Signals and Systems

Lab experience 1

Politecnico di Torino

1859

Introduction

- **Laboratory experience**
  - Ann Margareth Rosa Brusin
    - DET, old building, 3rd floor (above room 12)
    - Email: ann.rosabrusin@polito.it
  - Two teaching assistants:
    - Enes Vojka
    - Hamze Ghorbani Koujani

Dates and report deadlines

- **Lab experience 1: discrete-time signals**
  - 25th October 2024 (3h)
  - 30th October 2024 (1.5h)
  - Report deadline: 14th November 2024
- **Lab experience 2: LTI systems**
  - 15th November 2024 (3h)
  - 22nd November 2024 (3h)
  - Report deadline : 5th December 2024
- **Lab experience 3: DTFT and DFT/FFT**
  - 6th December 2024 (3h)
  - 13th December 2024 (3h)
  - Report deadline : 7th January 2025

Rules

- Each lab experience and its report can be submitted independently one from each other
  - Up to 1 point for each report
  - If you submit all the three reports → up to 3 points will be added to the final grade

- Reports MUST be submitted individually in the «Portale della Didattica»
  - ZIP file containing
    - Pdf file of the report (no other formats)
    - All codes (Matlab or Python)
  - The ZIP folder MUST be renamed as the string «s*XXXXXX_LaboratoryY*», where sXXXXXX is the student number and *Y* is the number of the lab experience

- Up to 5 pages for each report → not only figures, but also comments and analysis of the results
- We MUST be able to run your codes and they should produce the same figures present in the report

Exercise 1: Signal energy and average power evaluation

Exercise 1

Consider the signal $x[n] = a^{|n|}$ with n integer from $-\infty$ to ∞ and:

a) $|a| < 1$

b) $|a| \geq 1$

For both cases:

- Plot the signal assuming a and finite extension of your choice (you can use the `stem` function to plot the signal);
- Determine energy and average power and compare the results to the theory;
- Analyze the evolution of energy and average power for increasing extension size.

Required outputs:

- Plot of the signal;
- Report the values of energy and/or average power both from theory and numerical estimation;
- What is the minimum extension size needed to estimate the energy/average power with a relative error lower than 0.001%?

Exercise 2: Signal energy and average power evaluation

Exercise 2

Repeat exercise 1 considering the signals $x[n] = exp(j4\pi n/N)$ and $y[n] = exp(j8\pi n/N)$ for n in [0,N-1], N integer and >1. Verify if x[n] and y[n] are orthogonal in [0,N-1].

Required outputs:

- Plot of the signals;
- Report the values of energy and/or average power both from both theory and numerical estimation;
- Show if the two signals are orthogonal in $[0, N-1]$.

Energy and average power

- For a discrete-time signal, the **energy** is defined as

$$E\{x[n]\} = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \langle x[n], x[n] \rangle$$

- Instead, the **average power** is given by

$$P\{x[n]\} = \lim_{N \to +\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Exercise 3: Orthonormal basis

Exercise 3

Given the signal $x[n] = 3\delta[n] + 2\delta[n-1] - \delta[n-2] + 4\delta[n-3]$, determine its extension and consequently the value of N.
Considering the following basis:

a) The canonical basis $\mathbf{U}_\delta$

b) The Fourier basis $\mathbf{U_F}$

c) An orthonormal basis invented by you

with size $N \times N$, compute the coefficients obtained by projecting $x[n]$ on the new basis. This corresponds to computation of the vector $\mathbf{c} = \mathbf{U}^\dagger \mathbf{x}$. Then write the signal using the new basis.
Suggestion: to generate $\mathbf{U}_\delta$ and $\mathbf{U_F}$, write a general function that works for any N. Be careful with the operation of conjugation of complex numbers.

Required output:

- Figure representing the computed vector, i.e. the signal projected on the new basis;
- The expression of the signal using the new basis;

Orthonormal basis for signal representation

- A set B of N orthonormal signals $B = \{u_0[n], u_1[n], \ldots, u_{N-1}[n]\}$ is said to be orthonormal if
  - All its signals have energy equal to 1
  - They are all orthogonal to each other
- We can define the matrix **U**

$$\mathbf{U} = \left(\begin{bmatrix} \\ \mathbf{u}_0 \\ \\ \end{bmatrix} \begin{bmatrix} \\ \mathbf{u}_1 \\ \\ \end{bmatrix} \cdots \begin{bmatrix} \\ \mathbf{u}_{N-1} \\ \\ \end{bmatrix} \right) = \begin{pmatrix} u_0[0] & \cdots & u_{N-1}[0] \\ \vdots & \ddots & \vdots \\ u_0[N-1] & \cdots & u_{N-1}[N-1] \end{pmatrix}$$

- This allows to write: $\boldsymbol{x} = \boldsymbol{U}\boldsymbol{c}$ and $\boldsymbol{c} = \boldsymbol{U}^{\dagger}\boldsymbol{x}$

where $\boldsymbol{c}$ is the vector of coefficients c_k given by projecting the vector $\boldsymbol{x}$ on the basis $\boldsymbol{u_k}$

Canonical basis

$$\mathbf{U}_\delta = \mathbf{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$with\ size\ N \times N$

$$B_\delta = \{\delta[n], \delta[n-1], \dots, \delta[n-N+1]\}$$

Fourier basis

$$\mathbf{U}_F = \mathbf{F} = \left(\begin{bmatrix} \\ \mathbf{u}_0 \\ \\ \end{bmatrix} \begin{bmatrix} \\ \mathbf{u}_1 \\ \\ \end{bmatrix} \cdots \begin{bmatrix} \\ \mathbf{u}_{N-1} \\ \\ \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{N-1} \\ 1 & w^2 & w^4 & \cdots & w^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{N-1} & w^{2(N-1)} & \cdots & w^{(N-1)(N-1)} \end{pmatrix}$$

$where\ w = e^{\frac{j2\pi}{N}}$ and size $N \times N$

- The k-th column of the matrix is the signal $u_k[n]$:

$$u_k[n] = \frac{1}{\sqrt{N}} e^{j2\pi kn/N}$$

$$= \frac{1}{\sqrt{N}} \left[\cos(2\pi kn / N) + j \sin(2\pi kn / N) \right], \quad n = 0, 1, \ldots, N-1$$

Exercise 4: Orthonormal basis

Exercise 4

Write a function that generates the Walsh basis $\mathbf{U_W}$ in case of N=128.
Given the sequence $x[n] = r_{88}[n-5]$ defined in $[0, N\text{-}1]$ with $N = 128$, compute the vector of coefficients $\mathbf{c}$ considering:

a) The canonical basis $\mathbf{U}_\delta$

b) The Fourier basis $\mathbf{U_F}$

c) The Walsh basis $\mathbf{U_W}$

In addition, verify the Parseval equality.

Required output:

- Figure of the original signal;
- Figure of the computed vector, i.e. the signal projected on the new basis;
- Output if the Parseval equality is verified or not.

Walsh basis

- It has applications in UMTS systems (3G)

$$\boldsymbol{U}_{w,2} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \boldsymbol{U}_{w,4} = \frac{\sqrt{2}}{\sqrt{4}}\begin{bmatrix} \boldsymbol{U}_{w,2} & \boldsymbol{U}_{w,2} \\ \boldsymbol{U}_{w,2} & -\boldsymbol{U}_{w,2} \end{bmatrix} = \frac{1}{\sqrt{4}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

- It can be generalized as

$$\boldsymbol{U}_{w,2N} = \frac{\sqrt{N}}{\sqrt{2N}}\begin{bmatrix} \boldsymbol{U}_{w,N} & \boldsymbol{U}_{w,N} \\ \boldsymbol{U}_{w,N} & -\boldsymbol{U}_{w,N} \end{bmatrix}$$