

Numerical methods for Extreme Mass Ratio Inspirals of Black Hole Binaries

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Toward LISA EMRI templates

- ▶ Generating LISA EMRI templates requires evolving 10^6 orbits with precision on the order of $\delta P/P \sim 10^{-6}$ *Danzmann, Karsten. LISA: A proposal in response to the ESA call for L3 mission concepts (2017)*
- ▶ For EMRI's, the self-force approximation is used in the limit where the mass ratio is large (10^4 to 10^6)
- ▶ EMRI's have an orbital evolution timescale that scales as M/μ , and a period that scales as M . These two widely different timescales necessitate a different numerical approach than numerical relativity.
- ▶ Self-force prescribes a perturbative approximation in the mass ratio.

Self-force in general relativity

- ▶ In general relativity, test particles move along geodesics
- ▶ A compact object is not a test particle
- ▶ Motion \rightarrow radiation \rightarrow energy and angular momentum loss \rightarrow inspiral
- ▶ Applies to scalar, electromagnetic, and tensor fields on a gravitational background
- ▶ Perturbative expansion in powers of the mass ratio

Goals for the self-force community and this project

The long term goal for the field is to generate extremely precise EMRI gravitational wave templates for LISA.

- ▶ Scalar rather than tensor waves (Ψ rather than $h_{\mu\nu}$)
- ▶ Non-rotating black holes: Schwarzschild spacetime

The state of the art is

- ▶ Kerr eccentric inclined geodesics for scalar fields
- ▶ Kerr geodesics for gravitational fields with nearly extremal spin
- ▶ Adiabatic evolution in Schwarzschild or Kerr spacetime in the orbital plane
- ▶ Scalar unbound orbits in Schwarzschild spacetime
- ▶ Scalar Schwarzschild self-consistent evolution to low accuracy

Intermediate goals

1. To port a sample self-force simulation provided by Peter Diener from Fortran to C++ and parallelize it in HPX for the purpose of helping to develop the HPX parallelization system and numerical methods in the self-force field.
2. To assess the accuracy of Peter Diener's existing Discontinuous Galerkin eccentric orbit simulation.

A generic wave equation solver

Wave equation:

$$\square \Psi = RHS(r, t) \quad (1)$$

Can be written in state-vector form:

$$\frac{\partial u}{\partial t} = A \frac{\partial u}{\partial r} + Bu = RHS(u, t) \quad (2)$$

Use Discontinuous Galerkin method: truncation error scales as h^{N+1}

Flat spacetime evolution

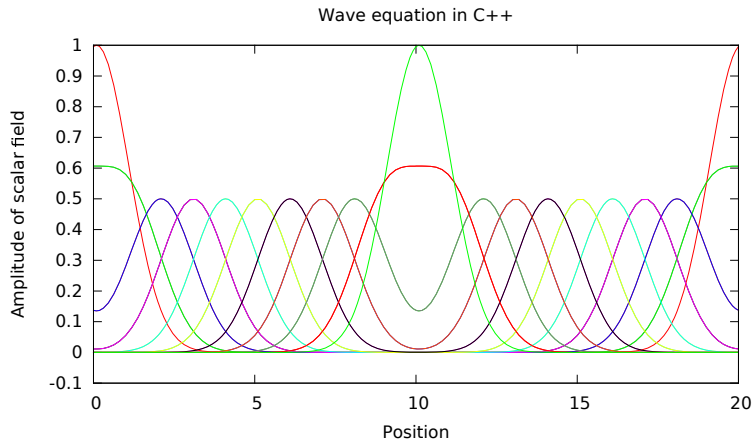


Figure: Gaussian initial conditions, flat spacetime

Flat spacetime error convergence

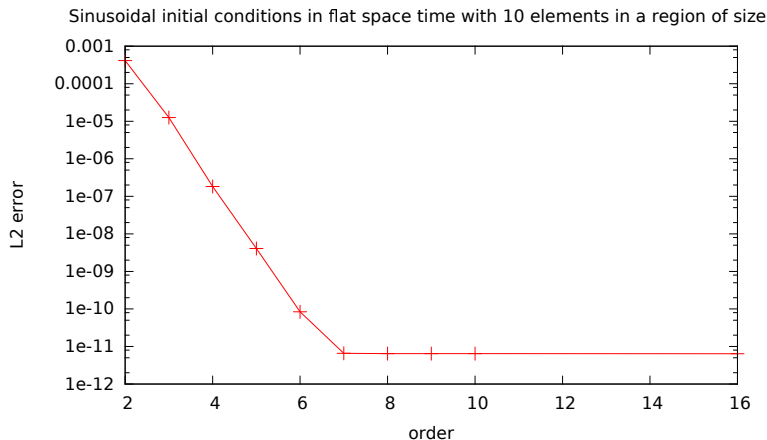


Figure: L_2 error converges exponentially until it hits roundoff noise with DG order for sinusoidal initial conditions with ten elements of size $h = 0.01$.

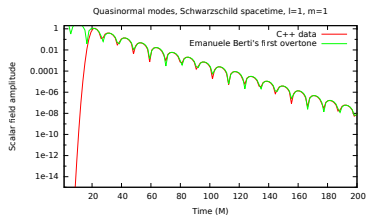
Schwarzschild spacetime without a source

- ▶ Wave equation: $\square \Psi = \frac{1}{\sqrt{-g}} \partial_\mu (g^{\mu\nu} (\partial_\nu \Psi) \sqrt{-g}) = 0$
- ▶ Multipole moment decomposition to account for angular dependence

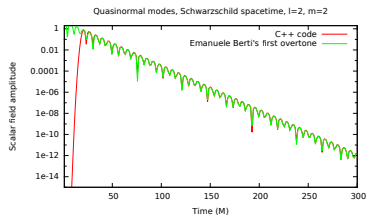
Quasinormal mode (QNM) ringing

- ▶ Similar to black hole ringdown phase after a merger
- ▶ Due to interactions of the scalar field with the background at the peak of the potential
- ▶ Higher frequencies and faster decay for higher l

Quasinormal modes



(a) $l = 1$



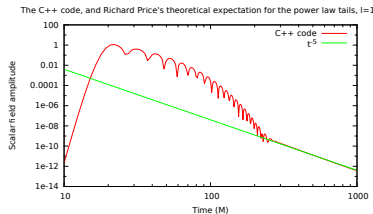
(b) $l = 2$

Figure: $l = 2$ has a higher frequency and a faster decay rate than $l = 1$

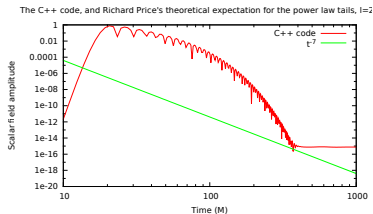
Power law tails

- ▶ Due to scattering off spacetime far from the central black hole
- ▶ Follow the QNM
- ▶ Go as $t^{-(2l+3)}$

Power law tails



(a) $l = 1$



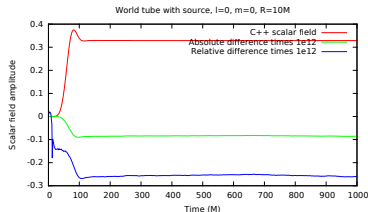
(b) $l = 2$

Figure: $l = 1$ decays as t^{-5} as expected; however, $l = 2$ has no power law tail due to truncation error

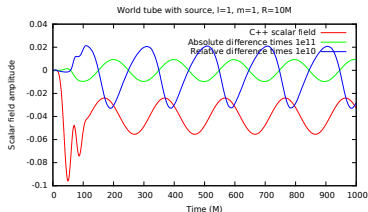
A source on a geodesic

- ▶ $\square\psi^{ret} = -4\pi q\delta_4(x, z(\tau'))d\tau'$
- ▶ Detweiler-Whiting singular field: *Steven Detweiler, Bernard F. Whiting (2002). Phys. Rev. D 67, 024025*
- ▶ Source is singular at location of particle
- ▶ Regularize field: $\psi^R = \psi^{ret} - \psi^S$
- ▶ Only know approximation to DW singular field, $\tilde{\psi}^S$
- ▶ Limit the effect of the approximation using a world tube window function
- ▶ $\square\psi^R = S_{eff} = \square\psi^{ret} - \square(W\tilde{\psi}^S)$
- ▶ HOW effective source: *Anna Heffernan, Adrian Ottewill, Barry Wardell (2013). Phys. Rev. D 82 104023*

Circular orbit roundoff error comparison between languages



(a) $l=0$



(b) $l=1$

Figure: Relative and absolute errors are at the roundoff level– 10^{-10} to 10^{-12} . Oscillations do not appear in the $l = 0$ mode but appear with the orbital period in the $l = 1$ mode.

Eccentric orbits using Peter Diener's simulation

- ▶ χ : radial evolution angle, ϕ : angular evolution angle \rightarrow precession because they are out of phase
- ▶ The orbit is held fixed on a geodesic to counteract the self-force generating the scalar waves
- ▶ $r_{\text{periastron}} = \frac{pM}{1+e}$, $r_{\text{apastron}} = \frac{pM}{1-e}$
- ▶ Radial self-force: $F_r = q\partial_r\Psi^R$

Precession of the eccentric orbit

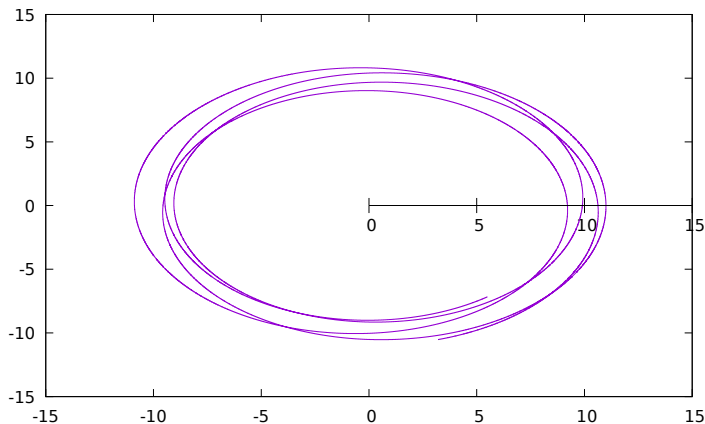


Figure: Precession of the eccentric orbit. $p = 9.9$, $e = 0.0$.

Evolution of the radial self-force for different initial conditions

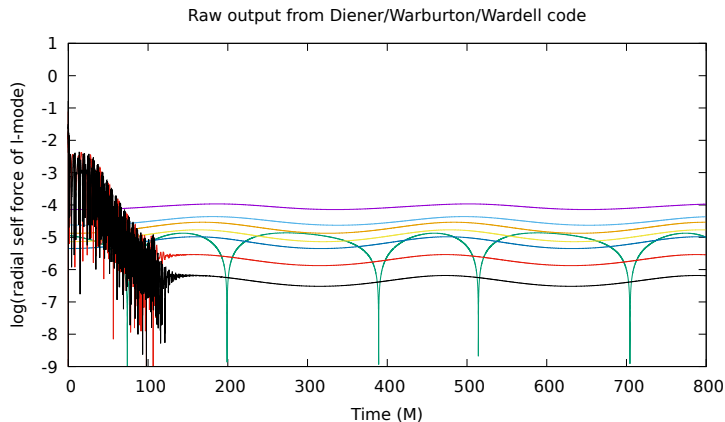


Figure: Low l behavior of the self-force for a given l -mode summed over m

Self-force spherical harmonic components depend on time

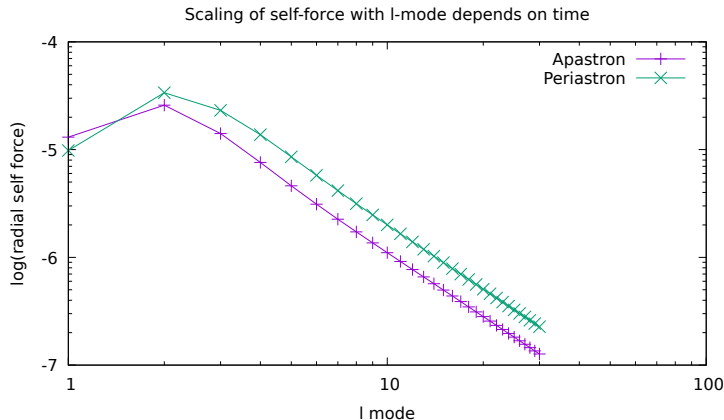


Figure: Periastron has a higher self-force amplitude for nearly every spherical harmonic than apastron.

The first order Richardson extrapolation

- ▶ Discontinuous Galerkin: ODE solver with truncation errors that scale as h^{n+1}
- ▶ Assume $F_r(n, l) = F_{inf}(l) + c(l) \exp(-\alpha n)$
- ▶ Obtain extrapolation by solving system of equations for $F_r(n_i, l)$, $i = 1, 2, 3$

Well-converging data

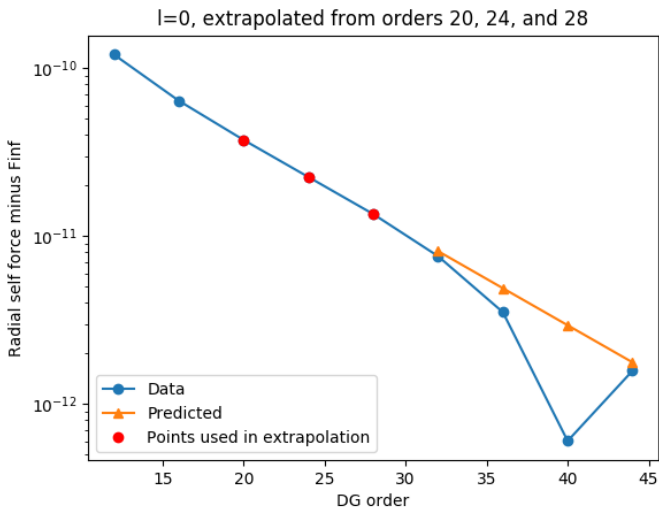


Figure: $l=0$ at $t=370$. This data converges very cleanly until it hits roundoff noise at high DG orders.

Error due to neglecting the first order Richardson extrapolation

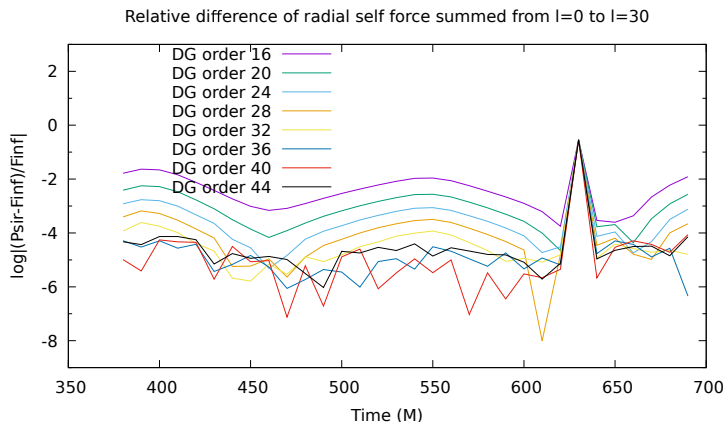


Figure: Relative error between DG starting orders and F_{inf} vs time. For DG order 36 where roundoff error sets in, the error is about 10^{-4} .

The l-mode sum and fit

- ▶ Use spherical harmonic decomposition of Ψ to encode angular information
- ▶ must sum over l and m at end to obtain self force
- ▶ Use fit to extend sum to $l = \infty$

$$F_r(l, t) = \frac{A(t)}{(2l-1)(2l+3)} + \frac{B(t)}{(2l-3)(2l-1)(2l+3)(2l+5)} \\ + \frac{C(t)}{(2l-5)(2l-3)(2l-1)(2l+3)(2l+5)(2l+7)} + \dots \quad (3)$$

Anna Heffernan, Adrian Ottewill, Barry Wardell (2012). Phys. Rev. D 86, 104023

- ▶ Fit from l_{min} to l_{max} .
- ▶ Sum numerically from zero to l_{max} then use fit coefficients to analytically sum to $l = \infty$

l-mode fit

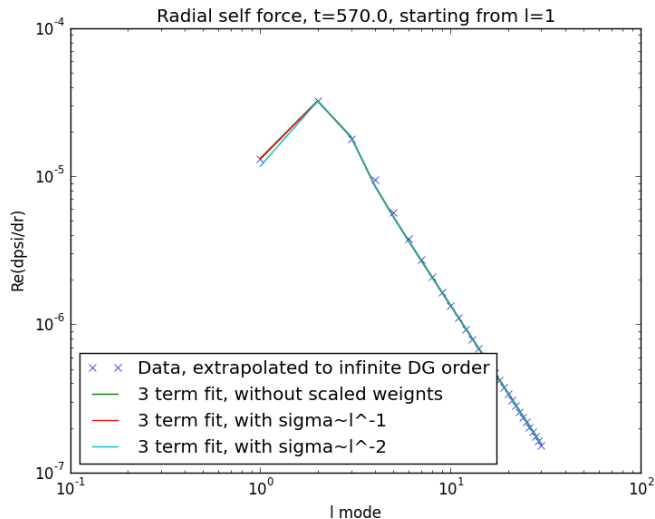
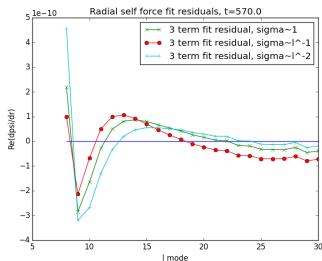
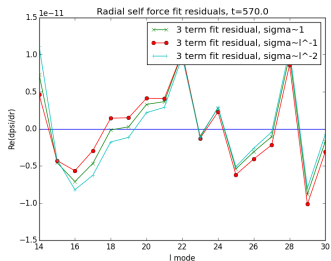


Figure: l -mode versus F_{inf} .

Residuals to the l-mode fit



(a) $l_{min} = 8$

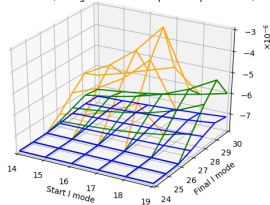


(b) $l_{min} = 14$

Figure: $l_{min} = 14$ is a better fit than $l_{min} = 8$ both because it is less systematically biased and because it has an amplitude an order of magnitude smaller. Both fits end at $l_{max} = 30$.

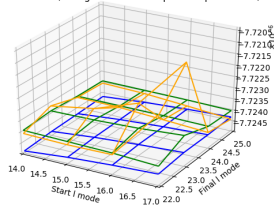
Roundoff noise in F_{inf} at high l_{max}

Total radial self force, using DG error extrapolation per l-mode, t=635



(a) Large range

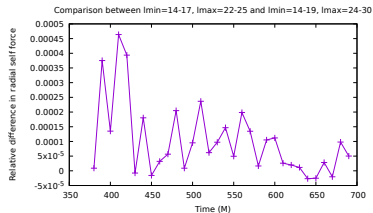
Total radial self force, using DG error extrapolation per l-mode, t=635



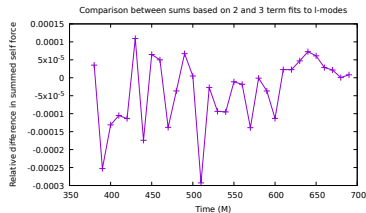
(b) Small range

Figure: $l_{min} = 14$ and $l_{max} = 25$ appear to be good start and stop values. Roundoff noise is evident at higher l .

Error due to l-mode selection, number of terms



(a) large versus small range



(b) 2 versus 3 terms

Figure: Relative errors in both of these effects appear to be at the 10^{-4} level.

Smooth evolution of total radial self-force

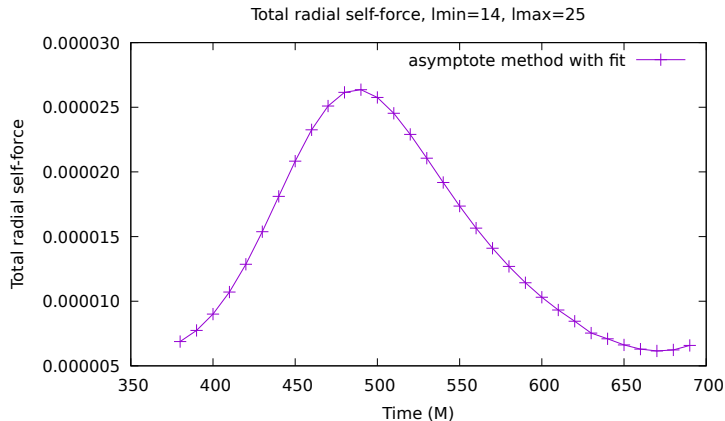


Figure: Total radial self-force including sum to $l = \infty$ over time

Error analysis conclusions

- ▶ The relative error due to neglecting the first order Richardson extrapolation is 10^{-4} . The best order at which to run is DG order 36. Limit dominated by roundoff error.
- ▶ The best $l_{min} = 14$ and $l_{max} = 25$. The relative error in these choice of values is 10^{-4} .
- ▶ The relative error due to the number of terms used in the fit is 10^{-4} .
- ▶ The error due to the use of weights in fitting is insignificant.
- ▶ These results are preliminary and need further investigation.