Introduction to Signals

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Cyberspectrum Melbourne

A Little Background

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- What is Fourier transform?

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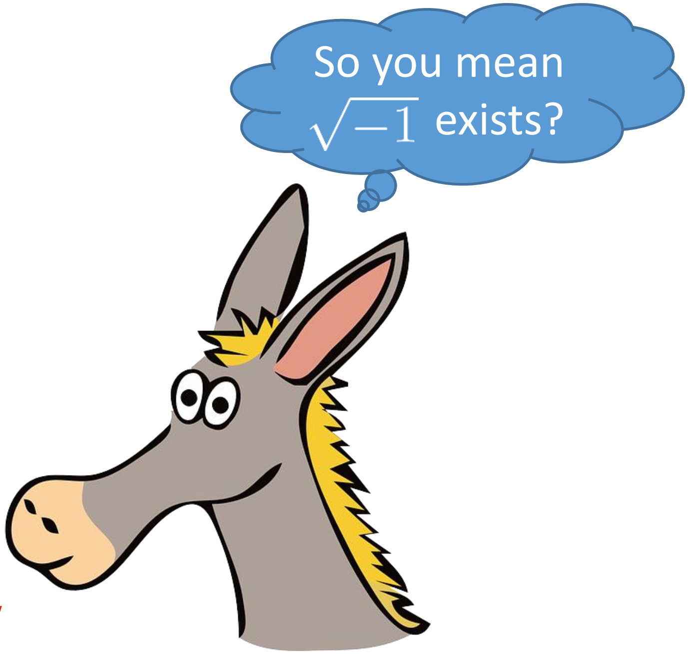

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- Disclaimer: I focus on DSP from the perspective of wireless communications

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  + After 2 such presentations (I think), we will be able to (almost) learn OFDM in the 3rd

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    - Modern powerline communication systems

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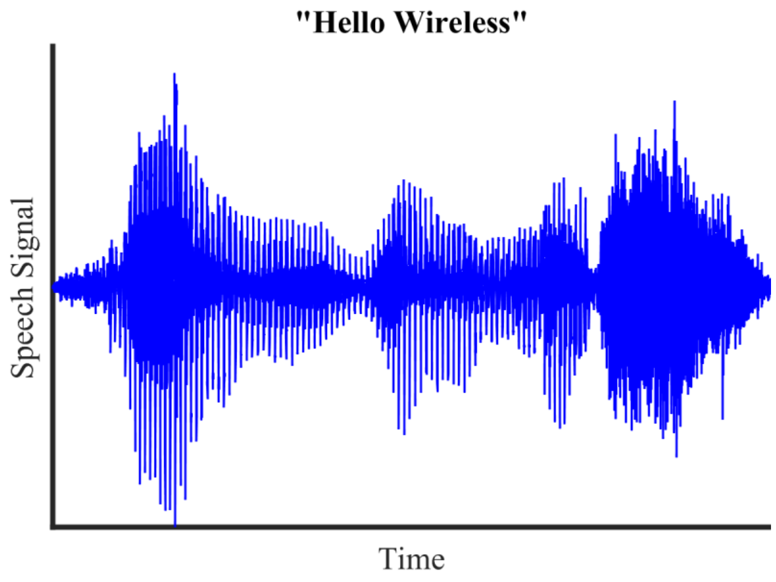

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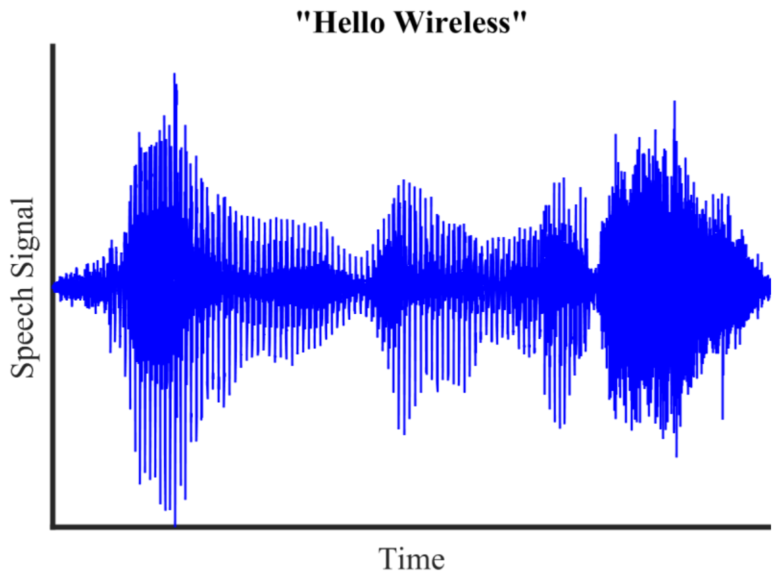

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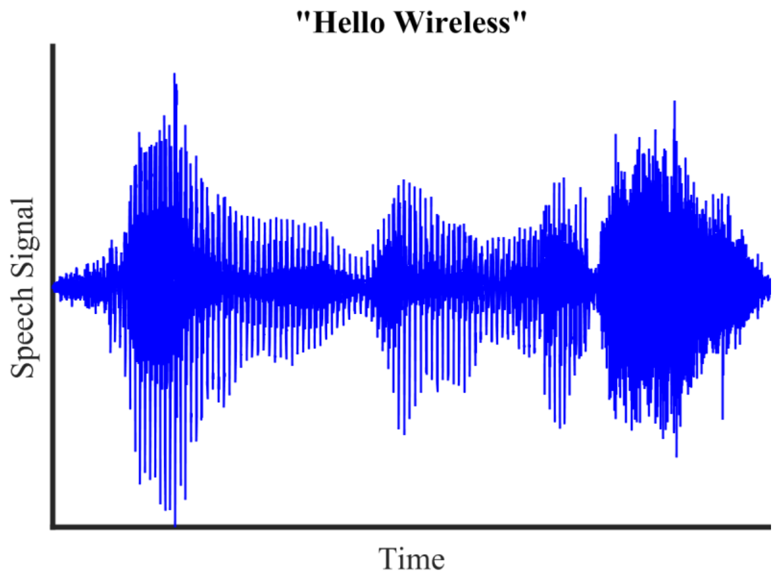

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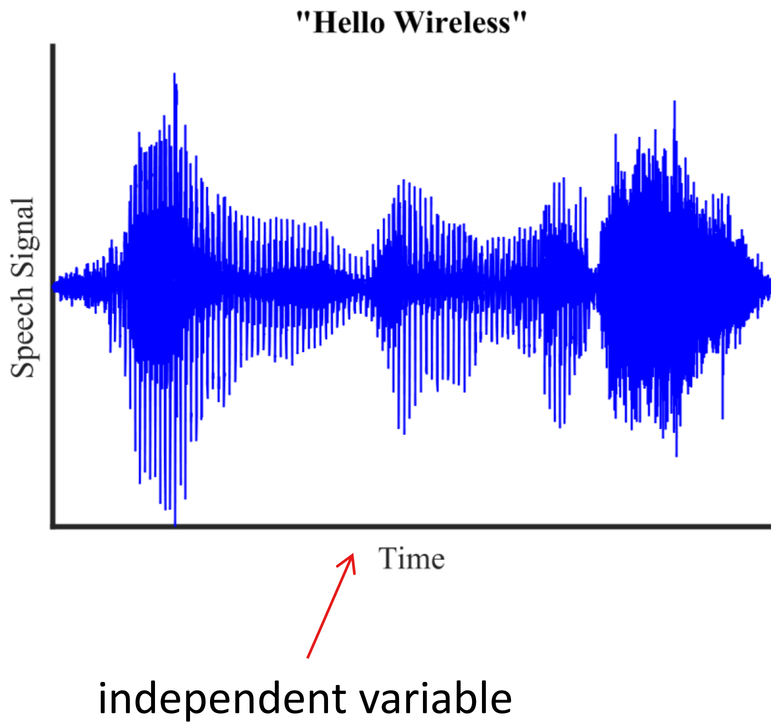

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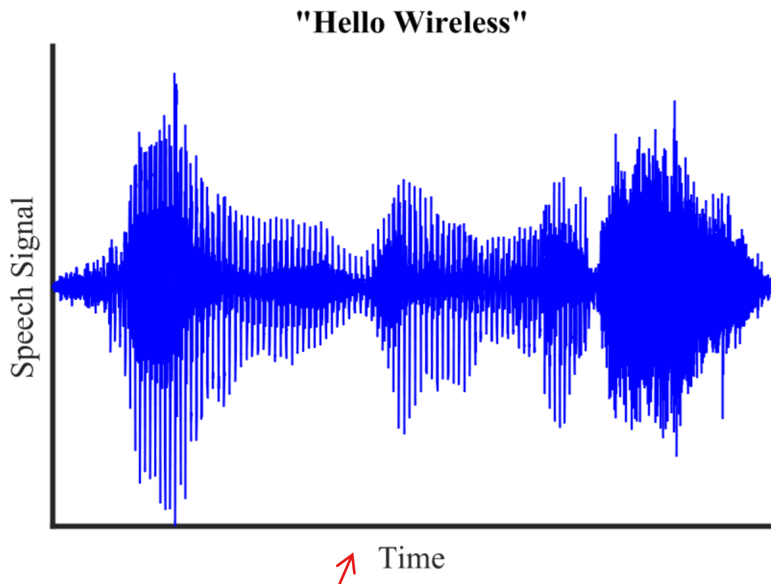
amplitude



independent variable

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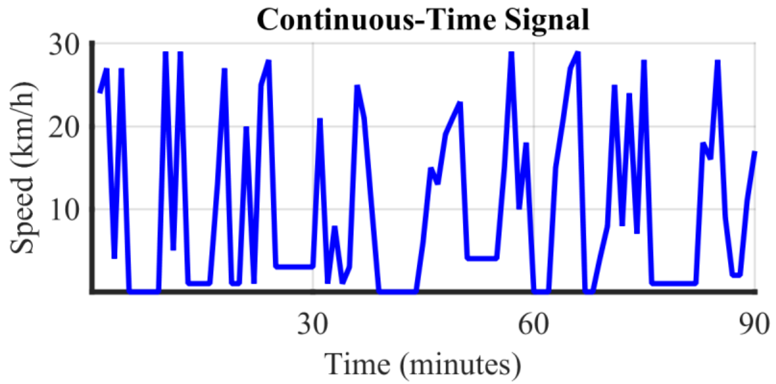

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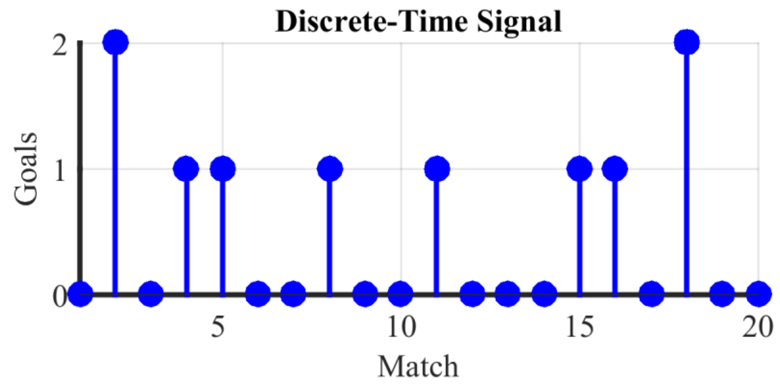

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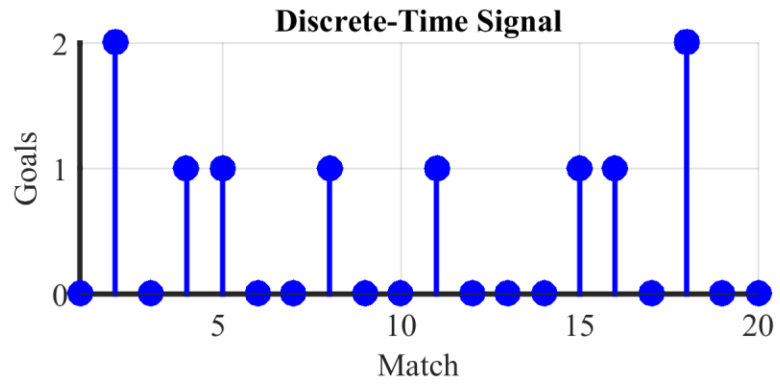

- DSP $\Rightarrow$ heavy focus on discrete-time signals

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    + It is simply undefined for non-integer values

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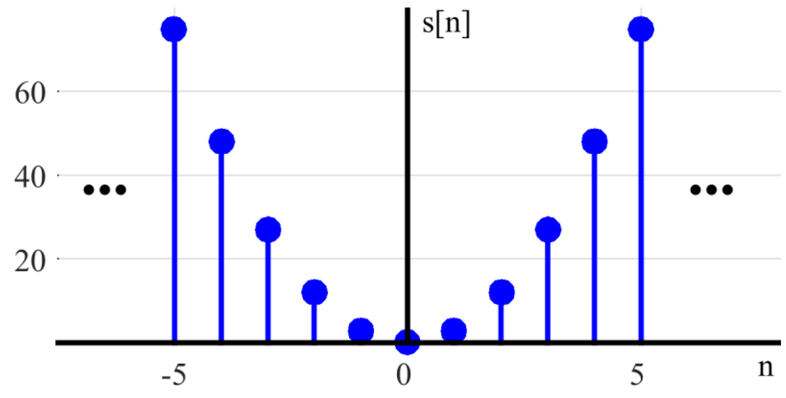

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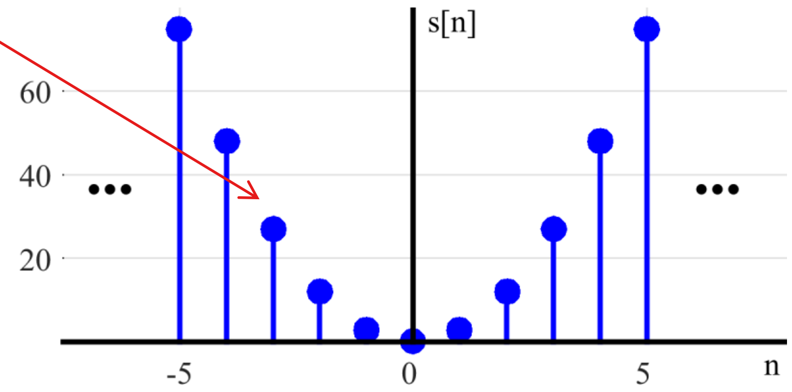

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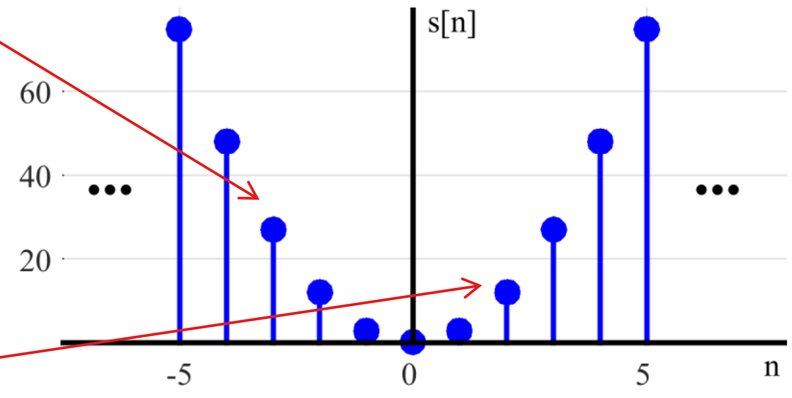

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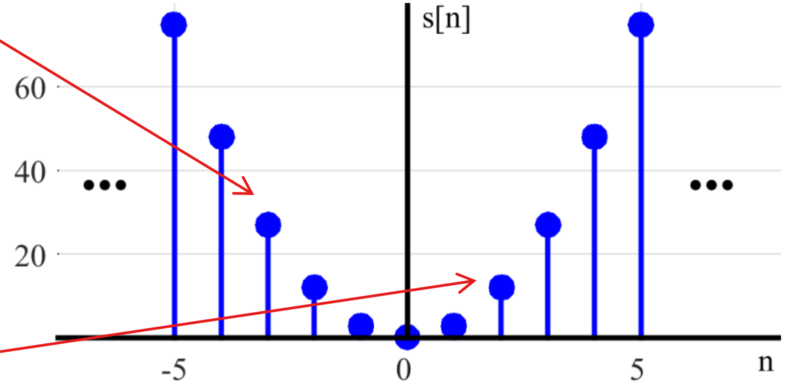

- Each member s[n] of a discrete-time signal is called a sample

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- Another way of representing a discrete-time signal

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$$s[n] = \{\cdots, 75, 48, 27, 12, 3, \underline{0}, 3, 12, 27, 48, 75, \cdots\}$$

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- Another way of representing a discrete-time signal

$$s[n] = \{\cdots, 75, 48, 27, 12, 3, \underline{0}, 3, 12, 27, 48, 75, \cdots\}$$

- In SDR, the signal is usually voltage (or sometimes current) changing over time

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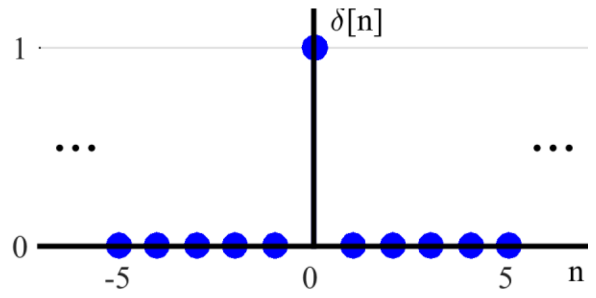

Unit Impulse Signal

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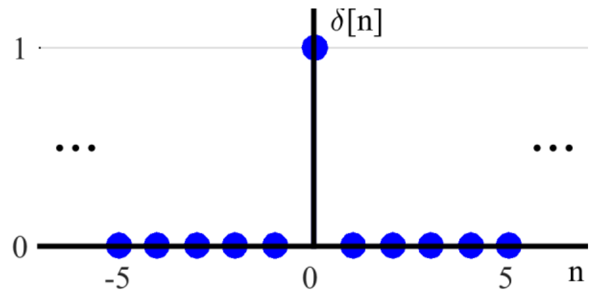
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



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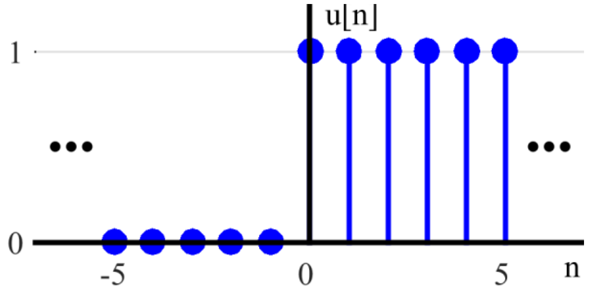

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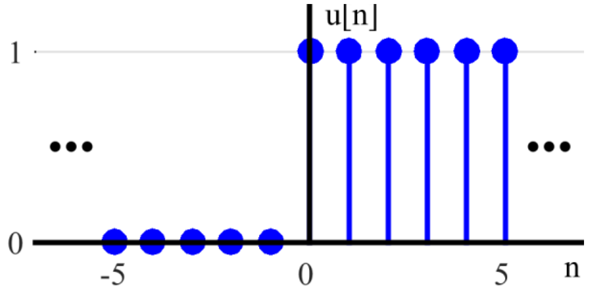
$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



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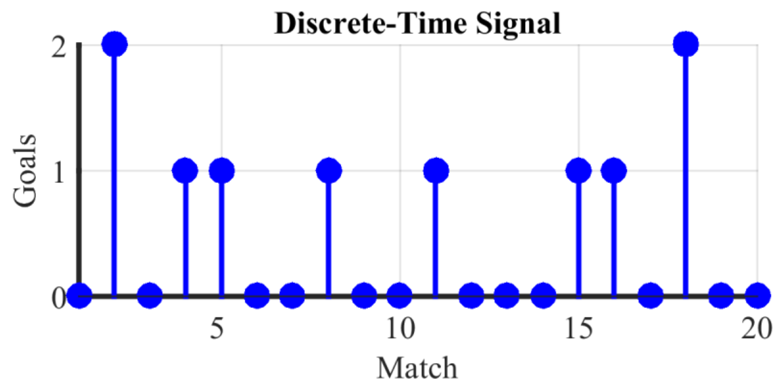

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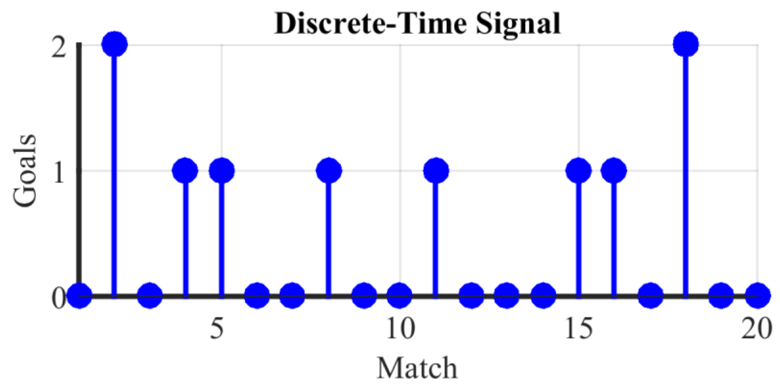
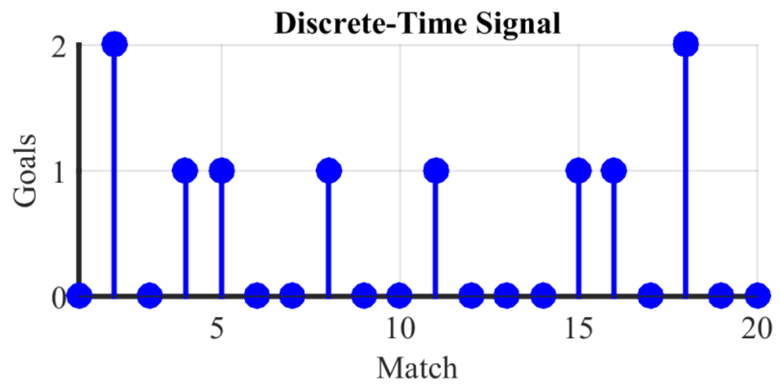
    + Total 10 goals in the tournament



Signal Energy

- Some measure of ``strength'' or ``size'' of a signal
- One simple method can be just adding all the values on the amplitude axis
  - + Total 10 goals in the tournament
- Now we can easily compare him with others, provided that amplitude > 0



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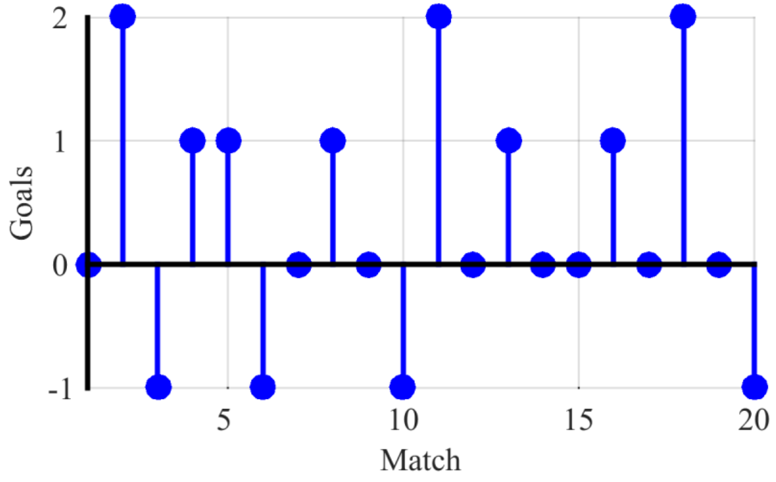

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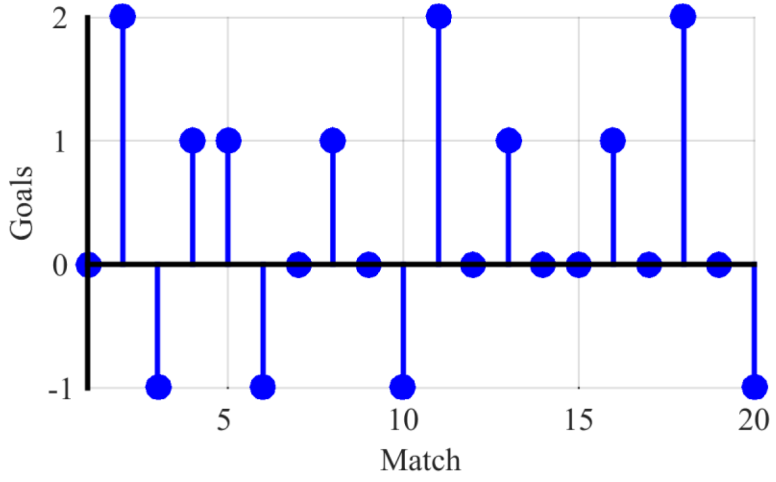

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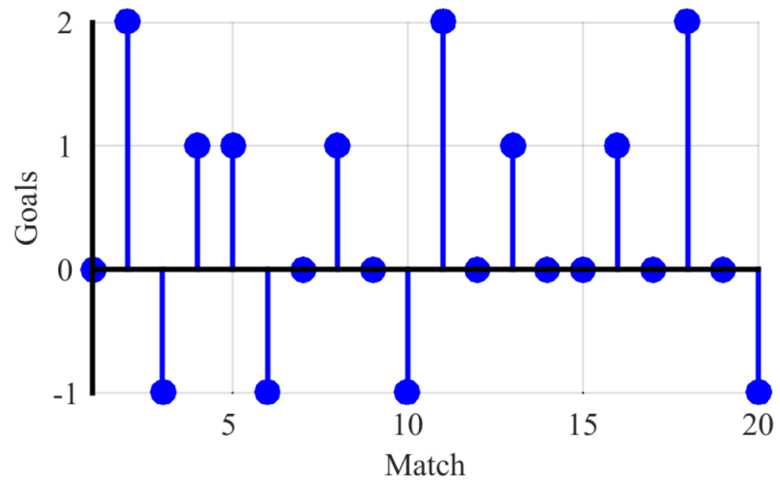

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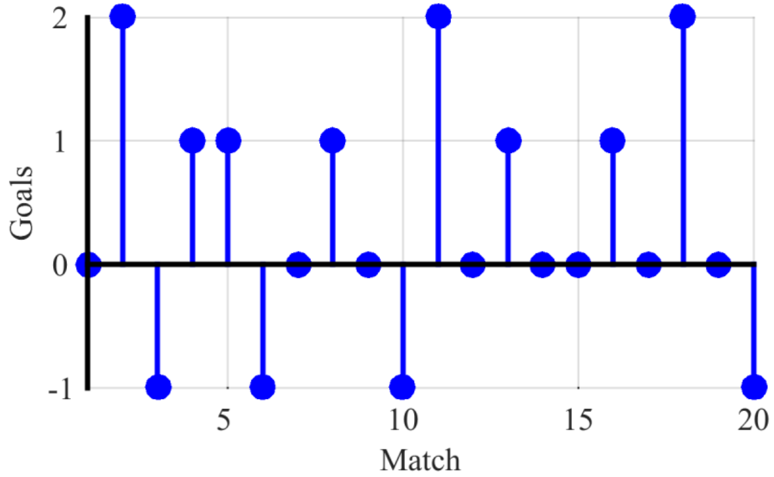

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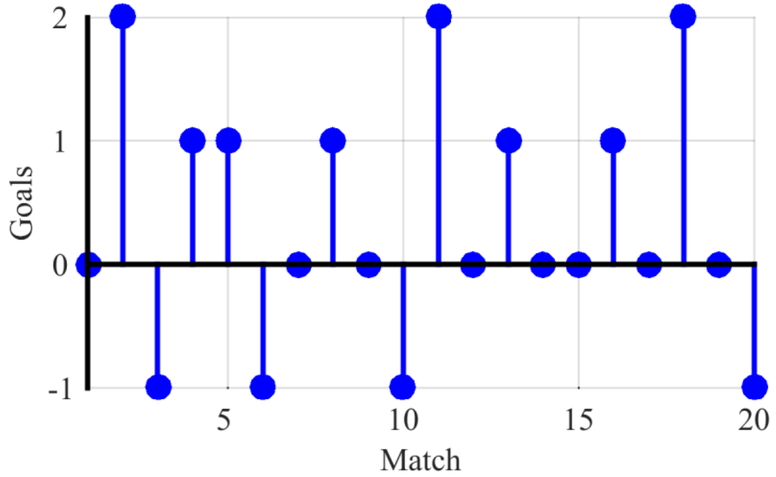

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- Strength of a signal can be measured by taking the absolute value of the signal and then adding all the values
  + Or square of the absolute value, or the fourth power of the absolute value

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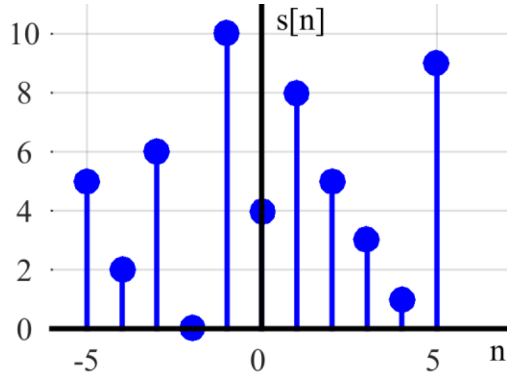

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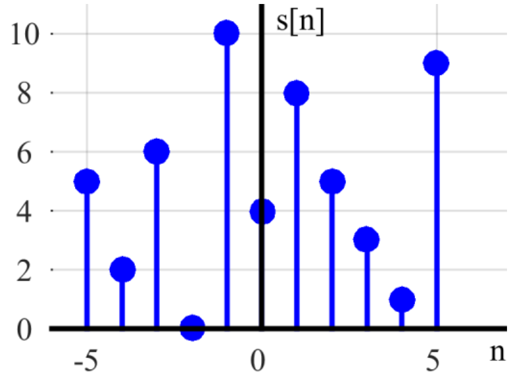

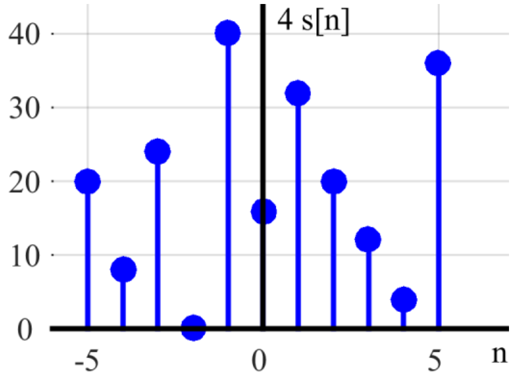

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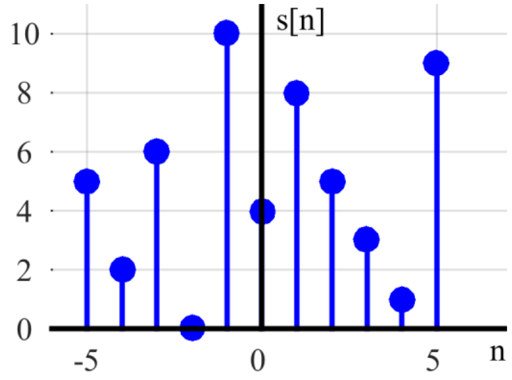

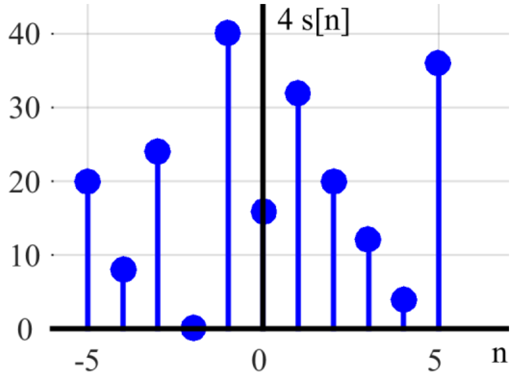

- Remember that the whole signal gets scaled by the same value

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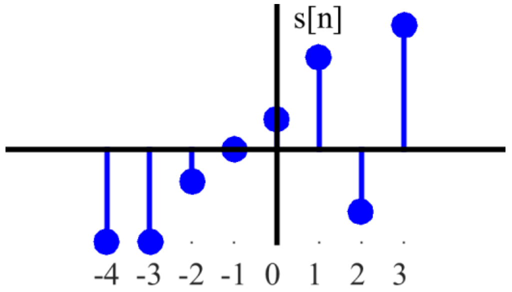

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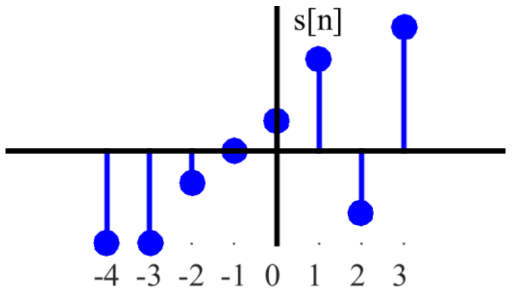

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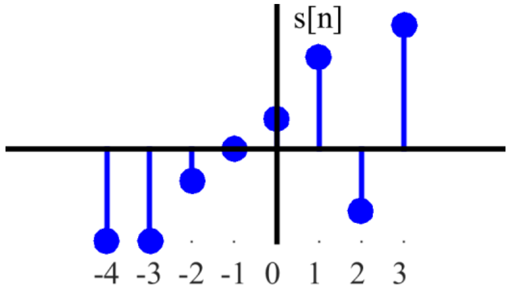

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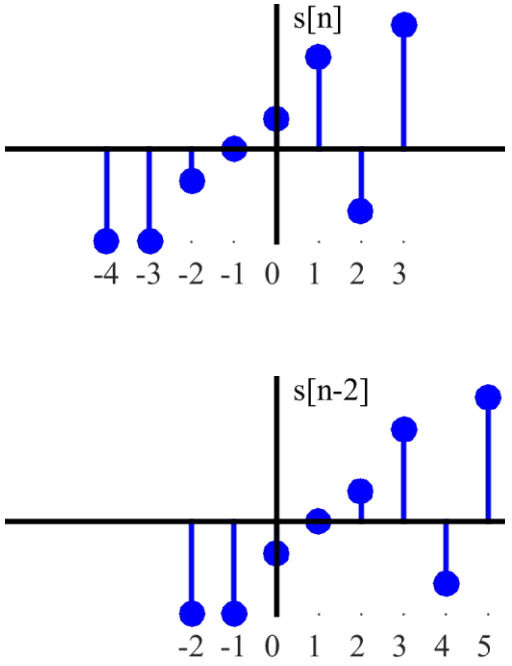

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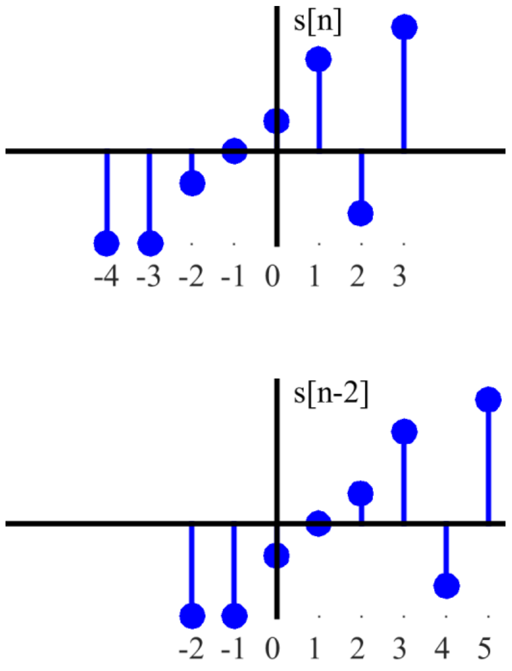

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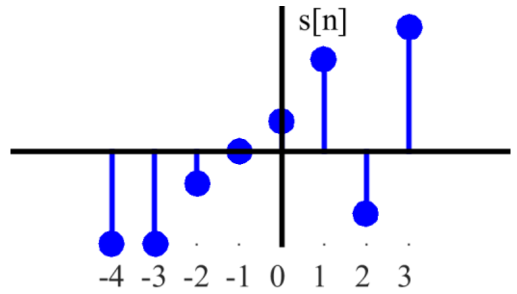

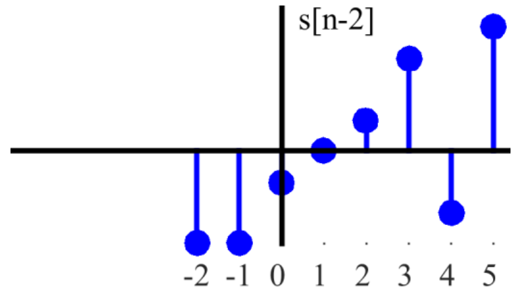

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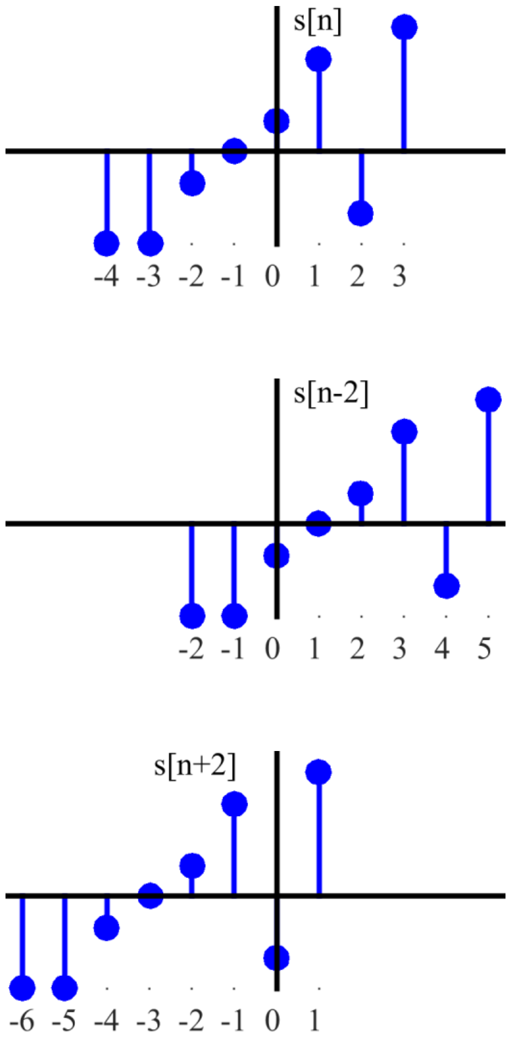

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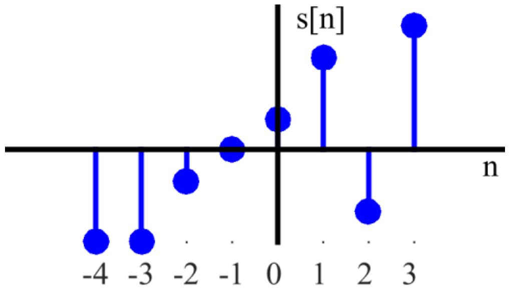

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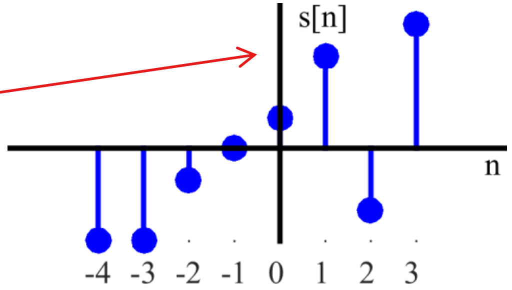

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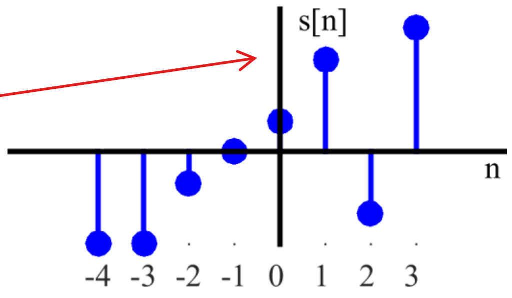

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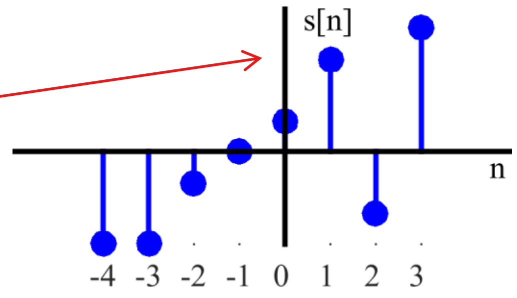

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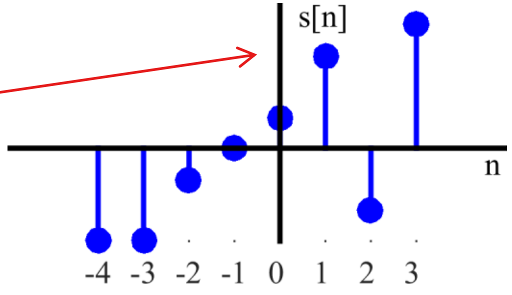

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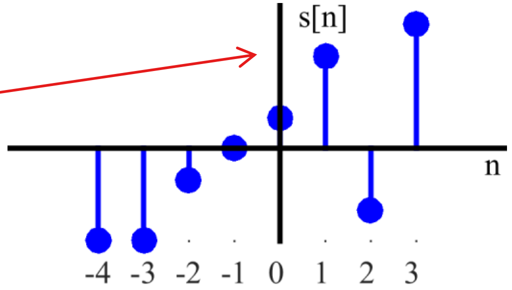

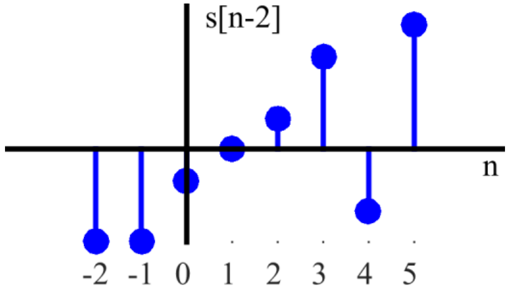

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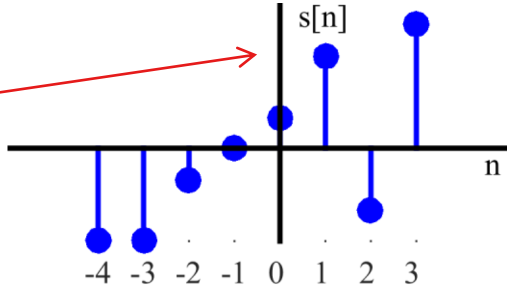

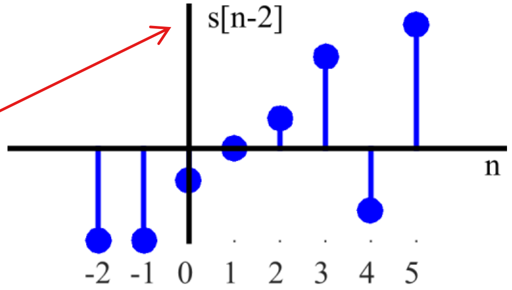

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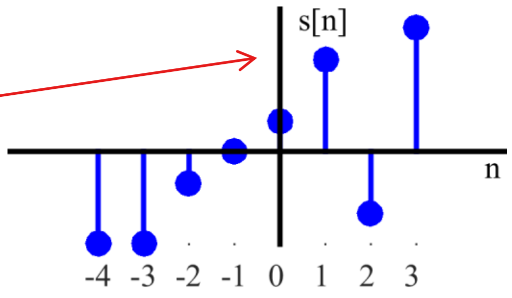

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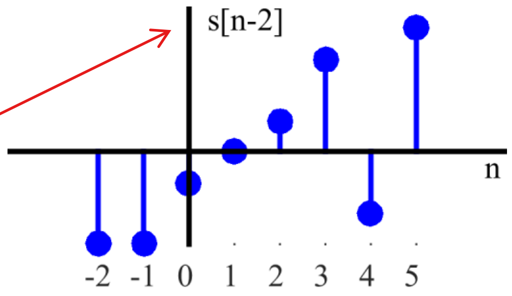

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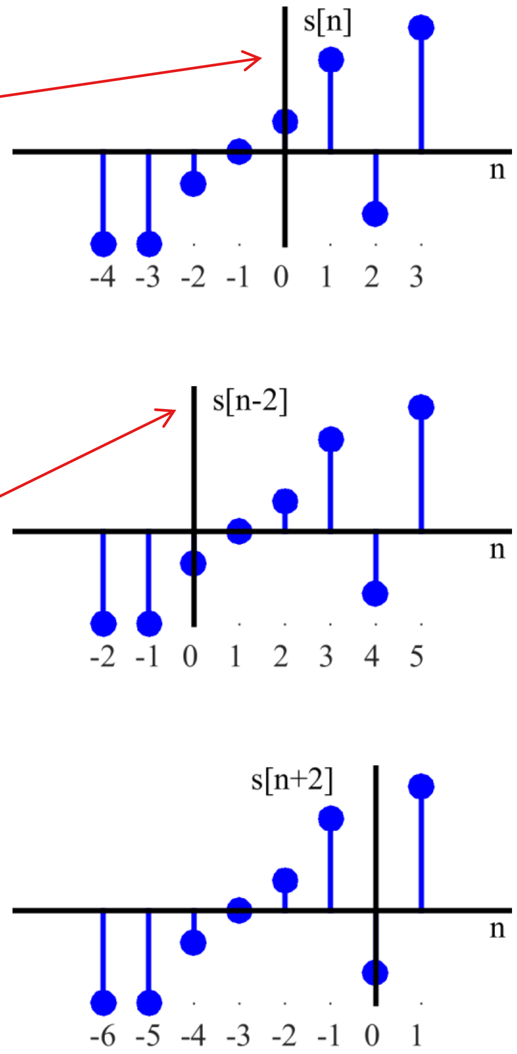

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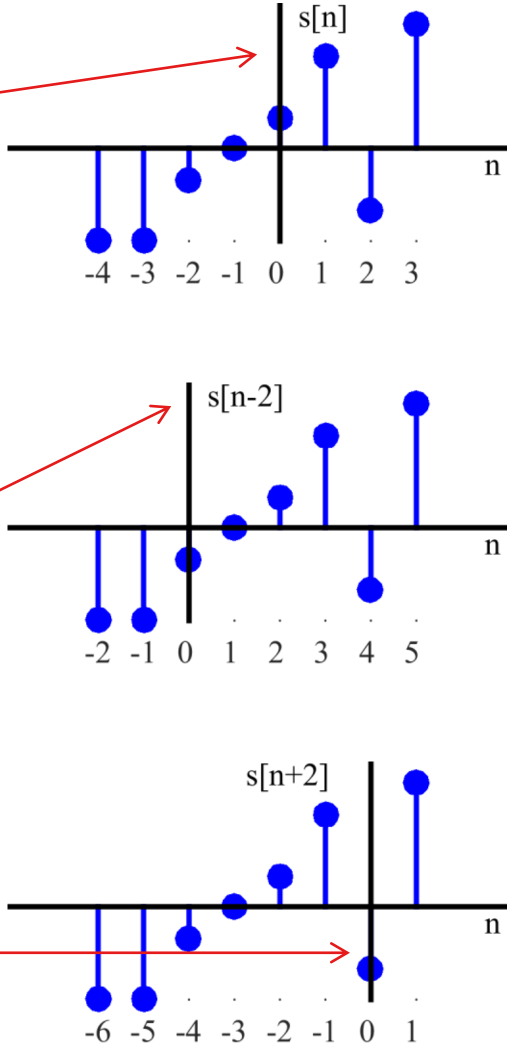

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- You will always encounter such equations in DSP

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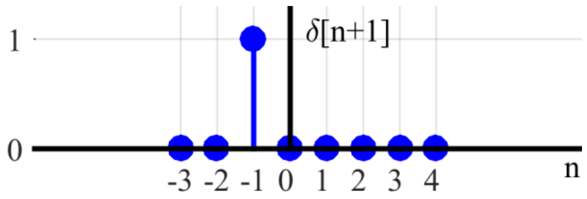

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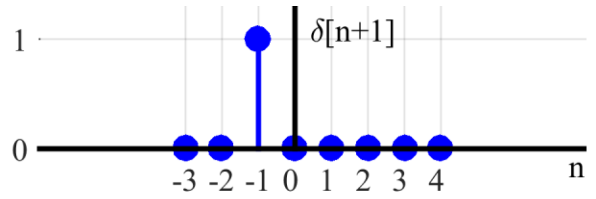

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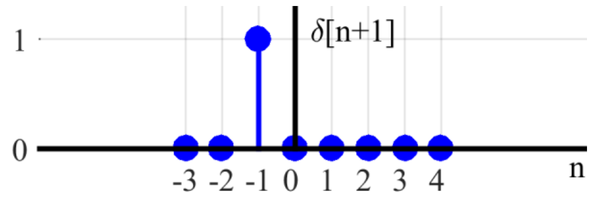

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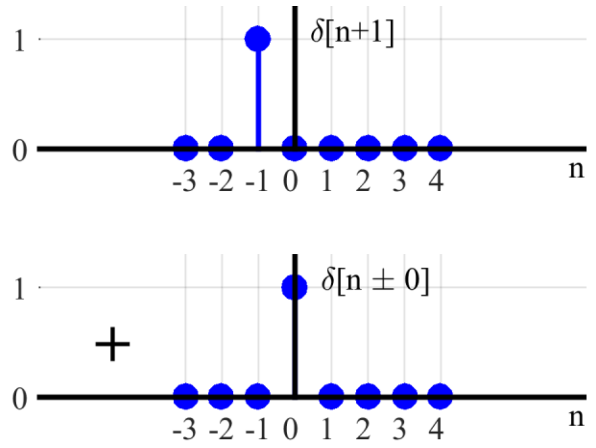

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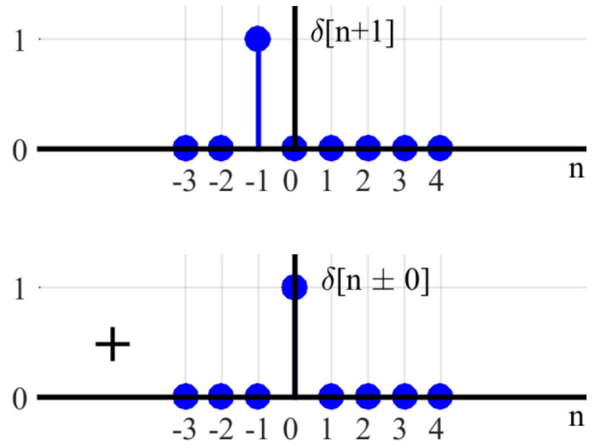

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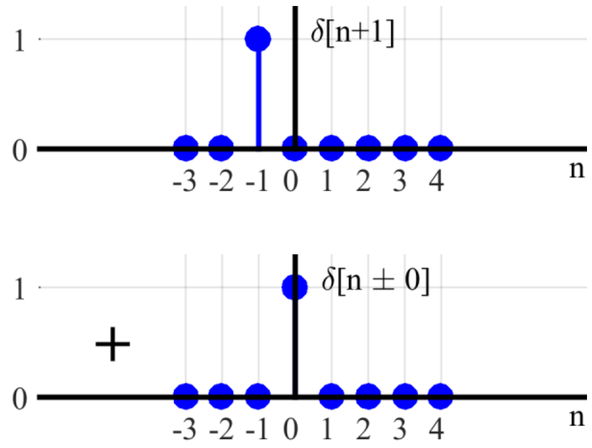

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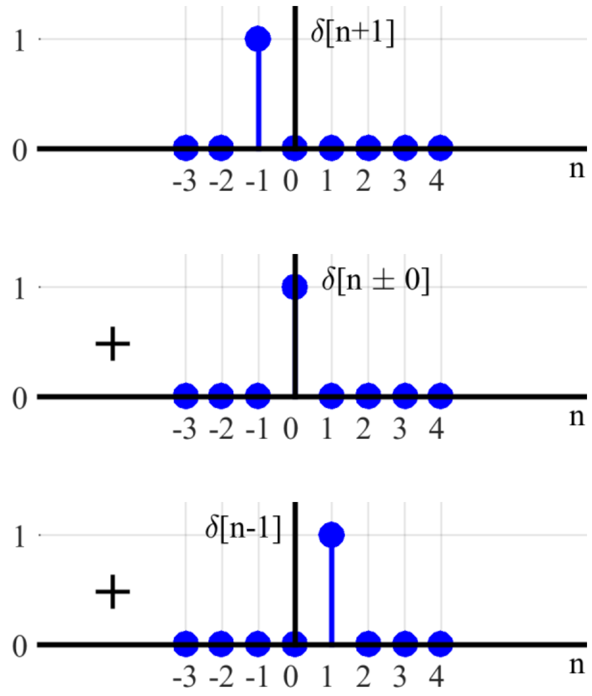

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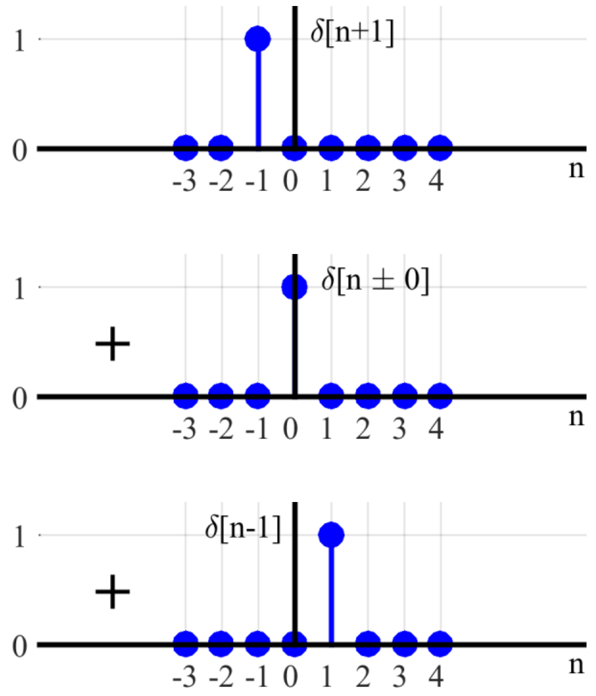

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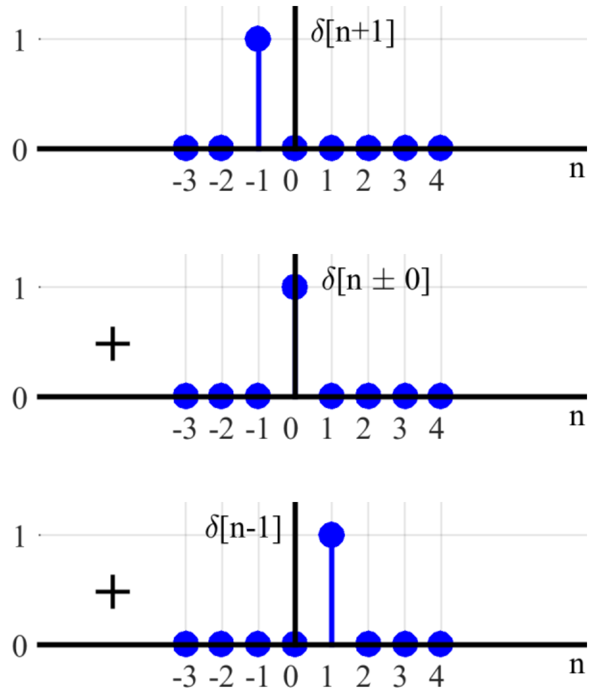

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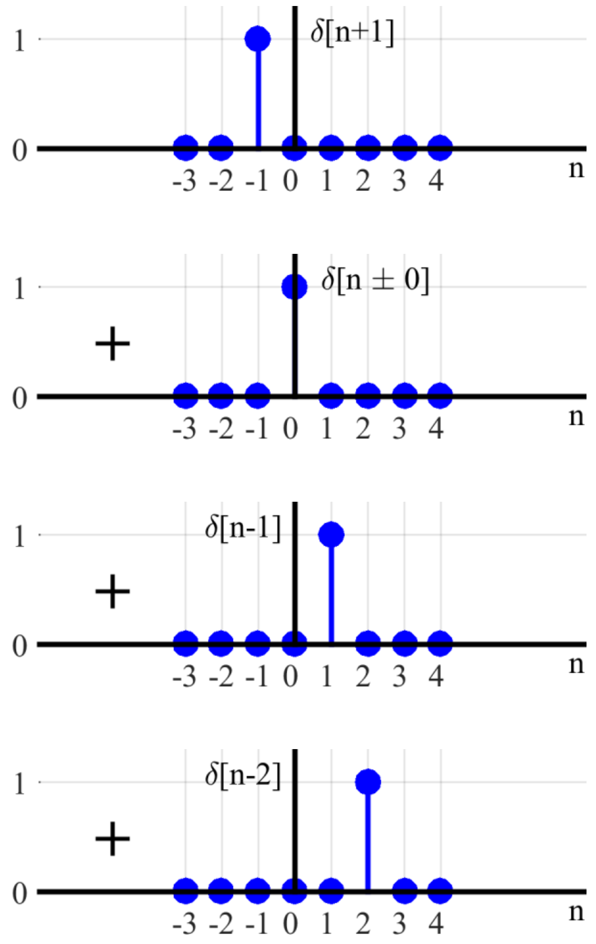
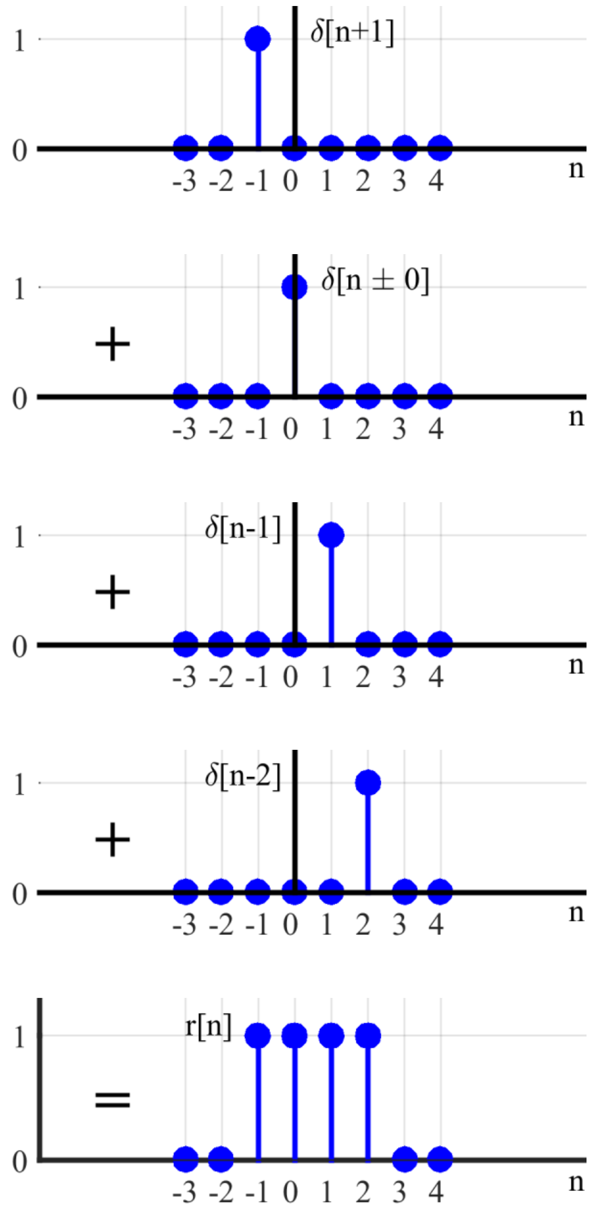


Addition of Time Shifted Signals

- Add time shifted versions of s[n] together
- Example: $r[n] = \sum_{m=-1}^{2} \delta[n-m]$

$$= \delta[n+1] + \delta[n-0]$$

$$+ \delta[n-1] + \delta[n-2]$$



Circular Shift

Circular Shift

- Concentrate on a segment of N samples **only**

Circular Shift

- Concentrate on a segment of N samples **only**
- “What goes around comes around”

Circular Shift

- Concentrate on a segment of N samples **only**
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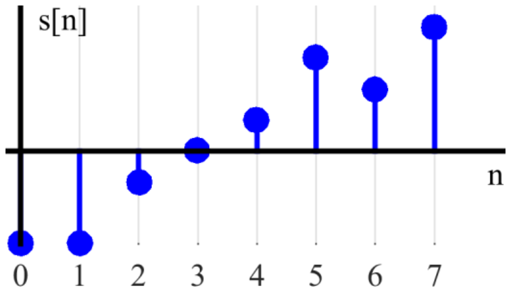

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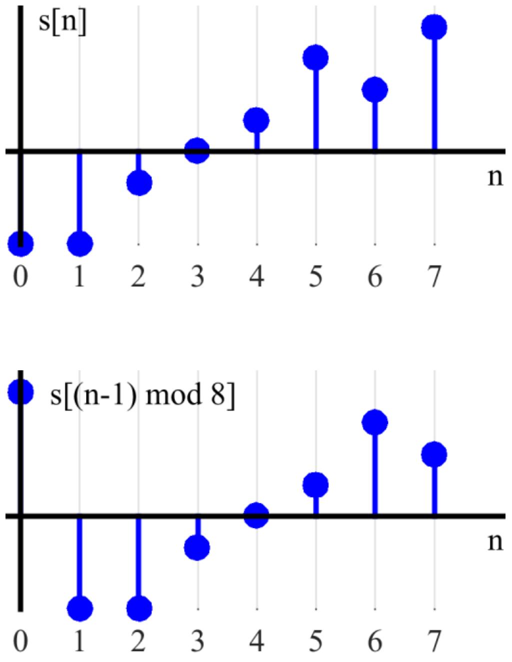

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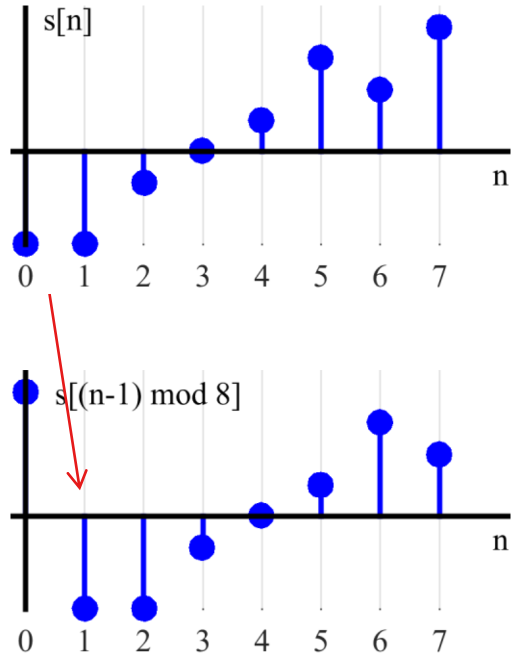

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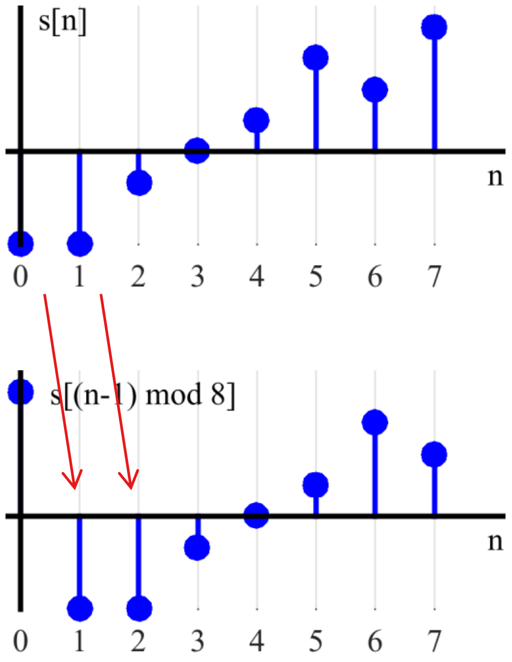

Circular Shift

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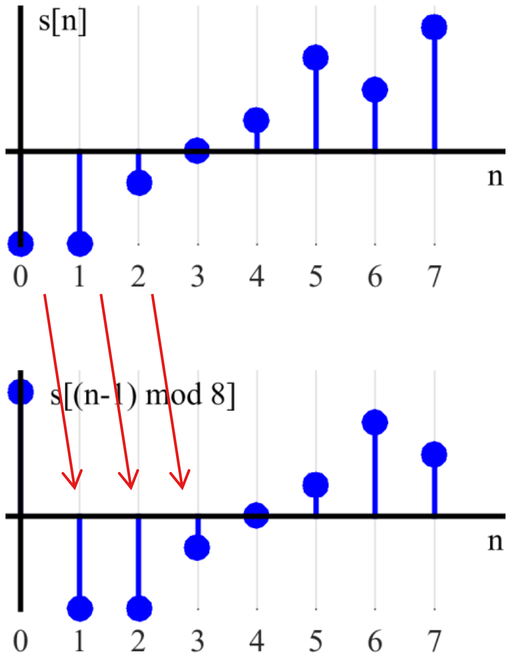

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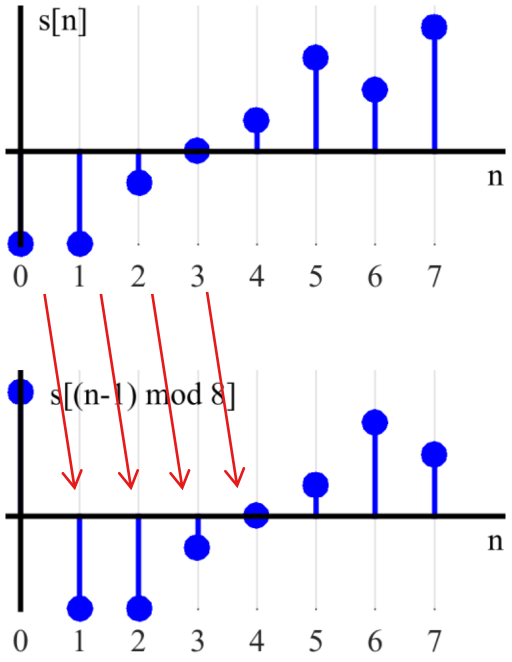

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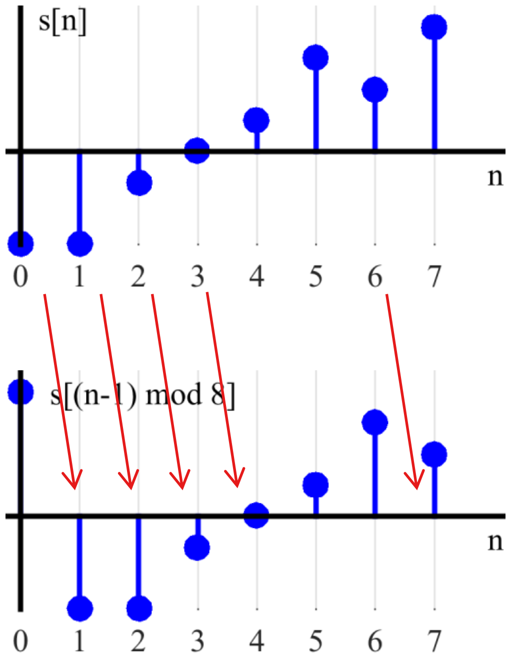
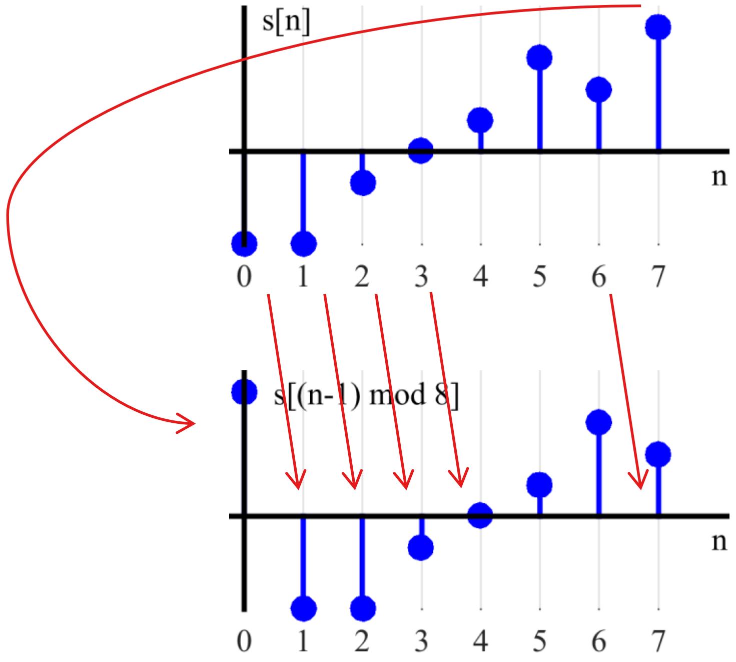


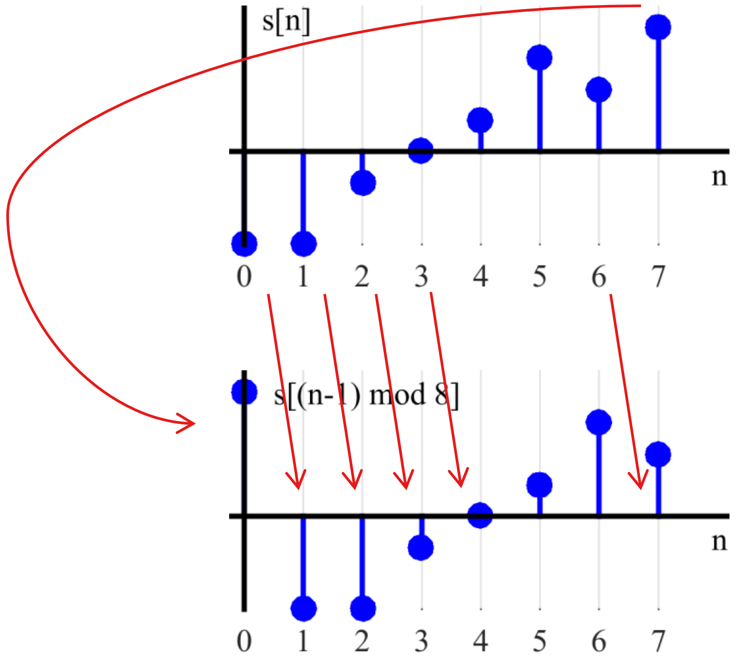
Circular Shift

- Concentrate on a segment of N samples only
- “What goes around comes around”



Circular Shift

- Concentrate on a segment of N samples only
- "What goes around comes around"
- Just like Pac-man



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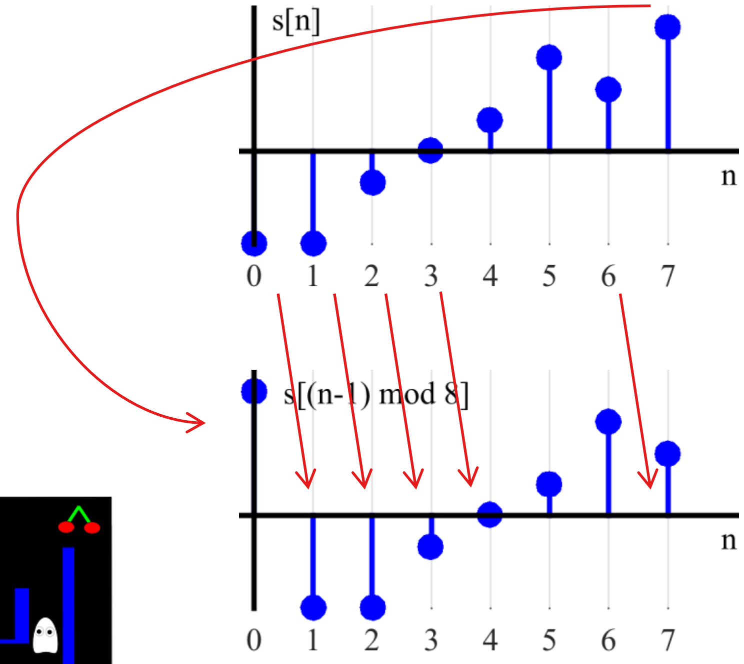

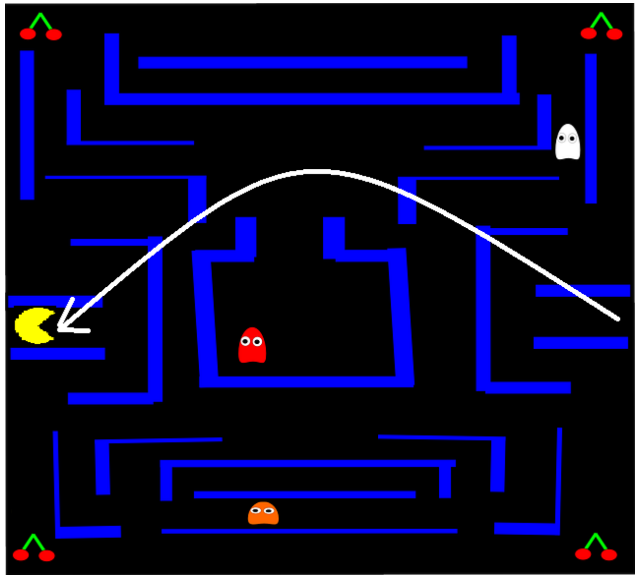

Circular Shift

- Concentrate on a segment of N samples only
- "What goes around comes around"
- Just like Pac-man