0.0000008
0.0000008
0.0000007
0.0000007
0.0000006
0.0000006
0.0000006
0.0000005
0.0000004
0.0000004
350
400
450
500
550
600
650
700

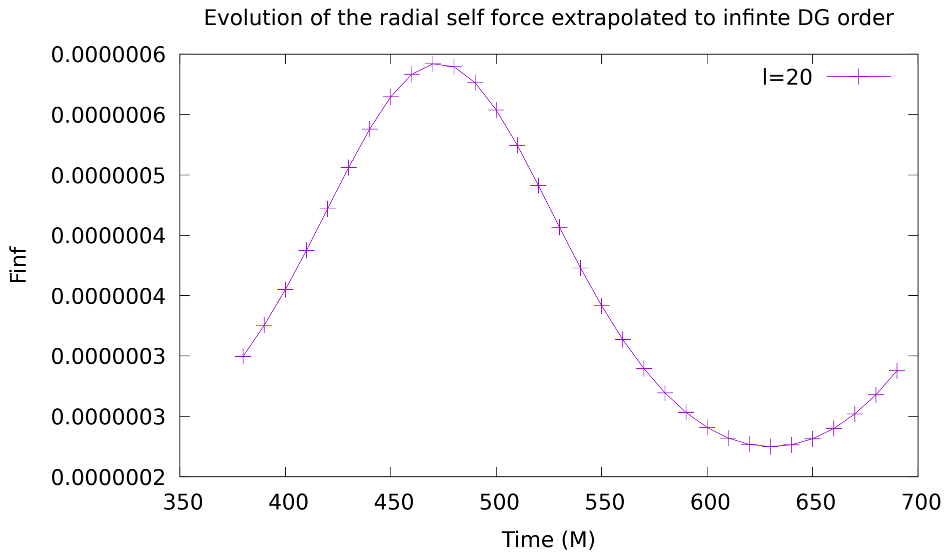

Evolution of the radial self force extrapolated to infinte DG order
l=17
Finf
Time (M)
0.0000008
0.0000008
0.0000007
0.0000007
0.0000006
0.0000006
0.0000006
0.0000005
0.0000004
0.0000004
0.0000003
350
400
450
500
550
600
650
700

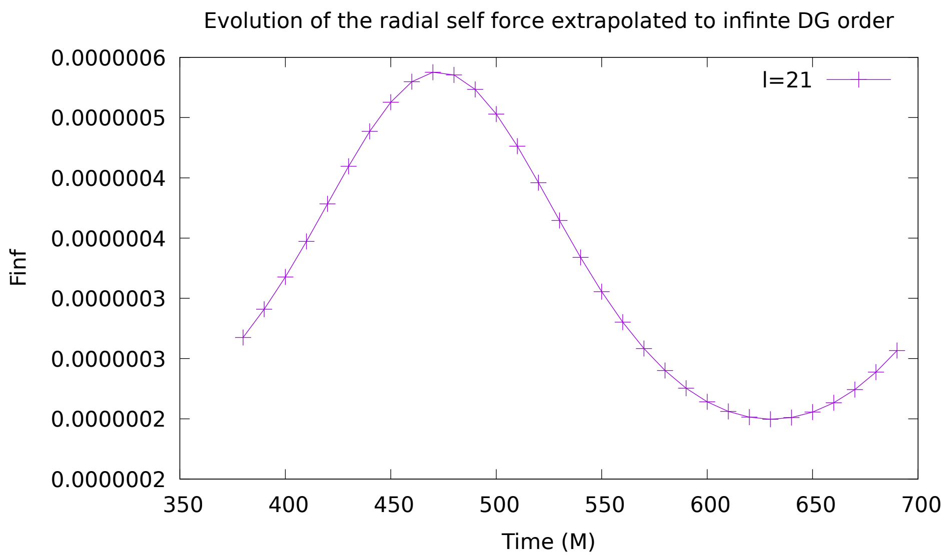


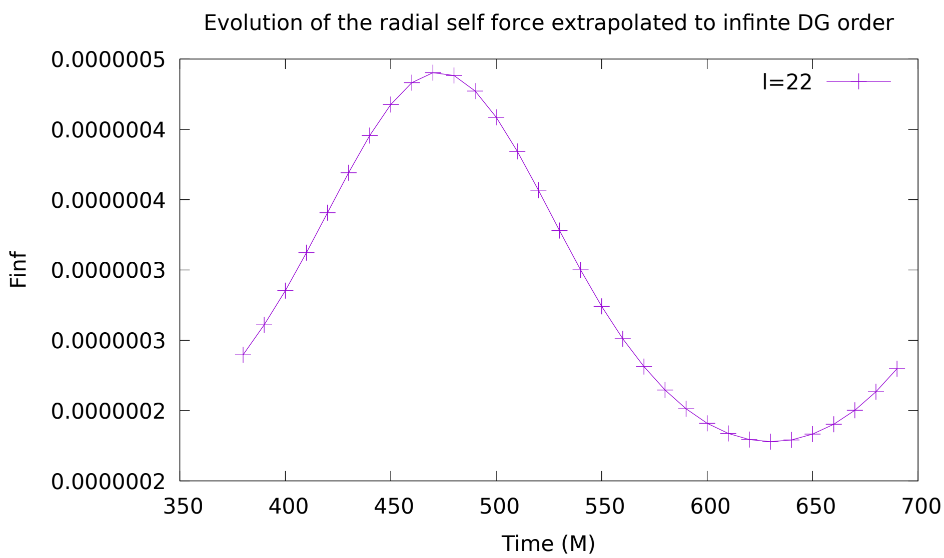
Evolution of the radial self force extrapolated to infinte DG order
0.0000007
0.0000007
0.0000006
0.0000006
0.0000006
0.0000005
0.0000004
0.0000004
0.0000003
0.0000003
Finf
350
400
450
500
550
600
650
700
Time (M)
l=18



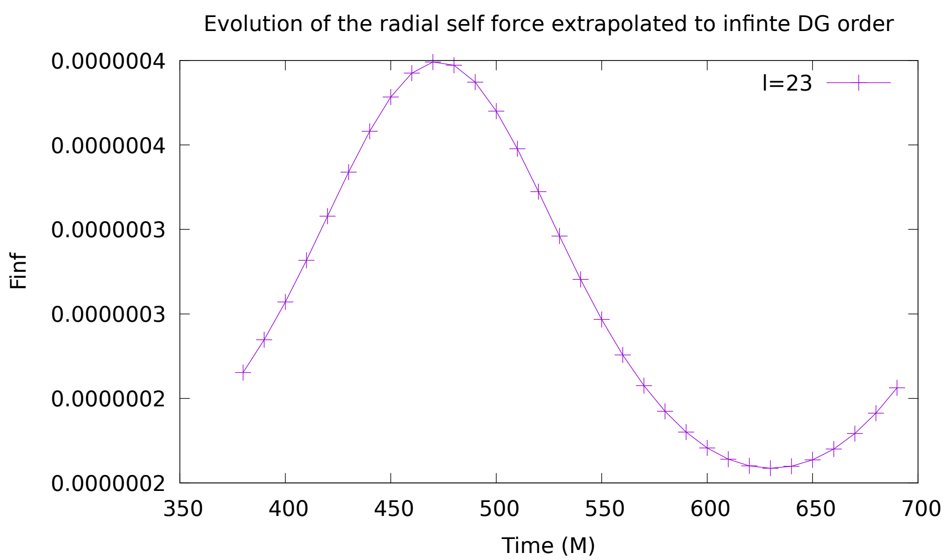
Evolution of the radial self force extrapolated to infinte DG order
0.0000007
0.0000006
0.0000006
0.0000006
0.0000005
0.0000004
0.0000004
0.0000003
0.0000003
Finf
350
400
450
500
550
600
650
700
Time (M)
l=19


Evolution of the radial self force extrapolated to infinte DG order
l=20
Finf
0.0000006
0.0000006
0.0000005
0.0000004
0.0000004
0.0000003
0.0000003
0.0000002
350
400
450
500
550
600
650
700
Time (M)

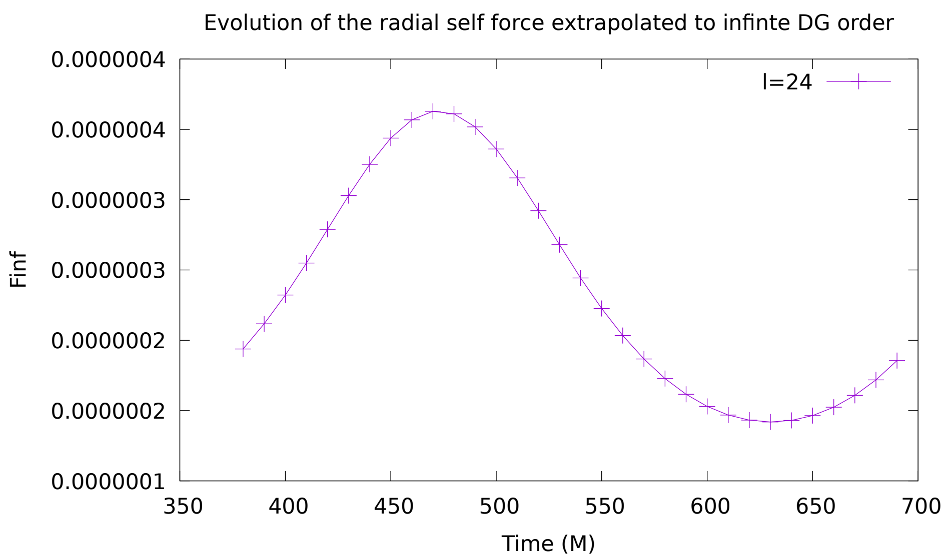

Evolution of the radial self force extrapolated to infinte DG order
l=21
Finf
0.0000006
0.0000005
0.0000004
0.0000004
0.0000003
0.0000003
0.0000002
0.0000002
350
400
450
500
550
600
650
700
Time (M)

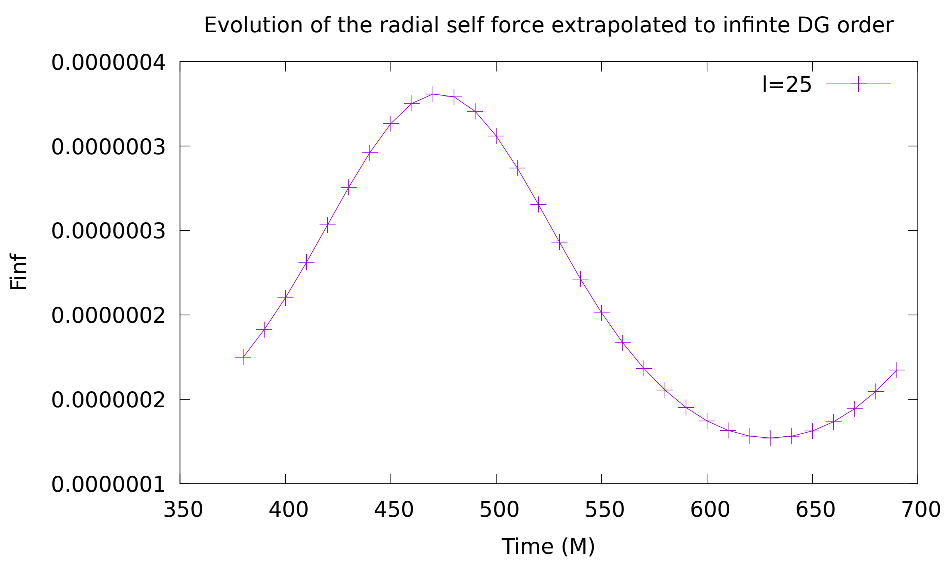


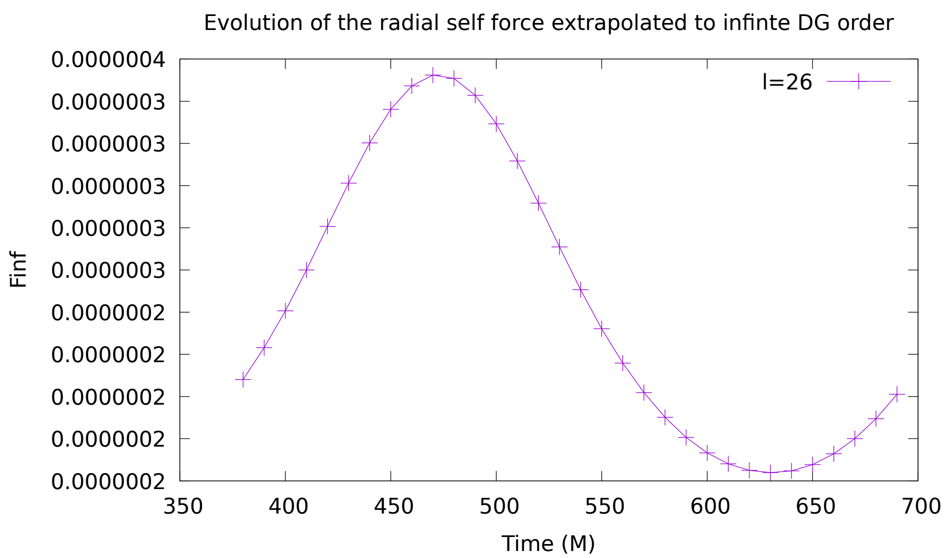
Evolution of the radial self force extrapolated to infinte DG order
l=22
Finf
0.0000005
0.0000004
0.0000004
0.0000003
0.0000003
0.0000002
0.0000002
350
400
450
500
550
600
650
700
Time (M)

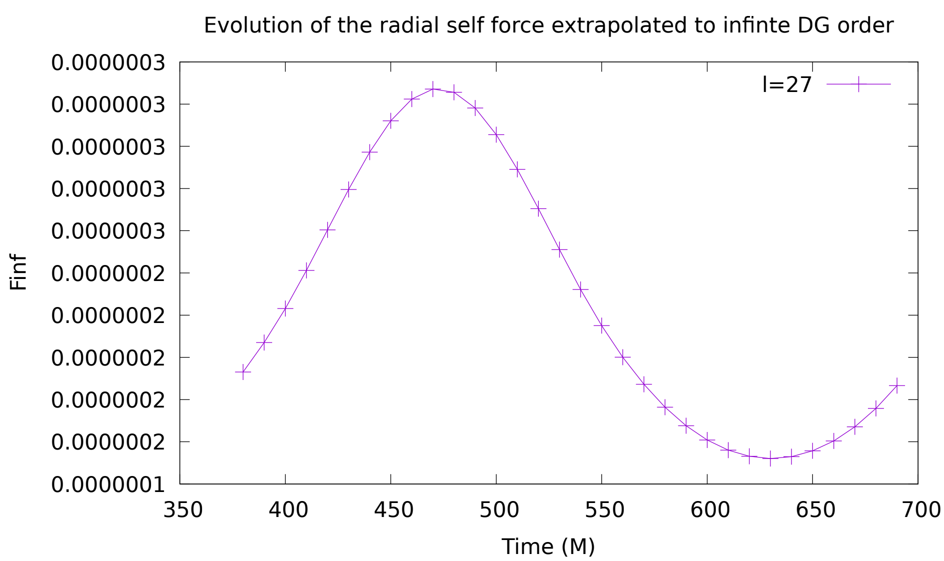


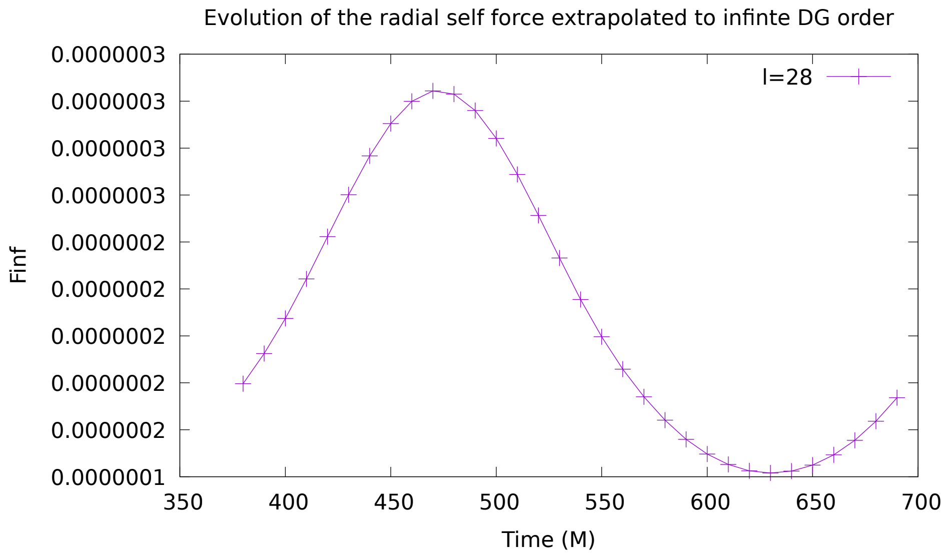
Evolution of the radial self force extrapolated to infinte DG order
l=23
Finf
0.0000004
0.0000004
0.0000003
0.0000003
0.0000002
0.0000002
350
400
450
500
550
600
650
700
Time (M)


Evolution of the radial self force extrapolated to infinte DG order
l=24
Finf
Time (M)
0.0000004
0.0000004
0.0000003
0.0000003
0.0000002
0.0000002
0.0000001
350
400
450
500
550
600
650
700

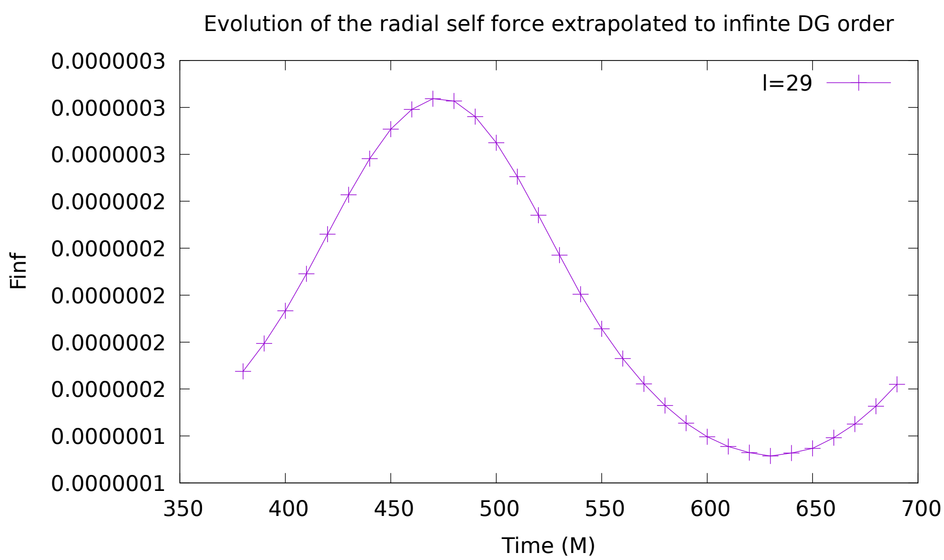

Evolution of the radial self force extrapolated to infinte DG order
l=25
Finf
Time (M)
0.0000004
0.0000003
0.0000003
0.0000002
0.0000002
0.0000001
350
400
450
500
550
600
650
700


Evolution of the radial self force extrapolated to infinte DG order
l=26
Finf
Time (M)
0.0000004
0.0000003
0.0000002
350
400
450
500
550
600
650
700

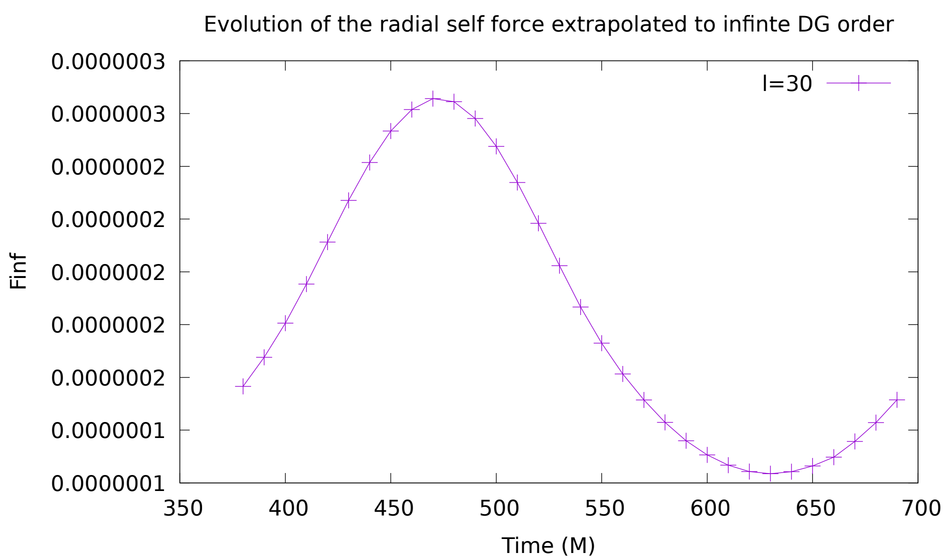

Evolution of the radial self force extrapolated to infinte DG order
l=27
Finf
Time (M)
0.0000003
0.0000002
0.0000001
350
400
450
500
550
600
650
700



Evolution of the radial self force extrapolated to infinte DG order
l=28
Finf
0.0000003
0.0000003
0.0000003
0.0000003
0.0000002
0.0000002
0.0000002
0.0000002
0.0000002
0.0000001
350
400
450
500
550
600
650
700
Time (M)



Evolution of the radial self force extrapolated to infinte DG order
l=29
Finf
0.0000003
0.0000003
0.0000003
0.0000002
0.0000002
0.0000002
0.0000002
0.0000002
0.0000001
0.0000001
350
400
450
500
550
600
650
700
Time (M)


Evolution of the radial self force extrapolated to infinte DG order
0.0000003
0.0000003
0.0000002
0.0000002
0.0000002
0.0000002
0.0000002
0.0000001
0.0000001
Finf
350
400
450
500
550
600
650
700
Time (M)
l=30

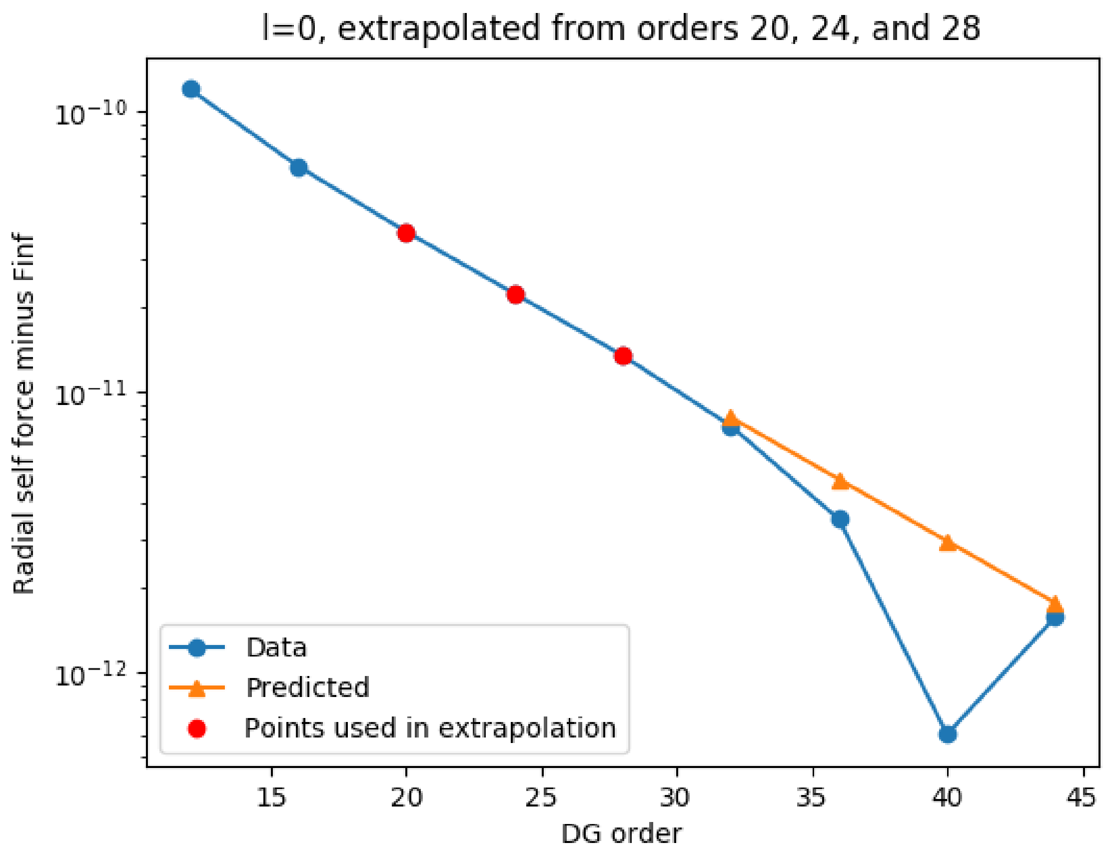
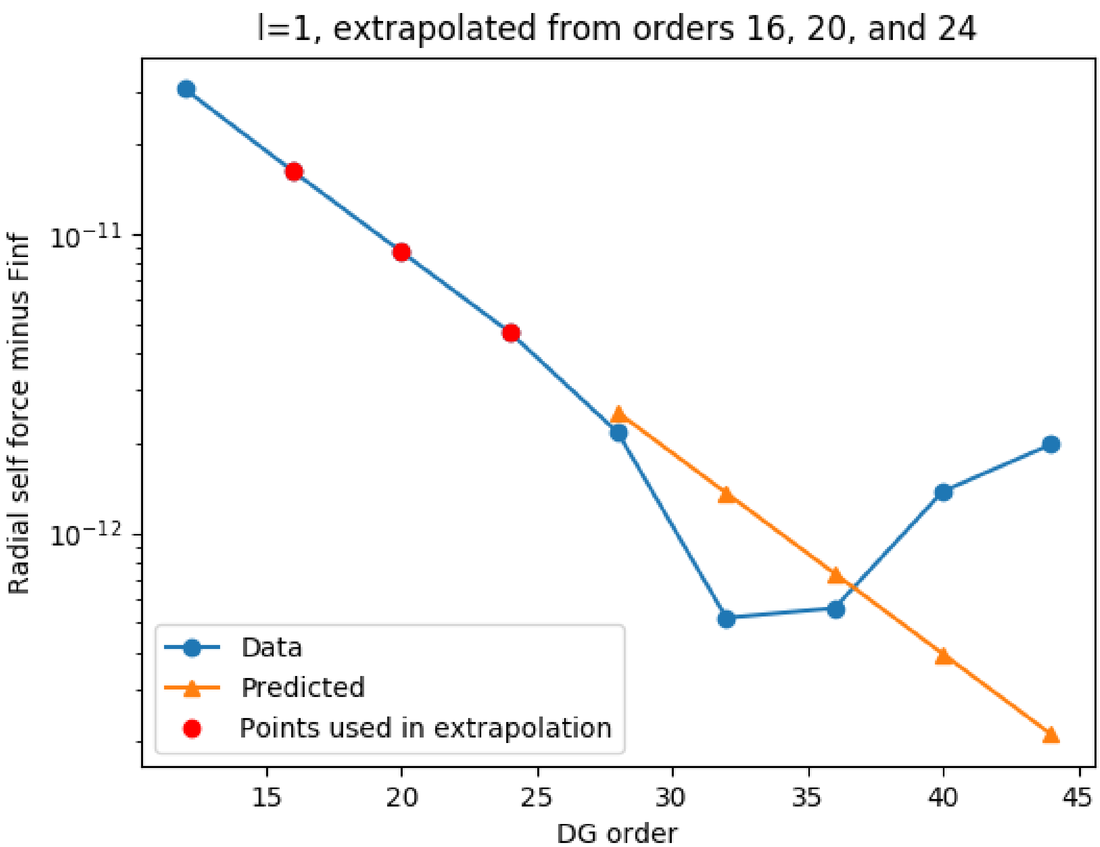
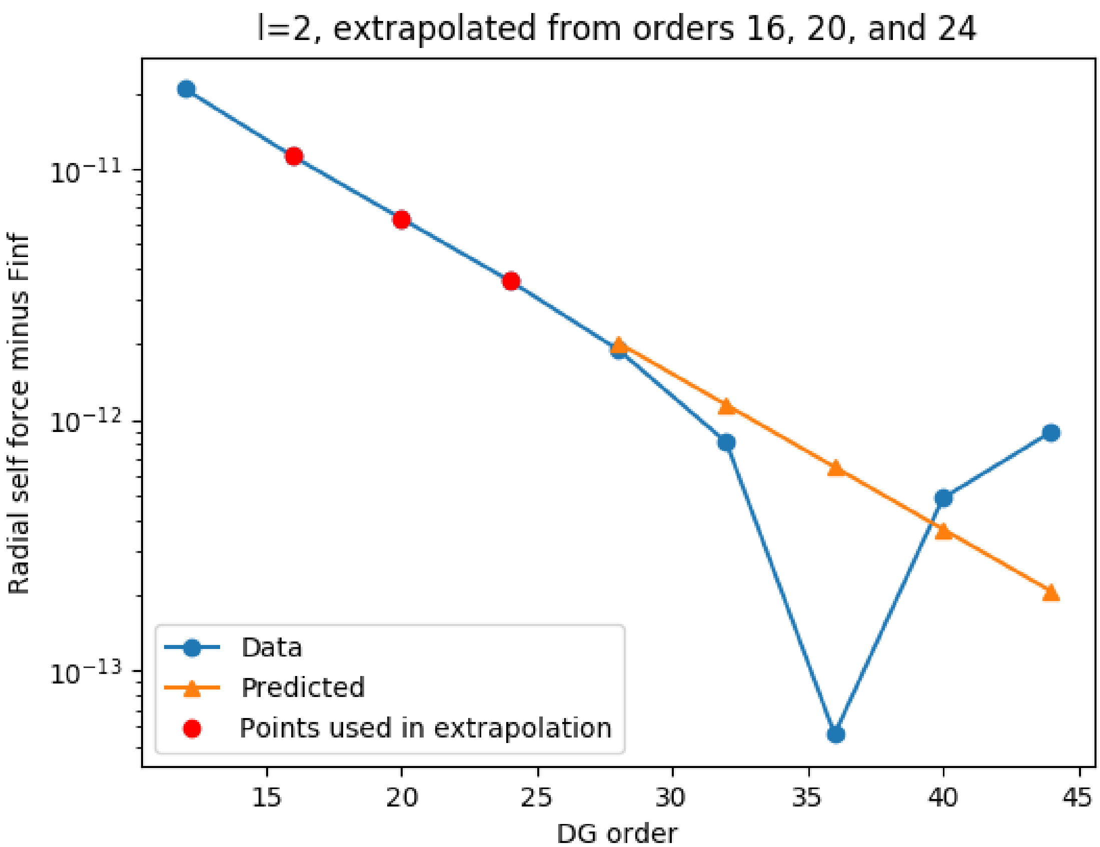
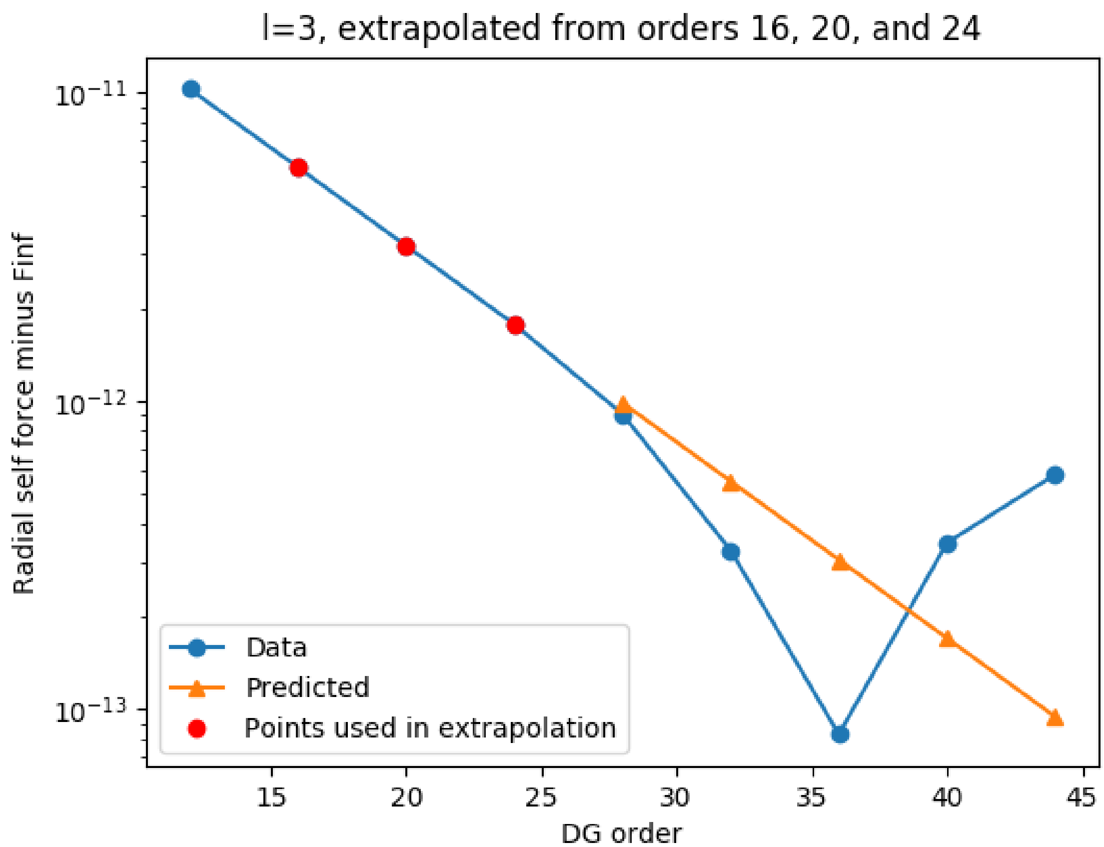


Figure 7: l=0 mode with fit-chosen starting index produces convergence plot with nice long exponentially converging region


l=1, extrapolated from orders 16, 20, and 24
Radial self force minus Finf
DG order
Data
Predicted
Points used in extrapolation


l=2, extrapolated from orders 16, 20, and 24
Radial self force minus Finf
10^{-11}
10^{-12}
10^{-13}
15
20
25
30
35
40
45
DG order
Data
Predicted
Points used in extrapolation


l=3, extrapolated from orders 16, 20, and 24
10^{-11}
10^{-12}
10^{-13}
Radial self force minus Finf
15
20
25
30
35
40
45
DG order
Data
Predicted
Points used in extrapolation

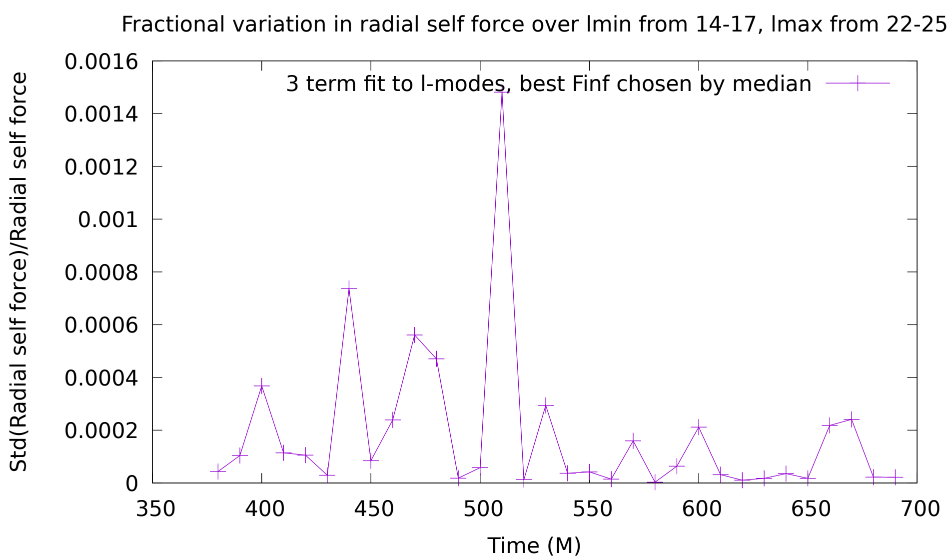


Figure 8: 3 term, median method

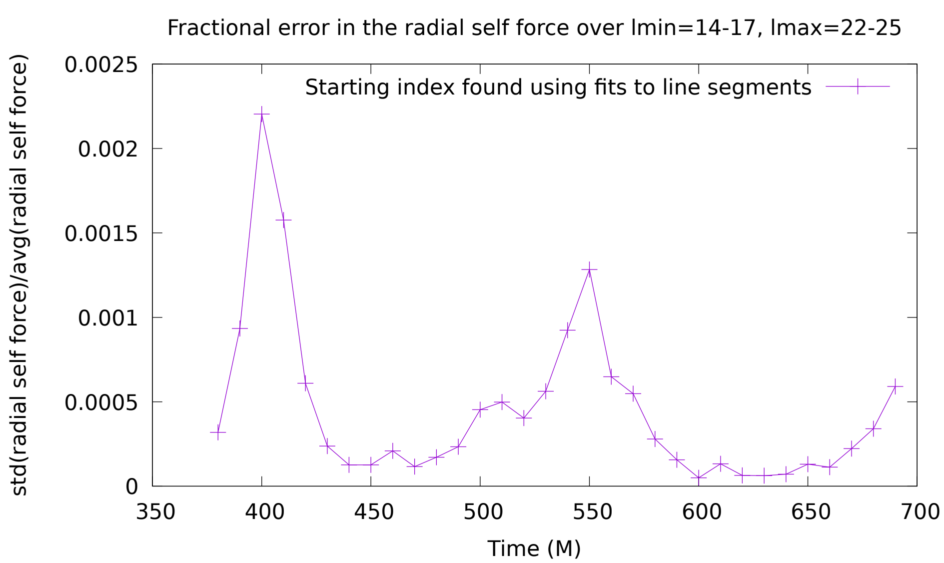


Figure 9: 3 term, fit method

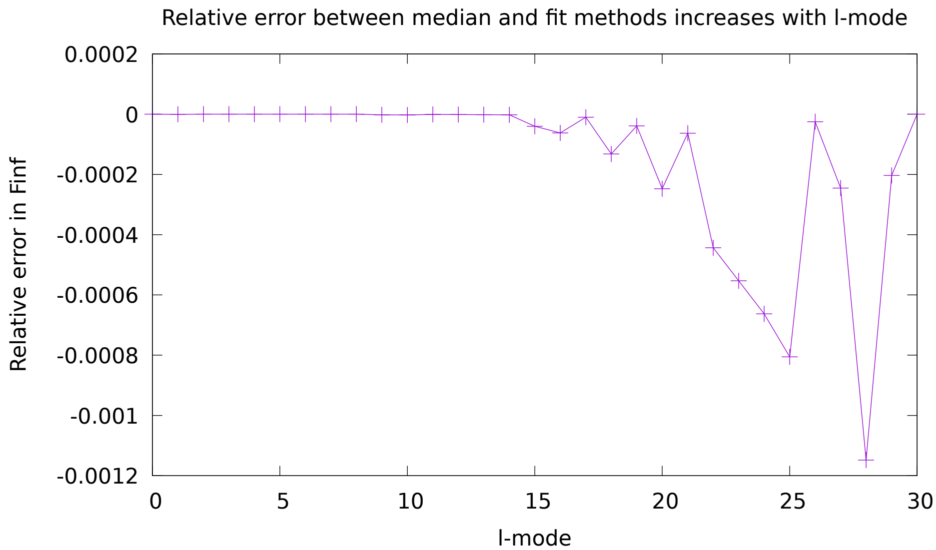


Figure 10: Relative error between fit and median techniques increases with l-mode

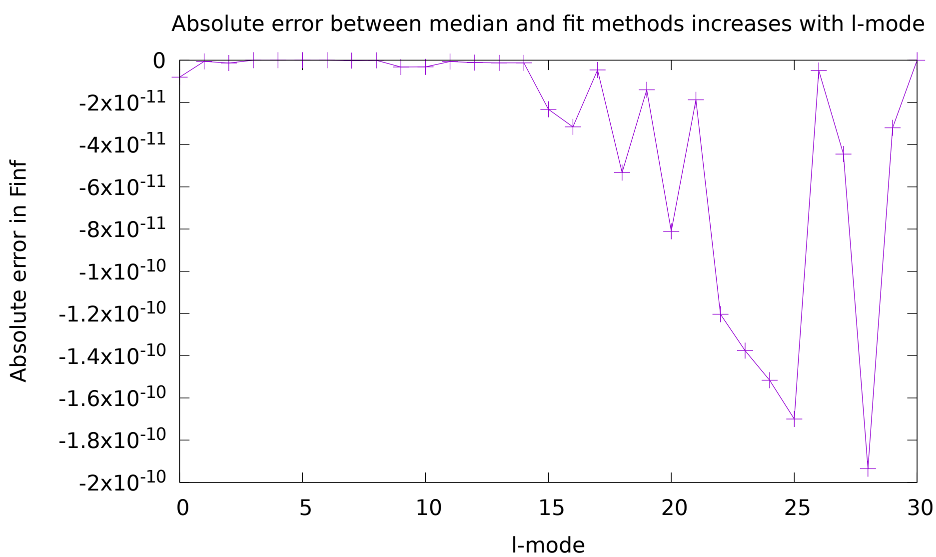


Figure 11: Absolute error between fit and median techniques increases with l-mode

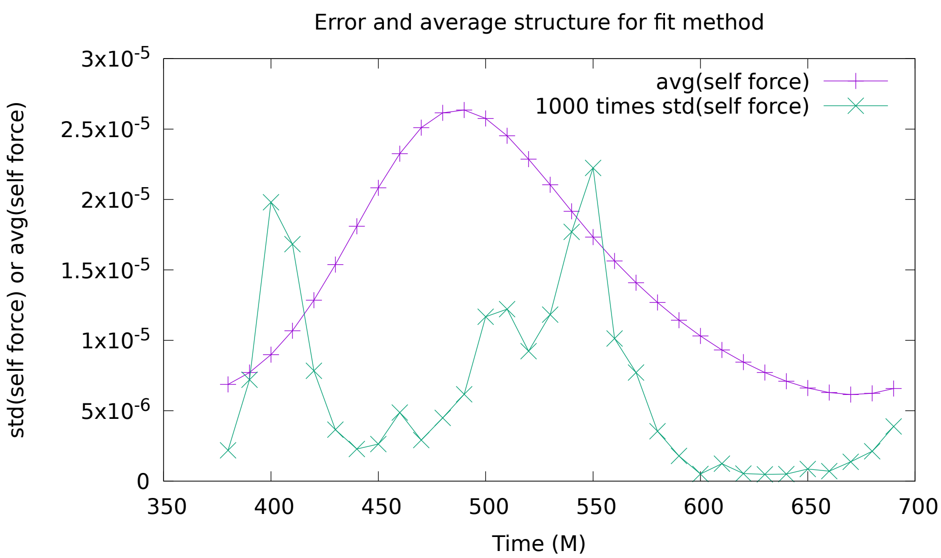


Figure 12: The structure of the absolute error in comparison to the evolution in time for the fit method