Strategies for computing the scalar self-force on a Schwarzschild background

Steven Dorsher

Louisiana State University

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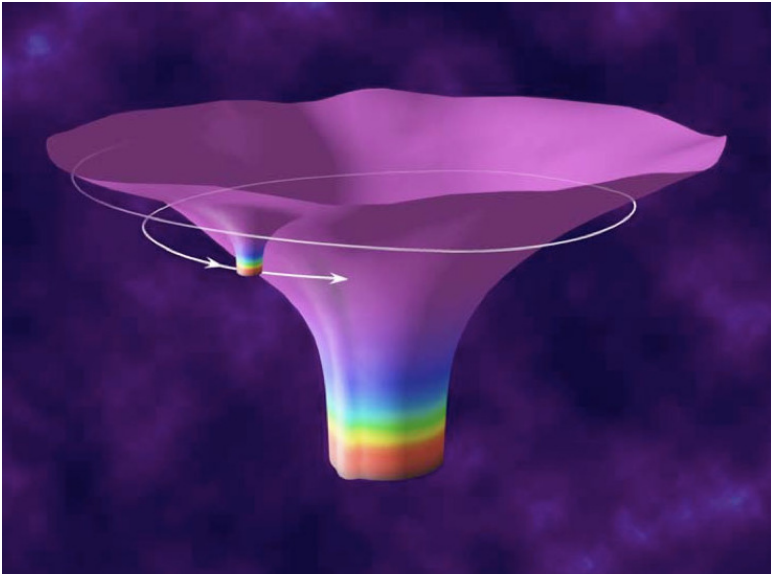
Overview

- Gravitational waves and Extreme Mass Ratio Inspirals
- The wave equation in flat spacetime
- Scalar waves on a Schwarzschild background without a source
- A scalar source on a Schwarzshild background on a circular orbit
- A scalar source on a Schwarzschild background on an eccentric orbit
- First order Richardson extrapolation
- The Discontinuous Galerkin method
- Fit to extend the mode sum to $\ell = \infty$
- Future work: a comparison of the self-consistent evolution and the geodesic evolution

Extreme Mass Ratio Inspirals



Laser Interferometer Space Antenna

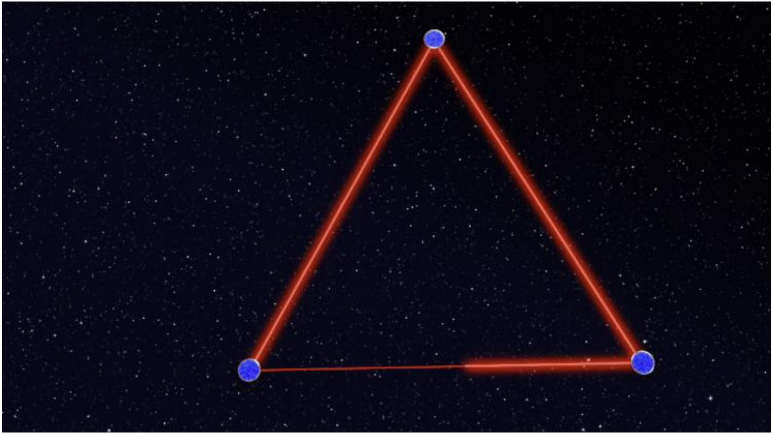
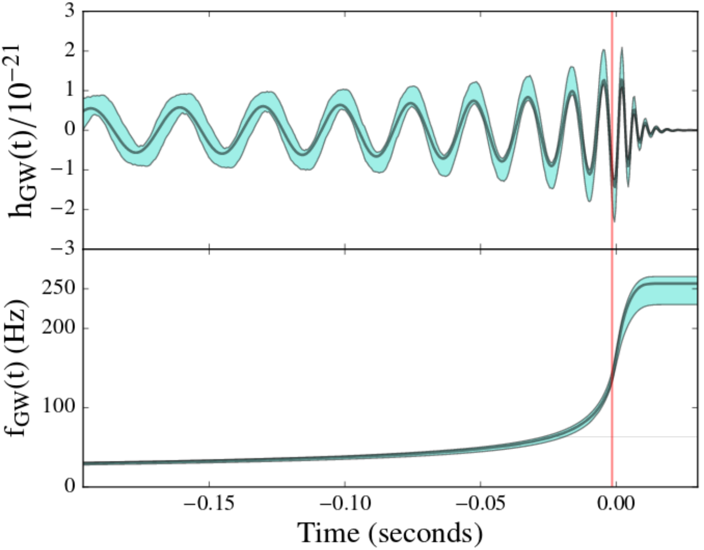


Figure: Laser Interferometer Space Antenna, which will operate around launches in early 2030's, ESA-NASA partnership, will detect EMRI's

Gravitational Waves



Self-force

- The self-force is a particle's interaction with its own field
- applies to scalar, electromagnetic, and tensor fields on a gravitational background
- motion $\rightarrow$ radiation $\rightarrow$ energy and angular momentum loss $\rightarrow$ inspiral
- electromagnetic or gravitational field: source from perturbation theory
- scalar: delta function source

Approximations and Goals

The long term goal for the field is to generate extremely precise EMRI gravitational wave templates for LISA.
Approximations:

- scalar rather than tensor waves (Ψ rather than $h_{\mu\nu}$)
- non-rotating black holes: Schwarzschild spacetime
- self-force causes a particle to inspiral as it emits radiation
- we use the Detweiler-Whiting effective source as implemented by Barry Wardell
- Niels Warburton assumes: the particle has been on the same geodesic for all time when he calculates the self-force

Our goal is to implement a highly accurate self-consistent scalar evolution of an EMRI using the self-force method and do a comparison study of the self-force at the location of the particle with Niels Warburton.
First: some simpler wave equation and self-force problems, in flat and curved spacetime

Wave equation in flat spacetime

For no source, the D'Alembertial equals zero:

$$\Box\Psi = 0 \tag{1}$$

In 1-dimension:

$$\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial x^2} = 0 \tag{2}$$

Rewrite as ODE

$$\begin{aligned} \partial_t\psi &= \rho \\ \partial_t\rho &= \partial_r\phi \\ \partial_t\phi &= \partial_r\rho \end{aligned} \tag{3}$$

Refer to $u = (\psi, \rho, \phi)$ as the state vector.

Flat space evolution

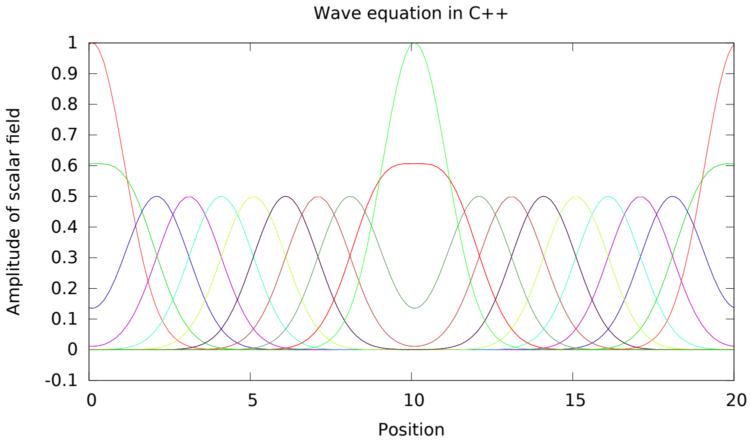


Figure: Gaussian initial conditions, flat spacetime

The Discontinuous Galerkin (DG) method

- Method for solving ODEs dependent on space and time.
- Break space into evenly spaced elements.
- Each element of order N has $N+1$ unevenly spaced nodes.
- $N+1$ interpolating polynomials run through these nodes to represent the values of the state vector.
- Numerical fluxes handle the flow of information through the ends of elements and naturally account for discontinuities that are allowed to occur there.
- The DG method returns a derivative matrix that takes a derivative across an element and a lift matrix that accounts for numerical fluxes,
- Beneficial because it has truncation error that decreases exponentially with DG order N and because it naturally handles discontinuities in ϕ and ρ.

Flat spacetime error convergence

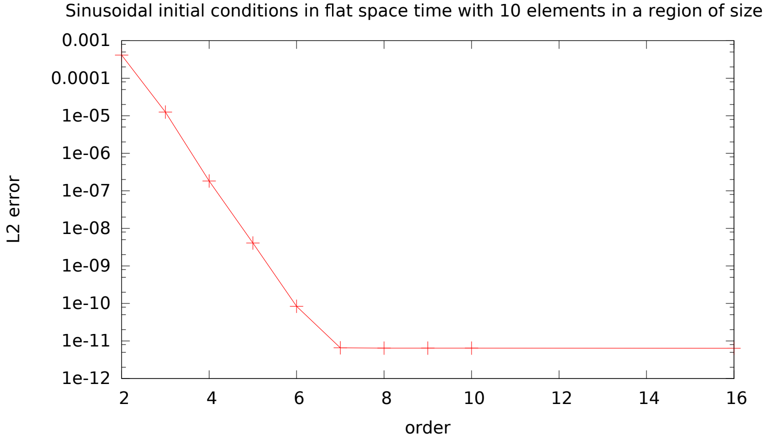


Figure: L_2 error converges exponentially until it hits roundoff noise with DG order for sinusiodal initial conditions with ten elements of size $h = 0.01$.