Complex (IQ) Signals

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Why DSP?

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- Software in Software Defined Radio (SDR)

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- Writing your own software
  - + Knowledge of communication systems and DSP algorithms is required
- Can be understood with basic mathematical skills

Complex Numbers

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- We will avoid the use of j and e

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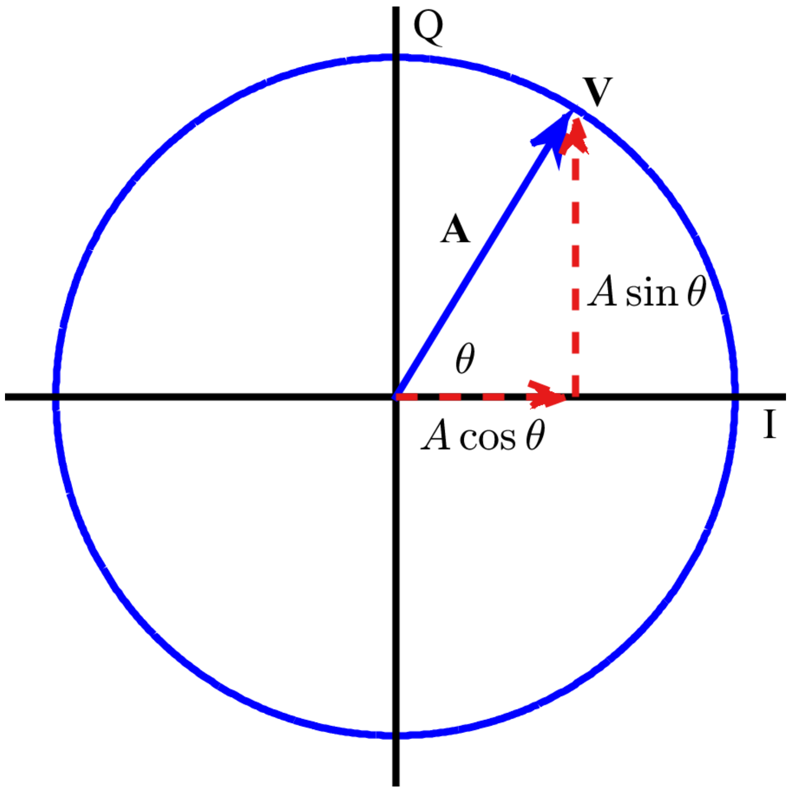

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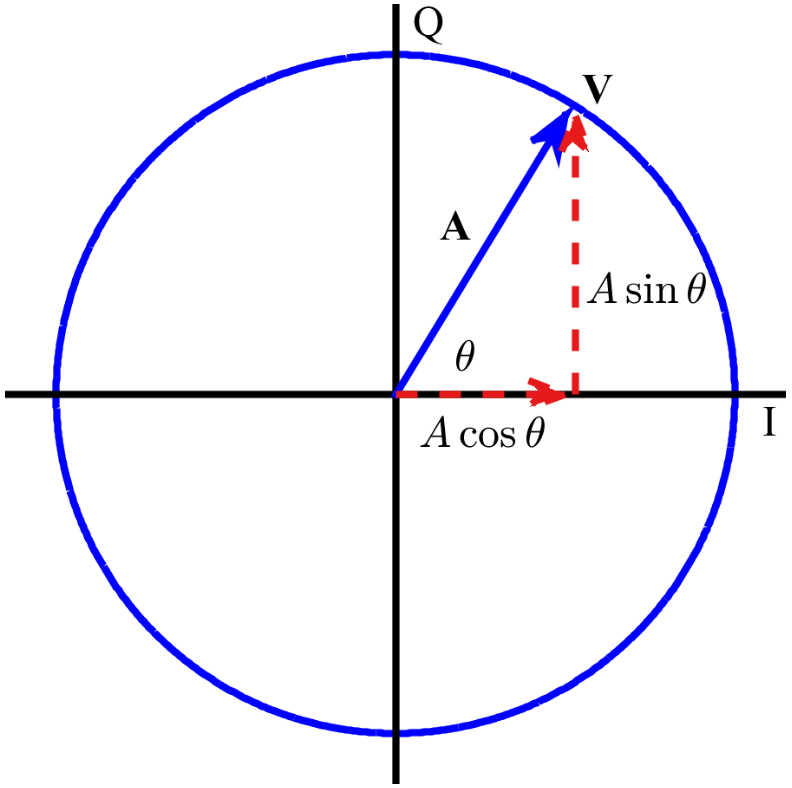
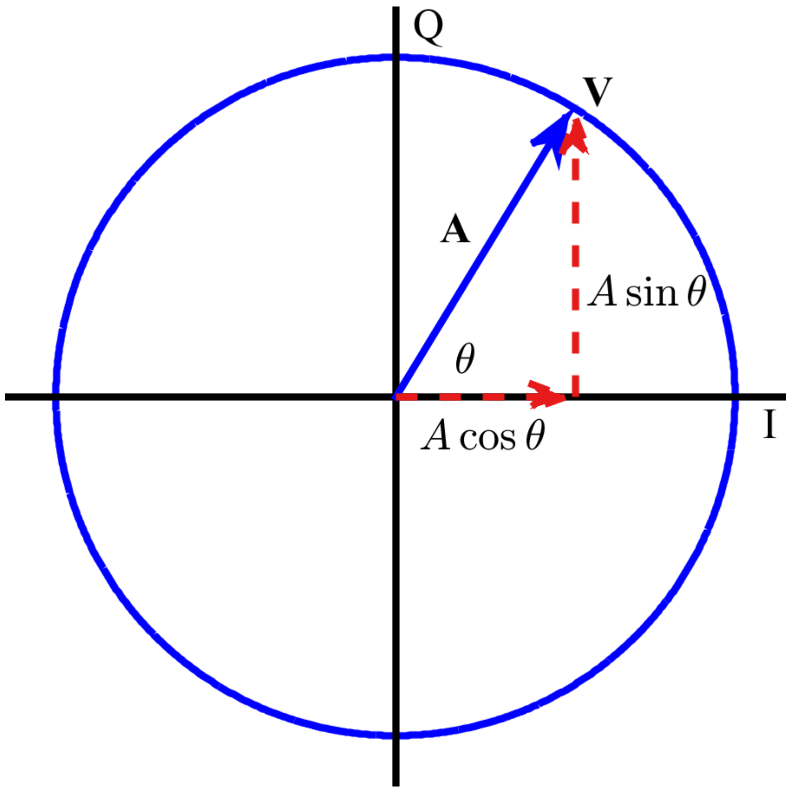


Complex Numbers …

- Start with an ordered pair of real numbers
- A complex number can be considered as a vector with initial point at (0,0)
- Problem with vectors: all arithmetic operations cannot be applied
  + Addition of two vectors is another vector in (x,y)-plane – that's good



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- Product of complex numbers is a complex number -- an extremely useful property

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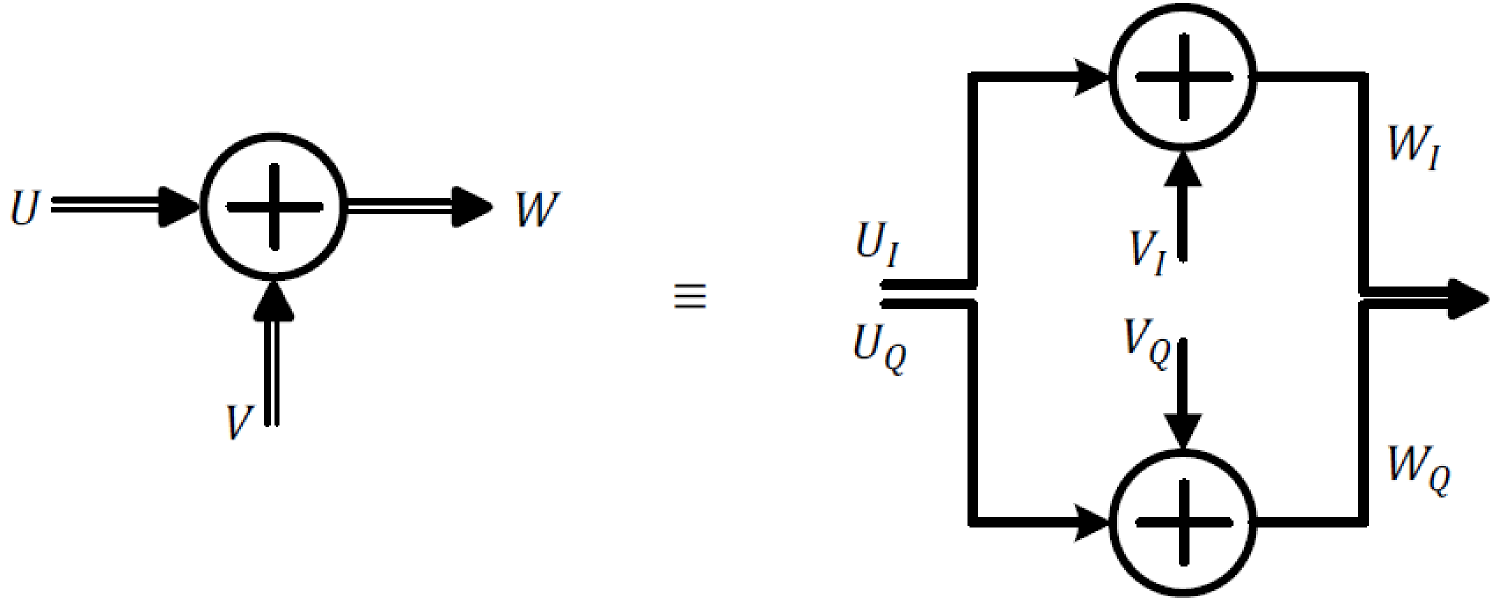

Concept of Frequency

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- What is a frequency?

Concept of Frequency …

and length A

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Concept of Frequency …

- Consider a complex number V in an IQ-plane with an angle θ and length A
- Now imagine V rotating with an angle continuously increasing with time
- Then, V can be treated as a signal with time as independent variable and we call it a complex sinusoid

Concept of Frequency …

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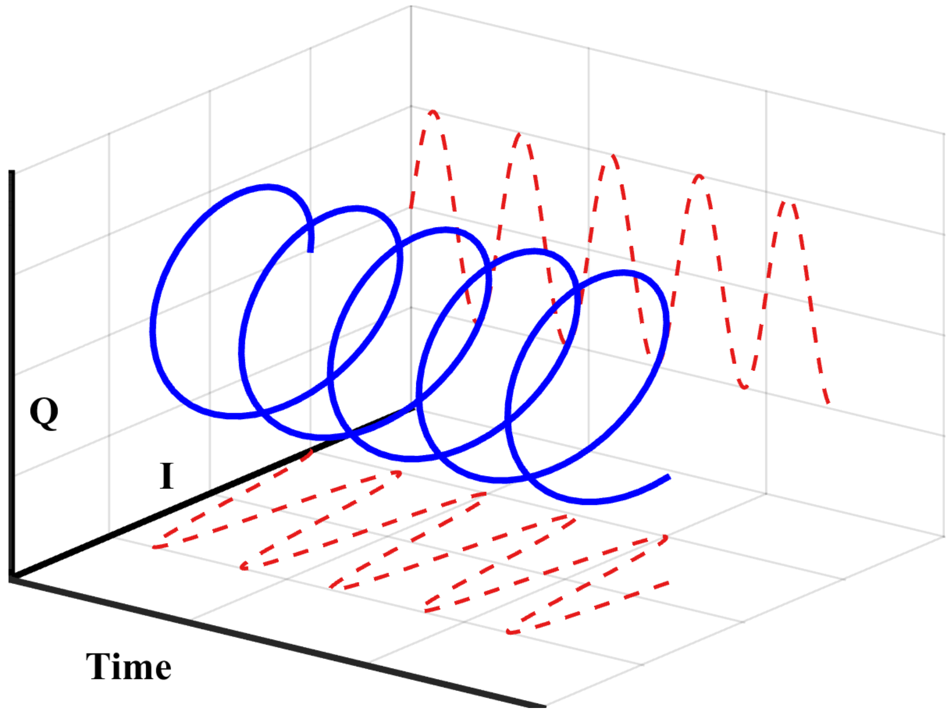
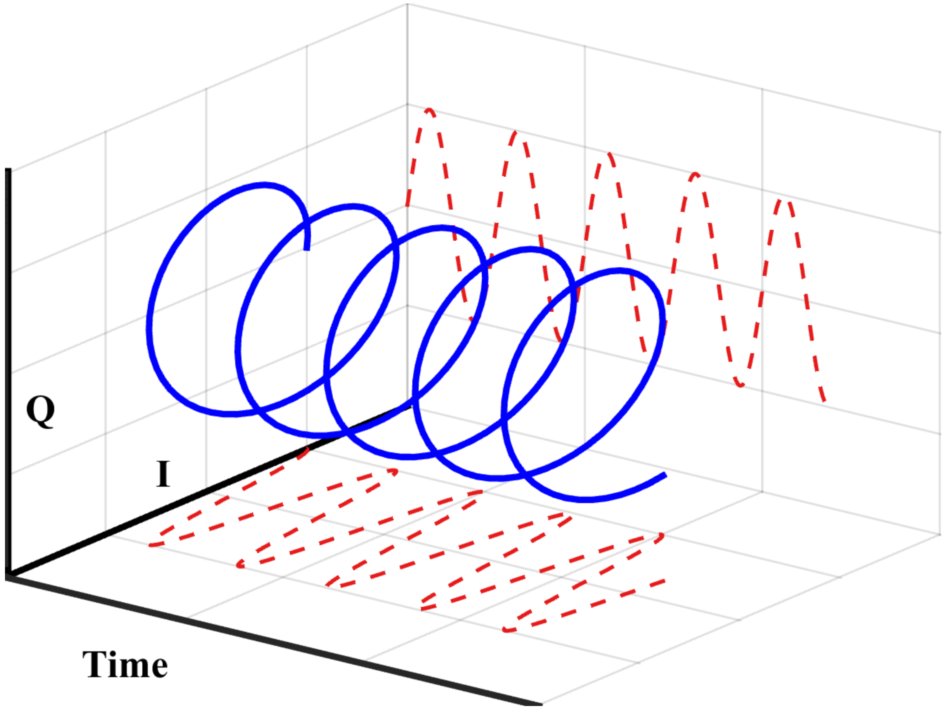


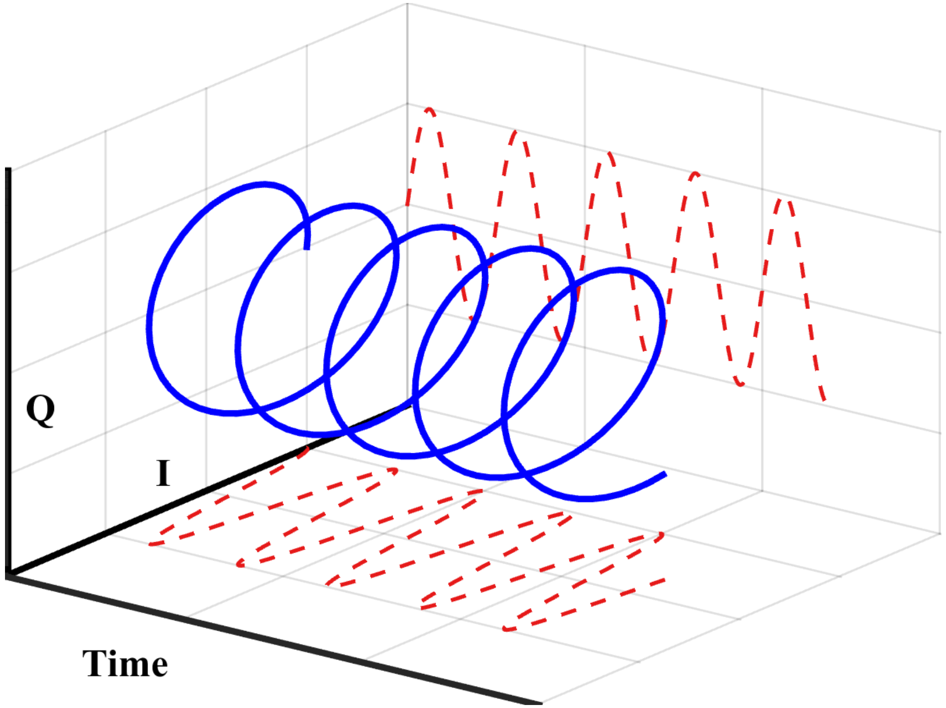
Concept of Frequency …

- Just like velocity is the rate of change of displacement, frequency is the rate of change in phase of a complex sinusoid



Concept of Frequency …

- Just like velocity is the rate of change of displacement, frequency is the rate of change in phase of a com$p\pi F$
- sinusoid
  - + This rate of change of phase results in V rotating in the time IQ-plane at an angular velocity $2\pi F$



Concept of Frequency

+

$$\cos(2\pi F t + \theta)$$

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Concept of Frequency

- As time passes, V is shown as coming out of the page

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Concept of Frequency

- os cos $2\pi Ft+\theta$ $2\pi Ft+\theta$ $2\pi\pi FFtt+\theta\theta$ $2\pi Ft+\theta$ cos $2\pi Ft+\theta$
- As time passes, V is shown as coming out of the page
  - When its projection from a 3-dimensional plane to a 2-dimensional plane formed by time and I-axis is drawn, we get $A\cos\ 2\pi Ft+\theta$
  -

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Concept of Frequency

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- As time passes, V is shown as coming out of the page
  - Similarly, when the projection is drawn on a 2-dimensional plane formed by time and Q-axis, it generates $A\sin\ 2\pi Ft+\theta$
  -

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    - Q component is called qaudrature because sin is at 90^o with cos()

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- In conclusion, a complex sinusoid with frequency F is composed of two real sinusoids

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Concept of Frequency …

- $V\ Q \quad VV \quad V\ Q\ QQ \quad V$

  $Q = AA$ sin $2\pi Ft + \theta$ sin sin $2\pi Ft + \theta$ $2\pi Ft + \theta$ $2\pi\pi FFtt + \theta\theta$ 2π

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Amplitude

Phase

- $V\ I \quad VV \quad V\ I\ II \quad V\ I = A$ $A \cos\ 2\pi Ft + \theta\ \cos \cos\ 2\pi Ft + \theta\ 2\pi Ft + \theta\ 2\pi\pi FFtt + \theta\theta\ 2\pi Ft +$ $\theta\ \cos\ 2\pi Ft + \theta$

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- Define it through the rate of rotation of a complex sinusoid V
  - A negative frequency simply implies rotation of V in a clockwise direction
  - Negative frequencies are real, just like negative numbers are real