Signals and Systems

Lab experience 2

Politecnico di Torino

1859

Exercise 1: convolution

Exercise 1

Write a code (Matlab or Python) that computes the convolution between two signals $x[n]$ and $y[n]$:

a) Applying the definition formula: $z[n] = x[n] * y[n]$

b) Using the matrix computation: $\mathbf{z} = \mathbf{M}_y \mathbf{x}$

c) Verify the obtained results for the convolution using the built-in function: Matlab's `conv` function or Python's `numpy.convolve`.

The two signals are defined as follows:

- $x[n] = Aa^n u[n]$, where $n \in]-\infty, \infty[$, $A = 4$ and $a = -3/4$
- $y[n] = B \cdot r_{10}[n-5]$, where $B = 2$

Required outputs:

- Plot the signals and the result of the convolution;
- Compare the results obtained with the different convolution implementations;
- What is the extension of the convolution?

Discrete convolution: definition

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

- Commutative property holds:

$$y[n] = x[n] * h[n] = h[n] * x[n]$$

Discrete convolution: matrix computation

- Given $x[n]$ and $h[n]$, with finite extensions $[0, N_x - 1]$ and $[0, N_h - 1]$, respectively
- We can compute the vector $\mathbf{y}$ corresponding to $y[n]$ as:

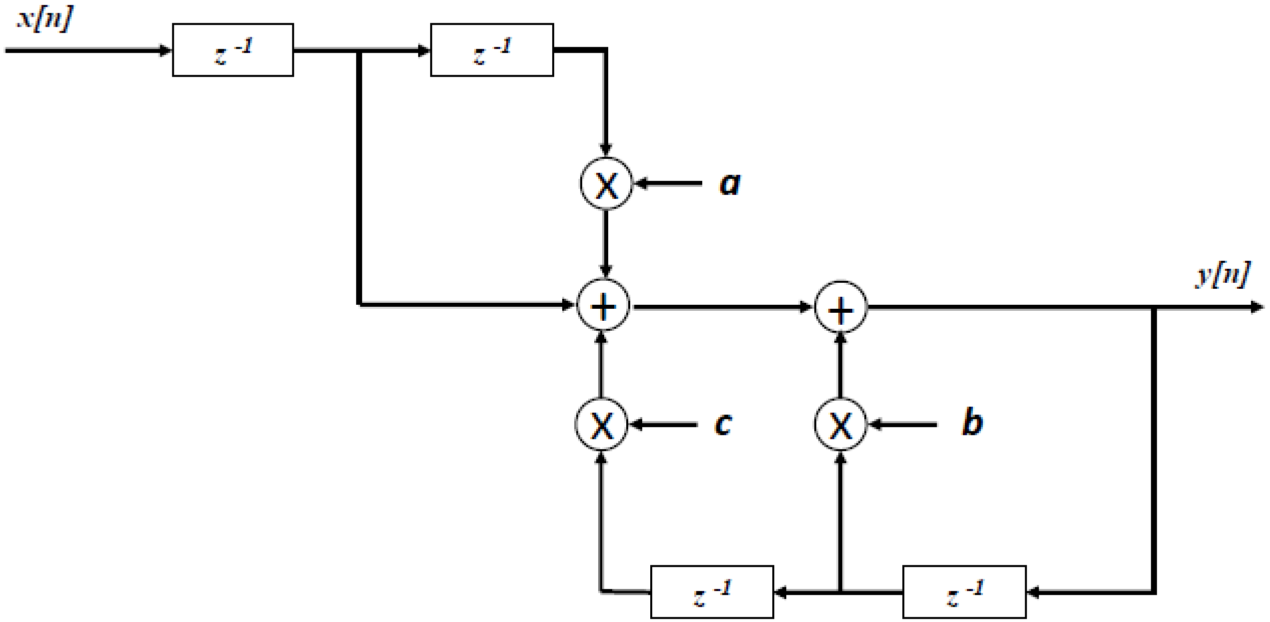
$$\boldsymbol{y} = \begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ \vdots \\ y[N_y - 1] \end{bmatrix} = \begin{bmatrix} h[0] & 0 & \cdots & 0 \\ h[1] & h[0] & \vdots & 0 \\ \vdots & h[1] & & 0 \\ h[N_h - 1] & \vdots & \ddots & 0 \\ 0 & h[N_h - 1] & & h[0] \\ 0 & 0 & \vdots & h[1] \\ 0 & 0 & & \vdots \\ 0 & 0 & \cdots & h[N_h - 1] \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N_x - 1] \end{bmatrix} = \boldsymbol{M_h x}$$

- $y[n]$ has finite extension $[0, N_y - 1]$, where $N_y = N_x + N_h - 1$

Exercise 2: LTI systems

Exercise 2

Consider the block diagram of the following LTI system:



Exercise 2: LTI systems

Required outputs

- Find the transfer function in the Z domain;
- Write the difference equation;
- Given $a = 2$, $b = 3/5$ and $c = 1/9$, determine the zeros, the poles and the region of convergence (ROC) of the LTI system;
- Determine the impulse response analytically and verify if it is correct through a script;
- Considering the input signal $x[n] = (1/a)^n u[n]$, compute the output $y[n]$ both analytically using the inverse Z transform and using a script. For a you can assume the same value defined above. Plot $x[n]$, $y[n]$ and the impulse response $h[n]$.

Exercise 3: LTI systems

Exercise 3

Consider the following difference equation $y[n] = k_1 x[n] + k_2 x[n-1] + k_3 x[n-2]$, with $k_1 = 1/4$, $k_2 = 3/4$ and $k_2 = -3/4$.

Required outputs

- Draw the block diagram and determine the transfer function in the Z domain;
- Determine the zeros and the poles;
- Determine the impulse response both analytically and using a script;
- Compute the output $y[n]$ assuming at the input the signal $x[n] = r_5[n-3]$ and verify the result using a script. Then plot $x[n]$, $y[n]$ and the impulse response $h[n]$.