Appendix I. Derivation of Buyer's Value Function

Buyer Utility. Following the platform competition literature (e.g., Kang & Downing, 2015; Nair et al., 2004; Zhu & Iansiti, 2012), we specify the buyer *i*'s utility from platform *k* in time *t* as a function of total non-size-based value, Q_{ikt}, and the bundle of goods purchased by the buyer, g_{kt}:

$$v_{ikt} = \ln(Q_{ikt} g_{kt})$$

where the bundle of goods $g_{kt} = \left(\sum_{j=1}^{J_{kt}} x_{jkt}^{\beta}\right)^{\frac{1}{\beta}}$ follows the constant elasticity of substitution (CES) form ($\beta > 1$) (Dixit & Stiglitz, 1977); x_{jkt} is the amount of goods that the buyer purchases from seller *j* in platform *k* at period *t*, and J_{kt} is a number of sellers in platform *k* in period *t*.

Utility Maximization Problem. The buyer's utility maximizing choice is which individual goods to buy on the platform that she adopts, seeking to maximize $\sum_{j=1}^{J_{kt}} x_{jkt}^{\beta}$ subject to the budget constraint $Y \geq \sum_{j=1}^{J_{kt}} p_{jkt}\, x_{jkt}$. This makes the problem equivalent to the following Lagrangian: $\mathcal{L} = \sum_{j=1}^{J_{kt}} x_{jkt}^{\beta} + \lambda\left(Y - \sum_{j=1}^{J_{kt}} p_{jkt}\, x_{jkt}\right)$, where λ is the Lagrange multiplier.

Marshallian Demand. Differentiating with respect to x_{jkt} gives $x_{jkt} = (\beta\lambda p_{jkt})^{\frac{\beta}{1-\beta}}$. We can substitute this into the budget constraint to yield $Y = \sum_{j=1}^{J_{kt}} p_{jkt}\,(\beta\lambda p_{jkt})^{\frac{\beta}{1-\beta}}$ and rearrange to show $(\beta\lambda)^{\frac{\beta}{1-\beta}} = \frac{Y}{\sum_{j=1}^{J_{kt}} p_{jkt}^{1/(1-\beta)}}$. Plugging this quantity back into the first order condition for x_{jkt}, we arrive at the optimal demand per seller: $x_{jkt}^{*} = (\beta\lambda)^{\frac{\beta}{1-\beta}}(p_{jkt})^{\frac{\beta}{1-\beta}} = \frac{Y p_{jkt}^{\beta/(1-\beta)}}{\sum_{j=1}^{J_{kt}} p_{jkt}^{1/(1-\beta)}}$. Following the literature (e.g., Nair et al., 2004; Zhu & Iansiti, 2012), we assume that the effect of a single product's price is negligible on the aggregate price index, denoted $\theta_{kt} = \left(\sum_{j=1}^{J_{kt}} p_{jkt}^{1/(1-\beta)}\right)^{1-\beta}$. Employing the derivations in (Zhu & Iansiti, 2007, 2012), we make use of the symmetric equilibrium that can be shown to exist for product prices between platforms, namely $p_{jkt} = p_k = \beta c_k^s$ (assuming no CSR or $\beta(c_k^s + \psi_k^s)$ with CSR), where c_k^s and ψ_k^s are the baseline and incremental CSR marginal costs, respectively, for sellers on platform *k*,. The demand for each product can then be shown by substituting the price index term back into the optimal demand expression above $x_{jkt}^{*} = \frac{Y\theta_{kt}}{p_{jkt}^{\beta/(\beta-1)}} = \frac{Y}{J_{kt} p_k}$, which gives the demand as a function of price and budget constraint.

Value Function. Substituting this optimal demand expression $x_{jkt}^{*} = \frac{Y}{J_{kt} p_k}$ into the buyer's utility function above yields the buyer's value (indirect utility) function, which equals $\left(\frac{Y}{p_{kt}}\right) Q_{ikt} J_{kt}^{\varepsilon}$, with the cross-group network effect term $\varepsilon = \beta - 1$. Finally, a log transform ensures a smooth twice-differentiable ($V' > 0, V'' < 0$) value function, which produces the form of Eq. (1) in the text: $V_{ikt} = \ln\left(\frac{Y}{p_{kt}}\right) + \ln Q_{ikt} + \varepsilon \ln J_{kt}$.

Appendix II. Derivation of CSR Price Premium

The CSR price premium for the rival platform 2 is defined as the price increase of platform 2 above platform 1, relative to platform 2's price, at which the demand is equal between platforms when platform 2 engages in CSR and platform 1 does not:

$$\overline{\zeta_{\iota 2t}} \equiv \left(\frac{p_{2t}-p_{1t}}{p_{2t}}\right)\Bigg|_{V_{1t}(\sigma_{1k}=0)=V_{2t}(\sigma_{2k}=1)}$$

Start with this definition and utilize the value function from Eq. (1) to exponentiated and rearrange the equivalency, yielding the price premium as a function of the unobserved buyer type:

$$V_{1t}(\sigma_{1k}=0)=V_{2t}(\sigma_{2k}=1) \Leftrightarrow \left(\frac{p_{2t}-p_{1t}}{p_{2t}}\right)=\left(\frac{u_2+\omega z_i}{u_1}\right)\left(\frac{J_{2t}}{J_{1t}}\right)^{\varepsilon}-1=\zeta_{i2t}$$

We can then take the expectation over the distribution of buyer types to yield the expected CSR price premium:

$$\overline{\zeta_{2t}} \equiv \mathbb{E}_{z_i}[\zeta_{i2t}|q]=\left(\frac{u_{2t}+\omega q}{u_{1t}}\right)\left(\frac{J_{2t}}{J_{1t}}\right)^{\varepsilon}-1$$

Appendix III. Derivation of Best Response Platform Strategy

Platform Profit. First, we must specify the platform's profit function. This is comprised of the platform's marginal profit multiplied by its demand share and the buyer's budget-price ratio of the platform, minus the platform's initial fixed cost: $\pi_{1it}^{p}=\pi(\sigma_{1t}^{p},z_i)=\Delta\pi(\sigma_{1t}^{p})\cdot s(\sigma_{1t}^{p},z_i)N\cdot\frac{Y}{p_{kt}(\sigma_{1t}^{p})}-F(\sigma_{1t}^{p})$. We can then specify the platform profit in terms of a given CSR response strategy facing an individual buyer type (ignoring for the moment the probability of that buyer type existing in the market):

$$\pi_{1it}^{p}=\begin{cases} s_{kt}^{CSR|H}\cdot\dfrac{\Delta\pi_{kt}^{CSR}}{p_{kt}^{CSR}}NY-(E_k+R_k)\,, & \text{if } \sigma_{kt}^{p}=\text{CSR}, z_i=\text{ C}\\ s_{kt}^{CSR|U}\cdot\dfrac{\Delta\pi_{kt}^{CSR}}{p_{kt}^{CSR}}NY-(E_k+R_k), & \text{if } \sigma_{kt}^{p}=\text{CSR}, z_i=\text{P}\\ -E_k, & \text{if } \sigma_{kt}^{p}=\text{No}, z_i=\text{ C}\\ \dfrac{\Delta\pi_{kt}^{No}}{p_{kt}^{No}}NY-E_k, & \text{if } \sigma_{kt}^{p}=\text{No}, z_i=\text{ P}\end{cases}$$

where E_k is an exogenous fixed cost and R_k is an incremental fixed cost resulting from CSR engagement. This assumes that if platform 1 doesn't use CSR (only platform 2 does), then a price-sensitive buyer will always choose platform 1 while a CSR buyer will always choose platform 2. In the case that both use CSR, the platforms split the buyer's demand in proportion to the conditional logit function of the buyer's value from each platform.

Platform Signaling Threshold: Pooling Strategy $A^C A^P$. Consider first the pooling stragy where a buyer would abstain from voting with the wallet regardless of her type, $A^C A^P$. Given this strategy profile for the buyer and the amount of observed voting with the wallet by buyer playing this strategy, the platform 1 seeking to maximize its profit only engages in CSR when the expected profit of a CSR strategy is greater than the expected profit of not engaging in CSR:

$$\mathbb{E}_z\left[\pi_{1t}^{p}\left(\sigma_{1t}^{p}=CSR,\sigma_{2t}^{p}=CSR\right)|A^C A^P,q_t^{post}\right]>\mathbb{E}_z\left[\pi_{1t}^{p}\left(\sigma_{1t}^{p}=No,\sigma_{2t}^{p}=CSR\right)|A^C A^P,q_t^{post}\right]\Leftrightarrow q_t^{post}>q_{kt}^{*}$$

Expanding this inequality by the definition of expectation, we can substitute in the platform profit values from above for each combination of buyer type and CSR strategy. Then we rearrange and simplify to show that the above inequality is equivalent to the following inequality $q_t^{post} > q_{kt}^*$, for $q_{kt}^* = \frac{(\phi_{kt}^{No} - s_k^{CSR|P}\phi_{kt}^{CSR})NY + R_k}{(\phi_{kt}^{No} + \Delta s_k^{CSR}\phi_{kt}^{CSR})NY}$, where the marginal profit per price-dollar ratios are $\phi_{kt}^{CSR} = \Delta\pi^{CSR}/p_{kt}^{CSR}$ when engaging in CSR, $\phi_{kt}^{NO} = \Delta\pi^{No}/p_{kt}^{No}$ otherwise, and the CSR demand difference is $\Delta s_{kt} = s_k^{CSR|C} - s_k^{CSR|P}$. Thus CSR is the optimal response when, following the observed voting with the wallet, an estimate of the CSR probability $\widehat{q_t} \equiv \mathbb{E}_X\left[q_t^{post}\right] = \frac{x_t + \alpha_{1t}}{N + \alpha_{1t} + \alpha_{2t}}$ is greater than the platform's threshold signaling proportion q_{kt}^* for buyer strategy profile $A^C A^P$, that is $\hat{q}_t > q_{kt}^*$.

Platform Signaling Threshold: Separating Strategy $W^C A^P$. Next, consider the separating strategy wherein a buyer would vote with the wallet if she were CSR type but abstain if price-sensitive, $W^C A^P$. Given this strategy profile and the observed voting with the wallet, platform1 seeking to maximize its profit engages in CSR only when the expected profit of doing so exceeds the expected profit of not engaging in CSR:

$$\mathbb{E}_z\left[\pi_{1t}^p\left(\sigma_{1t}^p = CSR, \sigma_{2t}^p = CSR\right)|W^C A^P, q^{post}\right] > \mathbb{E}_z\left[\pi_{1t}^p\left(\sigma_{1t}^p = No, \sigma_{2t}^p = CSR\right)|W^C A^P, q^{post}\right] \Leftrightarrow q^{post} > q_{kt}^*$$

By the same manner as the previous strategy profile we can expand the expectations and substitute in the profit values for each buyer type and CSR strategy. Then the CSR strategy is a best response when the posterior estimate of the CSR probability is greater than the platform's signaling threshold, $\hat{q}_t > q_{kt}^*$, where that separating threshold is $q_{kt}^* = \frac{\left(\phi_{kt}^{No} - \left(s_k^{CSR|C} + s_k^{CSR|P}\right)\phi_{kt}^{CSR}\right)NY + 2R_{kt}}{\left(\phi_{kt}^{No} - s_{kt}^{CSR|P}\phi_{kt}^{CSR}\right)NY + R_{kt}}$.

Appendix IV. Derivation of Optimal Buyer Strategy

Buyer Strategy Decision. A rational buyer will vote with the wallet when the net gain of doing so exceeds the value of not doing so, considering the unknown strategy of platform 1 and unknown strategy of "other buyer" b_{-i}. This can be expressed in the following inequality:

$$\mathbb{E}_{Z_{-i}\in\{C,P\}}\left[\mathbb{E}_{\sigma_{1t}^p\in\{CSR,No\}}\left[\bar{V}_{ikt}\left(\sigma_{it}^b = W, \sigma_{-it}^b, \sigma_{1t}^p\right)|\Delta m_{kt}\right]\middle|q\right] >$$
$$\mathbb{E}_{Z_{-i}\in\{C,P\}}\left[\mathbb{E}_{\sigma_{1t}^p\in\{CSR,No\}}\left[\bar{V}_{ikt}\left(\sigma_{it}^b = A, \sigma_{-it}^b, \sigma_{1t}^p\right)|\Delta m_{kt}\right]\middle|q\right]$$

By the linearity of expectation, we move outward the expectation over buyer types to be evaluated later:

$$\mathbb{E}_{Z_{-i}\in\{C,P\}}\left[\mathbb{I}_{\{\bar{V}_{ikt}\}}|q\right]$$

where we denote the CSR strategy inequality $\mathbb{I}_{\{\bar{V}_{ikt}\}} \equiv \left\{\mathbb{E}_{\sigma_{1t}^p\in\{CSR,No\}}\left[\bar{V}_{ikt}\left(\sigma_{it}^b = W, \sigma_{-it}^b, \sigma_{1t}^p\right)|\Delta m_{kt}\right] > \mathbb{E}_{\sigma_{1t}^p\in\{CSR,No\}}\left[\bar{V}_{ikt}\left(\sigma_{it}^b = A, \sigma_{-it}^b, \sigma_{1t}^p\right)|\Delta m_{kt}\right]\right\}$. We consider first this inequality's expectations over platform strategies. Let $m_{kt}^W \equiv \mathbb{P}\left(\hat{q}_t > q_{kt}^*|\sigma_{it}^b = W\right)$ be the probability of platform k engaging in CSR given that buyer i chose to vote with the wallet in period t. Let $m_{kt}^A \equiv \mathbb{P}\left(\hat{q}_t > q_{kt}^*|\sigma_{it}^b = A\right)$ be the probability of platform k engaging in CSR given that buyer i chose to abstain. Their complements, $(1 - m_{kt}^W)$ and $(1 - m_{kt}^A)$, represent the probabilities of the platform not engaging in CSR given the buyer chose to vote or abstain, respectively. These four probabilities can then be used to expand the inequality by the definition of expectation and then rearrange and simplify in terms of the exogenous signaling cost:

$$\mathbb{E}_{\sigma_{1t}^p\in\{CSR,No\}}\left[\bar{V}_{ikt}\left(\sigma_{it}^b = W, \sigma_{-it}^b, \sigma_{1t}^p\right)|\Delta m_{kt}\right] > \mathbb{E}_{\sigma_{1t}^p\in\{CSR,No\}}\left[\bar{V}_{ikt}\left(\sigma_{it}^b = A, \sigma_{-it}^b, \sigma_{1t}^p\right)|\Delta m_{kt}\right] \Leftrightarrow \kappa < \kappa_{kt}^*$$

where $\kappa_{kt}^{*} = \Delta V_{ikt} \Delta m_{kt}$. Here $\Delta V_{ikt} = V_{ikt}(\sigma_{kt}^{p} = CSR) - V_{ikt}(\sigma_{kt}^{p} = No), \forall z_i = C$, is the incremental value for a CSR buyer of platform k engaging in CSR, and $\Delta m_{kt} = m_{kt}^{W} - m_{kt}^{A}$ is analogous to the probability of buyer i's vote persuading platform k to engage in CSR. Thus, when the signaling cost is less than the threshold κ^{*}, a CSR buyer will vote with the wallet.

However, this still leaves the uncertainty over other buyers' types. As we just showed that the signaling cost threshold inequality is true if and only if the original inequality obtains ($\mathbb{I}_{\{\bar{V}_{ikt}\}} \Leftrightarrow \kappa < \kappa_{kt}^{*}$), then we evaluate the outer expectation over the "other buyer" b_{-i}'s type $\mathbb{E}_{Z_{-i} \in \{C,P\}}[\kappa < \kappa_{kt}^{*} | q]$. Since the exogenous signaling cost κ is a constant that's not dependent upon the buyer type, then we can move it to the outside, $\kappa < \mathbb{E}_{Z_{-i} \in \{C,P\}}[\kappa_{kt}^{*} | q]$, and we can show that a buyer will vote with the wallet when the signaling cost is less than the following expected signaling cost threshold:

$$\mathbb{E}_{Z_{-i} \in \{C,P\}}\left[\mathbb{I}_{\{\bar{V}_{ikt}\}} | q\right] \Leftrightarrow \kappa < \overline{\kappa_{kt}^{*}} \text{ for } \overline{\kappa_{kt}^{*}} = q \Delta V_{ikt} \Delta m_{kt}.$$

Value of Expected Signaling Cost Threshold. Finally, according to the structure of our probability model for buyers voting with the wallet in Eq. (5) we can use this to compute the specific value of the probability of persuading the platform to use CSR, Δm_{kt}. Assuming a mean point estimate of the posterior belief of CSR buyer probability, $\hat{q}_t \equiv \mathbb{E}_X\left[q_t^{post}\right]$, the platform's estimate is by definition $\hat{q}_t = \frac{x_t + \alpha_{1t}}{N + \alpha_{1t} + \alpha_{2t}}$. The CSR inducing event $\hat{q}_t > q_{kt}^{*}$ is therefore equivalent to the platform choosing a CSR strategy when the number of observed votes with the wallet is greater than one of the following values, depending upon buyer i's strategy:

$$X_t > \begin{cases} h_{kt}^{*}, & \text{if } \sigma_{it}^{b} = \mathrm{A} \\ h_{kt}^{*} - 1, & \text{if } \sigma_{it}^{b} = \mathrm{W} \end{cases}$$

where $h_{kt}^{*} = \lceil q_{kt}^{*} \times (N + \alpha_{1t} + \alpha_{2t}) - \alpha_{1t} + 1 \rceil$ is *the minimum number of votes with the wallet the platform needs to observe to choose a CSR strategy* (rounded up to integer values in the support of X_t). Substituting these inequalities into the definitions of probabilities m_{kt}^{W} and m_{kt}^{A} for the random variable of the number of observed votes with the wallet (X_t), we can use the definition of the cumulative distribution function and rearrange to show that Δm_{kt} is equal to the probability that X_t equals a specific value, $h_{kt}^{*} = \lfloor q_{kt}^{*} \times (N + \alpha_{1t} + \alpha_{2t}) - \alpha_{1t} + 1 \rfloor$, (note this is rounded to integer values in the support of X_t) which is a function of the platform's required signaling threshold (q_{kt}^{*}):

$$\begin{aligned}
\Delta m_{kt} &\equiv m_{kt}^{W} - m_{kt}^{A} \\
&\equiv \mathbb{P}\left(\hat{q}_t > q_{kt}^{*} | \sigma_{it}^{b} = W\right) - \mathbb{P}\left(\hat{q}_t > q_{kt}^{*} | \sigma_{it}^{b} = A\right) \\
&= \mathbb{P}(X_t > h_{kt}^{*} - 1) - \mathbb{P}(X_t > h_{kt}^{*}) \\
&= \left(1 - \mathbb{P}(X_t \leq h_{kt}^{*} - 1)\right) - \left(1 - \mathbb{P}(X_t \leq h_{kt}^{*})\right) \\
&= \mathbb{P}(X_t \leq h_{kt}^{*}) - \mathbb{P}(X_t \leq h_{kt}^{*} - 1) \\
&= \mathbb{P}(X_t = h_{kt}^{*}) \\
&= Binomial(X_t = h_{kt}^{*};\ N, \mu_{kt})
\end{aligned}$$

where μ_{kt} is the probability of buyer i voting with the wallet. Lastly, it is simple to show using Eq. (1) that $\Delta V_{ikt} = \ln\left(\frac{u_{kt} + \omega}{u_{kt}}\right)$. This yields the form of the expected signaling cost threshold in Eq. (9):

$$\overline{\kappa_{kt}^*} = q \cdot \ln\left(\frac{u_k + \omega}{u_k}\right) \cdot Binomial(X_t = h_{kt}^*;\ N, \mu_{kt})$$

Appendix V: Where is the Greatest Potential for Buyers Voting with the Wallet?

For the expected signaling cost threshold in Eq. (9):

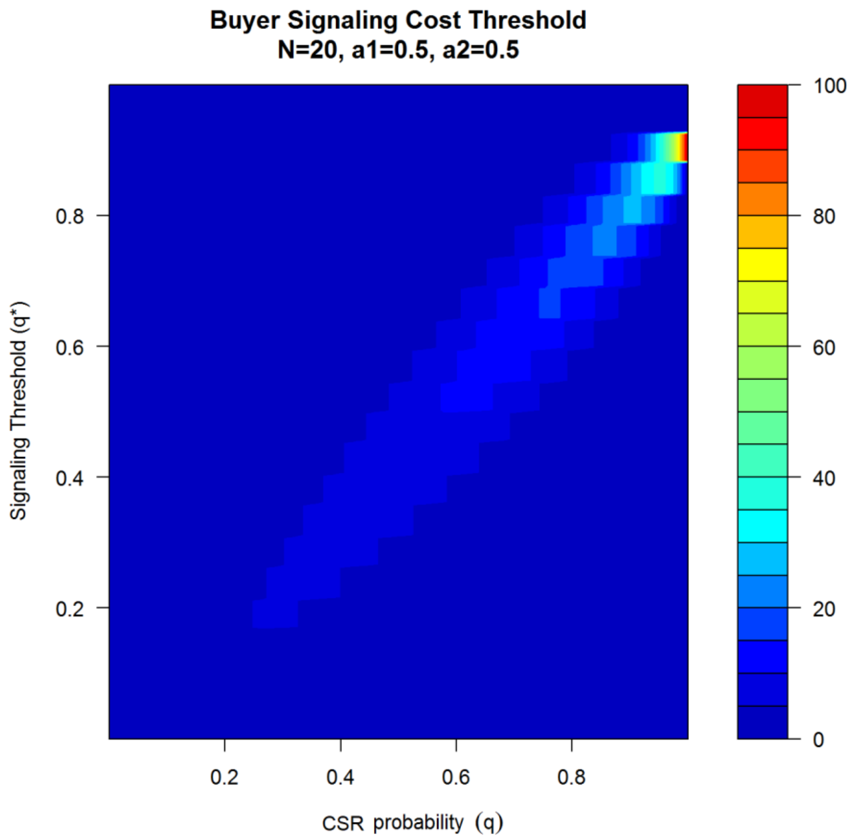
$$\overline{\kappa_{kt}^*} = \ln\left(\frac{u_{kt} + \omega}{u_{kt}}\right) \cdot q \cdot Binomial(h_{kt}^*;\ N,\ \mu_{kt})$$

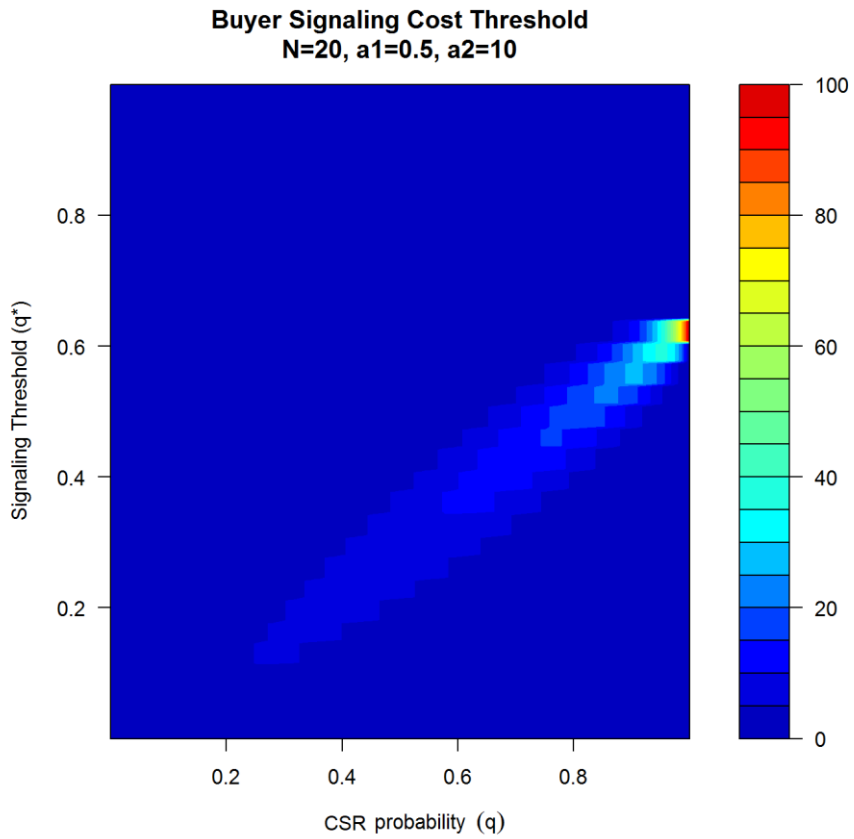
we interpret $\Delta m_{kt} \equiv Binomial(h_{kt}^*;\ N,\ \mu_{kt})$ as the probability of persuading the platform to choose CSR. Given the interpretation of h_{kt}^* from Appendix IV, this is then equivalent to *the probability that the platform observes exactly its minimum number of required votes with the wallet.*

Using Eq. (10) , $\mu_{kt} = \begin{cases} q, & \text{if } \kappa < \overline{\kappa_{kt}^*} \\ 0, & \text{if } \kappa \geq \overline{\kappa_{kt}^*} \end{cases}$, the buyer's expected signaling cost threshold can be expressed in terms of the CSR buyer probability q

$$\overline{\kappa_{kt}^*}(\kappa) = \begin{cases} \ln\left(\frac{u_{kt} + \omega}{u_{kt}}\right) \cdot \binom{N}{h_{kt}^*} \cdot\ q^{h_{kt}^*+1} \cdot (1-q)^{N-h_{kt}^*}, & \text{if } \kappa < \overline{\kappa_{kt}^*} \\ 0, & \text{if } \kappa \geq \overline{\kappa_{kt}^*} \end{cases}$$

Higher expected signaling cost and higher potential for voting with the wallet therefore depend upon the "alignment" between q and q_{kt}^*, as well as the platform's prior beliefs (i.e., past evidence of buyers voting or abstaining), and market size. Counterintuitively, higher q does *not* necessarily mean more voting with the wallet in the market -- and therefore does not necessarily mean more CSR occurring in the market. The figures below demonstrate this dependence. In the first scenario (left figure) with no prior evidence, the alignment between q and q_{kt}^* is roughly 1-to-1 where the highest potential of voting with the wallet occurs. However, if the platform has already observed lots of evidence of abstaining (right figure), implying a mostly price-sensitive market, then for each given value of CSR buyer probability q the highest potential for buyers to actually vote with the wallet occurs within a narrow range of q_{kt}^* that is *less* than if the platform had not observed any past abstaining. We might interpret this as the buyers in the market becoming more pessimistic about their chances to persuade the platform to engage in CSR after it has observed so much evidence of price-sensitive buyers. The buyers' growing pessimism entails that they only spend the signaling cost to vote if the platform has lower required signaling threshold to engage in CSR (i.e., lower than if they hadn't already observed buyers abstaining).


Buyer Signaling Cost Threshold
N=20, a1=0.5, a2=0.5
Signaling Threshold (q*)
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
CSR probability (q)
100
80
60
40
20
0


Buyer Signaling Cost Threshold
N=20, a1=0.5, a2=10
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CSR probability (q)
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0