http://stackoverflow.com/questions/15878325/what-are-the-possible-fast-ways-to-detect-circle-in-an-image

Standard algorithms to find circles are Hough (which jamk mentioned in the comments) and RANSAC. Parameterizing these algorithms will set a baseline speed for your application.

<http://en.wikipedia.org/wiki/Hough_transform>

<http://en.wikipedia.org/wiki/RANSAC>

To speed up these algorithms, you can look at your collection of images and decide whether limiting the search ranges will help speed up the search. That's straightforward enough: only search within a reasonable range for the radius. Since they take edge points as inputs, you can also look at methods to reduce the number of edge points checked.

However, there are a few other tricks to speed up processing.

* Carefully set the range or ranges over which radii are checked. For example, you might not simply check from the smallest possible radius to the largest possible radius, but instead you might split the search into two different ranges: from radius R1 to R2, and then from radius R3 to R4.
* Ditch the Canny edge detection in favor of the fastest possible edge detection your application can tolerate. (You can ditch Canny for lots of applications.)
* Preprocess your image of edge points to eliminate outliers. The appropriate algorithm to eliminate outliers will be specific to your image set, but you'll probably be able to find an algorithm that eliminates obvious outliers and thereby saves some search time in the more expensive circle fit algorithms.
* If your circles are very well defined, and all or nearly all points are present, figure out how you might match only a quarter circle or semicircle instead of a full circle.

Long story short: start with a complete implementation and benchmark it, then gradually tighten up parameter settings and limit search ranges while ensuring that you can still find circles for your application and your image set.

If your images are amenable to scaling, then one possibility is to create an image pyramid of images at different scales: 1/2 scale, 1/4 scale, 1/8 scale, etc. You'll need an edge-preserving scaling method at smaller scales.

Once you have your image pyramid, try the following:

1. Find circles at the very smallest scale. The image will be small and the range of possible radii will be limited, so this should be a quick operation.
2. If you find a circle using the initial fit at the small scale, improve the fit by testing in the next larger scale image -OR- go ahead and search in the full scale image.
3. Check the next largest scale. Circles that weren't visible in the smaller scale image may suddenly "appear" in the current scale.
4. Repeat the steps above through all scales in the image.

Image scaling will be a fast operation, and you can see that if at least one of your circles is present in a smaller scale image you should be able to reduce the total number of cycles by performing a rough circle fit in the small scale image and then optimizing the fit for those edge points alone in the full scale image.

Edge-preserving scaling can also make it possible to use correlation-type tools to find circles, but being able to do so depends on the content of your images, including the noise, how completely edge points represent circles, and so on.

**Canny Edge Detector**

  
**Common Names:** Canny edge detector

**Brief Description**

The http://homepages.inf.ed.ac.uk/rbf/HIPR2/mote.gifCanny operator was designed to be an optimal edge detector (according to particular criteria --- there are other detectors around that also claim to be optimal with respect to slightly different criteria). It takes as input a gray scale image, and produces as output an image showing the positions of http://homepages.inf.ed.ac.uk/rbf/HIPR2/mote.giftracked intensity discontinuities.

**How It Works**

The Canny operator works in a multi-stage process. First of all the image is smoothed by http://homepages.inf.ed.ac.uk/rbf/HIPR2/mote.gif[Gaussian convolution](http://homepages.inf.ed.ac.uk/rbf/HIPR2/gsmooth.htm). Then a simple 2-D first derivative operator (somewhat like the [Roberts Cross](http://homepages.inf.ed.ac.uk/rbf/HIPR2/roberts.htm)) is applied to the smoothed image to highlight regions of the image with high first spatial derivatives. Edges give rise to ridges in the gradient magnitude image. The algorithm then tracks along the top of these ridges and sets to zero all pixels that are not actually on the ridge top so as to give a thin line in the output, a process known as http://homepages.inf.ed.ac.uk/rbf/HIPR2/mote.gif*non-maximal suppression*. The tracking process exhibits hysteresis controlled by two thresholds: *T1* and *T2*, with *T1 > T2*. Tracking can only begin at a point on a ridge higher than *T1*. Tracking then continues in both directions out from that point until the height of the ridge falls below *T2*. This hysteresis helps to ensure that noisy edges are not broken up into multiple edge fragments.

http://homepages.inf.ed.ac.uk/rbf/HIPR2/mote.gif

**Guidelines for Use**

The effect of the Canny operator is determined by three parameters --- the width of the Gaussian [kernel](http://homepages.inf.ed.ac.uk/rbf/HIPR2/kernel.htm) used in the smoothing phase, and the upper and lower thresholds used by the tracker. Increasing the width of the Gaussian kernel reduces the detector's sensitivity to noise, at the expense of losing some of the finer detail in the image. The localization error in the detected edges also increases slightly as the Gaussian width is increased.

Usually, the upper tracking threshold can be set quite high, and the lower threshold quite low for good results. Setting the lower threshold too high will cause noisy edges to break up. Setting the upper threshold too low increases the number of spurious and undesirable edge fragments appearing in the output.

One problem with the basic Canny operator is to do with Y-junctions *i.e.* places where three ridges meet in the gradient magnitude image. Such junctions can occur where an edge is partially occluded by another object. The tracker will treat two of the ridges as a single line segment, and the third one as a line that approaches, but doesn't quite connect to, that line segment.

We use the image

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/cln1.gif)

to demonstrate the effect of the Canny operator on a natural scene.

Using a Gaussian kernel with standard deviation 1.0 and upper and lower thresholds of 255 and 1, respectively, we obtain

[cln1can1](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/cln1can1.gif)

Most of the major edges are detected and lots of details have been picked out well --- note that this may be too much detail for subsequent processing. The `Y-Junction effect' mentioned above can be seen at the bottom left corner of the mirror.

The image

[cln1can2](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/cln1can2.gif)

is obtained using the same kernel size and upper threshold, but with the lower threshold increased to 220. The edges have become more broken up than in the previous image, which is likely to be bad for subsequent processing. Also, the vertical edges on the wall have not been detected, along their full length.

The image

[cln1can3](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/cln1can3.gif)

is obtained by lowering the upper threshold to 128. The lower threshold is kept at 1 and the Gaussian standard deviation remains at 1.0. Many more faint edges are detected along with some short `noisy' fragments. Notice that the detail in the clown's hair is now picked out.

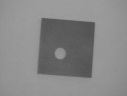
The image

[cln1can4](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/cln1can4.gif)

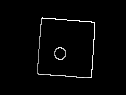
is obtained with the same thresholds as the previous image, but the Gaussian used has a standard deviation of 2.0. Much of the detail on the wall is no longer detected, but most of the strong edges remain. The edges also tend to be smoother and less noisy.

Edges in artificial scenes are often sharper and less complex than those in natural scenes, and this generally improves the performance of any edge detector.

The image

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/wdg2.gif)

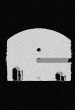
shows such an artificial scene, and

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/wdg2can1.gif)

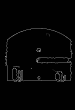
is the output from the Canny operator.

The Gaussian smoothing in the Canny edge detector fulfills two purposes: first it can be used to control the amount of detail that appears in the edge image and second, it can be used to suppress noise.

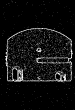
To demonstrate how the Canny operator performs on noisy images we use

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/ufo2noi2.gif)

which contains Gaussian noise with a standard deviation of *15*. Neither the [Roberts Cross](http://homepages.inf.ed.ac.uk/rbf/HIPR2/roberts.htm) nor the [Sobel](http://homepages.inf.ed.ac.uk/rbf/HIPR2/sobel.htm) operator are able to detect the edges of the object while removing all the noise in the image. Applying the Canny operator using a standard deviation of *1.0* yields

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/ufo2can1.gif)

All the edges have been detected and almost all of the noise has been removed. For comparison,

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/ufo2sob6.gif)

is the result of applying the Sobel operator and [thresholding](http://homepages.inf.ed.ac.uk/rbf/HIPR2/threshld.htm) the output at a value of *150*.

We use

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/ren1.gif)

to demonstrate how to control the details contained in the resulting edge image. The image

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/ren1can1.gif)

is the result of applying the Canny edge detector using a standard deviation of *1.0* and an upper and lower threshold of *255* and *1*, respectively. This image contains many details; however, for an automated recognition task we might be interested to obtain only lines that correspond to the boundaries of the objects. If we increase the standard deviation for the Gaussian smoothing to *1.8*, the Canny operator yields

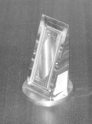
[ren1can2](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/ren1can2.gif)

Now, the edges corresponding to the uneveness of the surface have disappeared from the image, but some edges corresponding to changes in the surface orientation remain. Although these edges are `weaker' than the boundaries of the objects, the resulting pixel values are the same, due to the [saturation](http://homepages.inf.ed.ac.uk/rbf/HIPR2/wrap.htm) of the image. Hence, if we [scale down](http://homepages.inf.ed.ac.uk/rbf/HIPR2/pixmult.htm) the image before the edge detection, we can use the upper threshold of the edge tracker to remove the weaker edges. The image

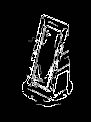
[ren1can3](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/ren1can3.gif)

is the result of first scaling the image with *0.25* and then applying the Canny operator using a standard deviation of *1.8* and an upper and lower threshold of *200* and *1*, respectively. The image shows the desired result that all the boundaries of the objects have been detected whereas all other edges have been removed.

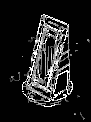
Although the Canny edge detector allows us the find the intensity discontinuities in an image, it is not guaranteed that these discontinuities correspond to actual edges of the object. This is illustrated using

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/prt2.gif)

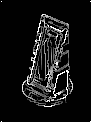
We obtain

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/prt2can1.gif)

by using a standard deviation of *1.0* and an upper and lower threshold of *255* and *1*, respectively. In this case, some edges of the object do not appear in the image and many edges in the image originate only from reflections on the object. It is a demanding task for an automated system to interpret this image. We try to improve the edge image by decreasing the upper threshold to *150*, as can be seen in

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/prt2can2.gif)

We now obtain most of the edges of the object, but we also increase the amount of noise. The result of further decreasing the upper threshold to *100* and increasing the standard deviation to *2* is shown in

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/prt2can3.gif)

**Common Variants**

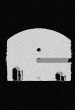
The problem with Y-junctions mentioned above can be solved by including a model of such junctions in the ridge tracker. This will ensure that no spurious gaps are generated at these junctions.

**Interactive Experimentation**

You can interactively experiment with this operator by clicking [here](http://homepages.inf.ed.ac.uk/rbf/HIPR2/cannydemo.htm).

**Exercises**

1. Adjust the parameters of the Canny operator so that you can detect the edges of

[](http://homepages.inf.ed.ac.uk/rbf/HIPR2/images/ufo2noi2.gif)

while removing *all* of the noise.

1. What effect does increasing the Gaussian kernel size have on the magnitudes of the gradient maxima at edges? What change does this imply has to be made to the tracker thresholds when the kernel size is increased?
2. It is sometimes easier to evaluate edge detector performance after [thresholding](http://homepages.inf.ed.ac.uk/rbf/HIPR2/threshld.htm) the edge detector output at some low gray scale value (*e.g.* 1) so that all detected edges are marked by bright white pixels. Try this out on the third and fourth example images of the clown mentioned above. Comment on the differences between the two images.
3. How does the Canny operator compare with the [Roberts Cross](http://homepages.inf.ed.ac.uk/rbf/HIPR2/roberts.htm) and [Sobel](http://homepages.inf.ed.ac.uk/rbf/HIPR2/sobel.htm) edge detectors in terms of speed? What do you think is the slowest stage of the process?
4. How does the Canny operator compare in terms of noise rejection and edge detection with other operators such as the Roberts Cross and Sobel operators?
5. How does the Canny operator compare with other edge detectors on simple artificial 2-D scenes? And on more complicated natural scenes?
6. Under what situations might you choose to use the Canny operator rather than the Roberts Cross or Sobel operators? In what situations would you definitely not choose it?

**References**

**R. Boyle and R. Thomas** *Computer Vision: A First Course*, Blackwell Scientific Publications, 1988, p 52.

**J. Canny** *A Computational Approach to Edge Detection*, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 8, No. 6, Nov. 1986.

**E. Davies** *Machine Vision: Theory, Algorithms and Practicalities*, Academic Press, 1990, Chap. 5.

**R. Gonzalez and R. Woods** *Digital Image Processing*, Addison-Wesley Publishing Company, 1992, Chap. 4.

**Local Information**

Specific information about this operator may be found [here.](http://homepages.inf.ed.ac.uk/rbf/HIPR2/local/canny.txt)

More general advice about the local HIPR installation is available in the [*Local Information*](http://homepages.inf.ed.ac.uk/rbf/HIPR2/local.htm) introductory section.

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Hough Transform

https://en.wikipedia.org/wiki/Hough\_transform

From Wikipedia, the free encyclopedia

|  |
| --- |
| [**Feature detection**](https://en.wikipedia.org/wiki/Feature_detection_(computer_vision)) |
| [**Edge detection**](https://en.wikipedia.org/wiki/Edge_detection) |
| * [Canny](https://en.wikipedia.org/wiki/Canny_edge_detector) * [Deriche](https://en.wikipedia.org/wiki/Deriche_edge_detector) * [Differential](https://en.wikipedia.org/wiki/Edge_detection#Differential_edge_detection) * [Sobel](https://en.wikipedia.org/wiki/Sobel_operator) * [Prewitt](https://en.wikipedia.org/wiki/Prewitt_operator) * [Roberts cross](https://en.wikipedia.org/wiki/Roberts_cross) |
| [**Corner detection**](https://en.wikipedia.org/wiki/Corner_detection) |
| * [Harris operator](https://en.wikipedia.org/wiki/Corner_detection#The_Harris_.26_Stephens_.2F_Plessey_.2F_Shi.E2.80.93Tomasi_corner_detection_algorithm) * [Shi and Tomasi](https://en.wikipedia.org/wiki/Corner_detection#The_Harris_.26_Stephens_.2F_Plessey_.2F_Shi.E2.80.93Tomasi_corner_detection_algorithm) * [Level curve curvature](https://en.wikipedia.org/wiki/Corner_detection#The_level_curve_curvature_approach) * [SUSAN](https://en.wikipedia.org/wiki/Corner_detection#The_SUSAN_corner_detector) * [FAST](https://en.wikipedia.org/wiki/Corner_detection#AST-based_feature_detectors) |
| [**Blob detection**](https://en.wikipedia.org/wiki/Blob_detection) |
| * [Laplacian of Gaussian (LoG)](https://en.wikipedia.org/wiki/Blob_detection#The_Laplacian_of_Gaussian) * [Difference of Gaussians (DoG)](https://en.wikipedia.org/wiki/Difference_of_Gaussians) * [Determinant of Hessian (DoH)](https://en.wikipedia.org/wiki/Blob_detection#The_determinant_of_the_Hessian) * [Maximally stable extremal regions](https://en.wikipedia.org/wiki/Maximally_stable_extremal_regions) * [PCBR](https://en.wikipedia.org/wiki/Principal_Curvature-Based_Region_Detector) |
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| **Hough transform** |
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| [**Scale space**](https://en.wikipedia.org/wiki/Scale_space) |
| * [Scale-space axioms](https://en.wikipedia.org/wiki/Scale-space_axioms) * [Implementation details](https://en.wikipedia.org/wiki/Scale_space_implementation) * [Pyramids](https://en.wikipedia.org/wiki/Pyramid_(image_processing)) |
| * [v](https://en.wikipedia.org/wiki/Template:Feature_detection_(computer_vision)_navbox) * [t](https://en.wikipedia.org/wiki/Template_talk:Feature_detection_(computer_vision)_navbox) * [e](https://en.wikipedia.org/w/index.php?title=Template:Feature_detection_(computer_vision)_navbox&action=edit) |

The **Hough transform** is a [feature extraction](https://en.wikipedia.org/wiki/Feature_extraction) technique used in [image analysis](https://en.wikipedia.org/wiki/Image_analysis), [computer vision](https://en.wikipedia.org/wiki/Computer_vision), and [digital image processing](https://en.wikipedia.org/wiki/Digital_image_processing).[[1]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-1) The purpose of the technique is to find imperfect instances of objects within a certain class of shapes by a voting procedure. This voting procedure is carried out in a[parameter space](https://en.wikipedia.org/wiki/Parameter_space), from which object candidates are obtained as local maxima in a so-called accumulator space that is explicitly constructed by the algorithm for computing the Hough transform.

The classical Hough transform was concerned with the identification of[lines](https://en.wikipedia.org/wiki/Line_(mathematics)) in the image, but later the Hough transform has been extended to identifying positions of arbitrary shapes, most commonly circles or ellipses. The Hough transform as it is universally used today was invented by [Richard Duda](https://en.wikipedia.org/wiki/Richard_Duda) and [Peter Hart](https://en.wikipedia.org/wiki/Peter_E._Hart) in 1972, who called it a "generalized Hough transform"[[2]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-2) after the related 1962 patent of Paul Hough.[[3]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-3)[[4]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-4) The transform was popularized in the[computer vision](https://en.wikipedia.org/wiki/Computer_vision) community by [Dana H. Ballard](https://en.wikipedia.org/wiki/Dana_H._Ballard) through a 1981 journal article titled "[Generalizing the Hough transform to detect arbitrary shapes](https://en.wikipedia.org/wiki/Generalised_Hough_Transform)".

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* [2Theory](https://en.wikipedia.org/wiki/Hough_transform#Theory)
* [3Implementation](https://en.wikipedia.org/wiki/Hough_transform#Implementation)
* [4Example](https://en.wikipedia.org/wiki/Hough_transform#Example)
* [5Variations and extensions](https://en.wikipedia.org/wiki/Hough_transform#Variations_and_extensions)
  + [5.1Using the gradient direction to reduce the number of votes](https://en.wikipedia.org/wiki/Hough_transform#Using_the_gradient_direction_to_reduce_the_number_of_votes)
  + [5.2Kernel-based Hough transform (KHT)](https://en.wikipedia.org/wiki/Hough_transform#Kernel-based_Hough_transform_.28KHT.29)
  + [5.33-D Kernel-based Hough transform for plane detection (3DKHT)](https://en.wikipedia.org/wiki/Hough_transform#3-D_Kernel-based_Hough_transform_for_plane_detection_.283DKHT.29)
  + [5.4Hough transform of curves, and its generalization for analytical and non-analytical shapes](https://en.wikipedia.org/wiki/Hough_transform#Hough_transform_of_curves.2C_and_its_generalization_for_analytical_and_non-analytical_shapes)
  + [5.5Circle detection process](https://en.wikipedia.org/wiki/Hough_transform#Circle_detection_process)
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History[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=1" \o "Edit section: History)]

It was initially invented for machine analysis of [bubble chamber](https://en.wikipedia.org/wiki/Bubble_chamber) photographs (Hough, 1959).

The Hough transform was patented as [U.S. Patent 3,069,654](https://www.google.com/patents/US3069654) in 1962 and assigned to the U.S. Atomic Energy Commission with the name "Method and Means for Recognizing Complex Patterns". This patent uses a slope-intercept parametrization for straight lines, which awkwardly leads to an unbounded transform space since the slope can go to infinity.

The rho-theta parametrization universally used today was first described in

Duda, R. O. and P. E. Hart, "Use of the Hough Transformation to Detect Lines and Curves in Pictures," *Comm. ACM, Vol. 15*, pp. 11–15 (January, 1972),

although it was already standard for the [Radon transform](https://en.wikipedia.org/wiki/Radon_transform) since at least the 1930s.

O'Gorman and Clowes' variation is described in

*O'Gorman, Frank; Clowes, MB (1976). "Finding Picture Edges Through Collinearity of Feature Points". IEEE Trans. Computers****25****(4): 449–456.*

The story of how the modern form of the Hough transform was invented is given in

Hart, P. E., "[How the Hough Transform was Invented](http://www.iro.umontreal.ca/~mignotte/IFT6150/ComplementCours/HoughTransform.pdf)" (PDF, 268 kB), *IEEE Signal Processing Magazine, Vol 26, Issue 6*, pp 18 – 22 (November, 2009).

Theory[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=2" \o "Edit section: Theory)]

In automated analysis of digital images, a subproblem often arises of detecting simple shapes, such as straight lines, circles or ellipses. In many cases an [edge detector](https://en.wikipedia.org/wiki/Edge_detection) can be used as a pre-processing stage to obtain image points or image pixels that are on the desired curve in the image space. Due to imperfections in either the image data or the edge detector, however, there may be missing points or pixels on the desired curves as well as spatial deviations between the ideal line/circle/ellipse and the noisy edge points as they are obtained from the edge detector. For these reasons, it is often non-trivial to group the extracted edge features to an appropriate set of lines, circles or ellipses. The purpose of the Hough transform is to address this problem by making it possible to perform groupings of edge points into object candidates by performing an explicit voting procedure over a set of parameterized image objects (Shapiro and Stockman, 304).

The simplest case of Hough transform is detecting straight lines. In general, the straight line *y = mx + b* can be represented as a point (*b*, *m*) in the parameter space. However, vertical lines pose a problem. They would give rise to unbounded values of the slope parameter *m*. Thus, for computational reasons, Duda and Hart[[5]](https://en.wikipedia.org/wiki/Hough_transform" \l "cite_note-5) proposed the use of the [Hesse normal form](https://en.wikipedia.org/wiki/Hesse_normal_form)

 ,

where  is the distance from the [origin](https://en.wikipedia.org/wiki/Origin_(mathematics)) to the closest point on the straight line, and (*theta*) is the angle between the  axis and the line connecting the origin with that closest point.

It is therefore possible to associate with each line of the image a pair . The  plane is sometimes referred to as *Hough space* for the set of straight lines in two dimensions. This representation makes the Hough transform conceptually very close to the two-dimensional[Radon transform](https://en.wikipedia.org/wiki/Radon_transform). (They can be seen as different ways of looking at the same transform.[[6]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-6))

Given a *single point* in the plane, then the set of *all* straight lines going through that point corresponds to a [sinusoidal](https://en.wikipedia.org/wiki/Sine_wave) curve in the (*r,θ*) plane, which is unique to that point. A set of two or more points that form a straight line will produce sinusoids which cross at the (*r,θ*) for that line. Thus, the problem of detecting [collinear points](https://en.wikipedia.org/wiki/Line_(geometry)#Collinear_points)can be converted to the problem of finding [concurrent](https://en.wikipedia.org/wiki/Concurrent_lines) curves.[[7]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-7)

Implementation[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=3" \o "Edit section: Implementation)]

The linear Hough transform [algorithm](https://en.wikipedia.org/wiki/Algorithm) uses a two-dimensional array, called an accumulator, to detect the existence of a line described by . The [dimension](https://en.wikipedia.org/wiki/Dimension) of the accumulator equals the number of unknown parameters, i.e., two, considering quantized values of r and θ in the pair (r,θ). For each pixel at *(x,y)*and its neighborhood, the Hough transform algorithm determines if there is enough evidence of a straight line at that pixel. If so, it will calculate the parameters (r,θ) of that line, and then look for the accumulator's bin that the parameters fall into, and increment the value of that bin. By finding the bins with the highest values, typically by looking for local maxima in the accumulator space, the most likely lines can be extracted, and their (approximate) geometric definitions read off. (Shapiro and Stockman, 304) The simplest way of finding these *peaks* is by applying some form of threshold, but other techniques may yield better results in different circumstances – determining which lines are found as well as how many. Since the lines returned do not contain any length information, it is often necessary, in the next step, to find which parts of the image match up with which lines. Moreover, due to imperfection errors in the edge detection step, there will usually be errors in the accumulator space, which may make it non-trivial to find the appropriate peaks, and thus the appropriate lines.

The final result of the linear Hough transform is a two-dimensional array (matrix) similar to the accumulator—one dimension of this matrix is the quantized angle θ and the other dimension is the quantized distance r. Each element of the matrix has a value equal to the number of points or pixels that are positioned on the line represented by quantized parameters (r, θ). So the element with the highest value indicates the straight line that is most represented in the input image.[[8]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-Jeppe_Jensen_2007-8)

Example[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=4" \o "Edit section: Example)]

Consider three data points, shown here as black dots.

* For each data point, a number of lines are plotted going through it, all at different angles. These are shown here as solid lines.
* For each solid line a line is plotted which is [perpendicular](https://en.wikipedia.org/wiki/Perpendicular) to it and which intersects the [origin](https://en.wikipedia.org/wiki/Origin_(mathematics)). These are shown as dashed lines.
* The length (i.e. perpendicular distance to the origin) and angle of each dashed line is measured. In the diagram above, the results are shown in tables.
* This is repeated for each data point.
* A graph of the line lengths for each angle, known as a Hough space graph, is then created.

The point where the curves intersect gives a distance and angle. This distance and angle indicate the line which intersects the points being tested. In the graph shown the lines intersect at the pink point; this corresponds to the solid pink line in the diagrams above, which passes through all three points.

The following is a different example showing the results of a Hough transform on a raster image containing two thick lines.

The results of this transform were stored in a matrix. Cell value represents the number of curves through any point. Higher cell values are rendered brighter. The two distinctly bright spots are the Hough parameters of the two lines. From these spots' positions, angle and distance from image center of the two lines in the input image can be determined.

Variations and extensions[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=5" \o "Edit section: Variations and extensions)]

**Using the gradient direction to reduce the number of votes**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=6" \o "Edit section: Using the gradient direction to reduce the number of votes)]

An improvement suggested by O'Gorman and Clowes can be used to detect lines if one takes into account that the local [gradient](https://en.wikipedia.org/wiki/Gradient) of the image intensity will necessarily be orthogonal to the edge. Since [edge detection](https://en.wikipedia.org/wiki/Edge_detection) generally involves computing the intensity [gradient](https://en.wikipedia.org/wiki/Gradient) magnitude, the gradient direction is often found as a side effect. If a given point of coordinates (*x,y*) happens to indeed be on a line, then the local direction of the gradient gives the *θ* parameter corresponding to said line, and the *r*parameter is then immediately obtained. (Shapiro and Stockman, 305) The gradient direction can be estimated to within 20°, which shortens the sinusoid trace from the full 180° to roughly 45°. This reduces the computation time and has the interesting effect of reducing the number of useless votes, thus enhancing the visibility of the spikes corresponding to real lines in the image.

**Kernel-based Hough transform (KHT)**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=7" \o "Edit section: Kernel-based Hough transform (KHT))]

Fernandes and Oliveira [[9]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-9) suggested an improved voting scheme for the Hough transform that allows a software implementation to achieve real-time performance even on relatively large images (e.g., 1280×960). The Kernel-based Hough transform uses the same  parameterization proposed by Duda and Hart but operates on clusters of approximately collinear pixels. For each cluster, votes are cast using an oriented elliptical-Gaussian kernel that models the uncertainty associated with the best-fitting line with respect to the corresponding cluster. The approach not only significantly improves the performance of the voting scheme, but also produces a much cleaner accumulator and makes the transform more robust to the detection of spurious lines.

**3-D Kernel-based Hough transform for plane detection (3DKHT)**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=8" \o "Edit section: 3-D Kernel-based Hough transform for plane detection (3DKHT))]

Limberger and Oliveira [[10]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-10) suggested a deterministic technique for plane detection in unorganized point clouds whose cost is  in the number of samples, achieving real-time performance for relatively large datasets (up to  points on a 3.4 GHz CPU). It is based on a fast Hough-transform voting strategy for planar regions, inspired by the Kernel-based Hough transform (KHT). This 3D Kernel-based Hough transform (3DKHT) uses a fast and robust algorithm to segment clusters of approximately co-planar samples, and casts votes for individual clusters (instead of for individual samples) on a () spherical accumulator using a trivariate Gaussian kernel. The approach is several orders of magnitude faster than existing (non-deterministic) techniques for plane detection in point clouds, such as [RHT](https://en.wikipedia.org/wiki/Randomized_Hough_Transform) and[RANSAC](https://en.wikipedia.org/wiki/RANSAC), and scales better with the size of the datasets. It can be used with any application that requires fast detection of planar features on large datasets.

**Hough transform of curves, and its generalization for analytical and non-analytical shapes**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=9" \o "Edit section: Hough transform of curves, and its generalization for analytical and non-analytical shapes)]

Although the version of the transform described above applies only to finding straight lines, a similar transform can be used for finding any shape which can be represented by a set of parameters. A circle, for instance, can be transformed into a set of three parameters, representing its center and radius, so that the Hough space becomes three dimensional. Arbitrary ellipses and curves can also be found this way, as can any shape easily expressed as a set of parameters.

The generalization of the Hough transform for detecting analytical shapes in spaces having any dimensionality was proposed by Fernandes and Oliveira.[[11]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-11) In contrast to other Hough transform-based approaches for analytical shapes, Fernandes' technique does not depend on the shape one wants to detect nor on the input data type. The detection can be driven to a type of analytical shape by changing the assumed model of geometry where data have been encoded (e.g., [euclidean space](https://en.wikipedia.org/wiki/Euclidean_space" \o "Euclidean space),[projective space](https://en.wikipedia.org/wiki/Projective_space), [conformal geometry](https://en.wikipedia.org/wiki/Conformal_geometry), and so on), while the proposed formulation remains unchanged. Also, it guarantees that the intended shapes are represented with the smallest possible number of parameters, and it allows the concurrent detection of different kinds of shapes that best fit an input set of entries with different dimensionalities and different geometric definitions (e.g., the concurrent detection of planes and spheres that best fit a set of points, straight lines and circles).

For more complicated shapes in the plane (i.e., shapes that cannot be represented analytically in some 2D space), the [Generalised Hough transform](https://en.wikipedia.org/wiki/Generalised_Hough_transform" \o "Generalised Hough transform) [[12]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-12) is used, which allows a feature to vote for a particular position, orientation and/or scaling of the shape using a predefined look-up table.

**Circle detection process**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=10" \o "Edit section: Circle detection process)]

The process of identifying possible [circular objects in Hough space](https://en.wikipedia.org/wiki/Circle_Hough_Transform) is relatively simple,

* First we create our accumulator space which is made up of a cell for each pixel, initially each of these will be set to 0.
* For each(edge point in image(i, j)): Increment all cells which according to the equation of a circle  could be the center of a circle, these cells are represented by the letter 'a' in the equation.
* For all possible value of a found in the previous step, find all possible values of b which satisfy the equation.
* Search for the local maxima cells, these are any cells whose value is greater than every other cell in its neighbourhood. These cells are the one with the highest probability of being the location of the circle(s) we are trying to locate.

Note that in most problems we will know the radius of the circle we are trying to locate beforehand, however if this is not the case we can use a three-dimensional accumulator space, this is much more computationally expensive. This method can also detect circles that are partially outside of the accumulator space if enough of its area is still present within it.

**Detection of 3D objects (Planes and cylinders)**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=11" \o "Edit section: Detection of 3D objects (Planes and cylinders))]

Hough transform can also be used for the detection of 3D objects in range data or 3D[point clouds](https://en.wikipedia.org/wiki/Point_cloud). The extension of classical Hough transform for plane detection is quite straightforward. A plane is represented by its explicit equation  for which we can use a 3D Hough space corresponding to ,  and . This extension suffers from the same problems as its 2D counterpart i.e., near horizontal planes can be reliably detected, while the performance deteriorates as planar direction becomes vertical (big values of  and  amplify the noise in the data). This formulation of the plane has been used for the detection of planes in the [point clouds](https://en.wikipedia.org/wiki/Point_cloud) acquired from airborne laser scanning [[13]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-13) and works very well because in that domain all planes are nearly horizontal.

For generalized plane detection using Hough transform, the plane can be parametrized by its normal vector  (using spherical coordinates) and its distance from the origin  resulting in a three dimensional Hough space. This results in each point in the input data voting for a sinusoidal surface in the Hough space. The intersection of these sinusoidal surfaces indicates presence of a plane.[[14]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-14) A more general approach for more than 3 dimensions requires search heuristics to remain feasible.[[15]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-15)

Hough transform has also been used to find cylindrical objects in [point clouds](https://en.wikipedia.org/wiki/Point_cloud) using a two step approach. The first step finds the orientation of the cylinder and the second step finds the position and radius.[[16]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-16)

**Using weighted features**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=12" \o "Edit section: Using weighted features)]

One common variation detail. That is, finding the bins with the highest count in one stage can be used to constrain the range of values searched in the next.

**Carefully chosen parameter space**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=13" \o "Edit section: Carefully chosen parameter space)]

A high-dimensional parameter space for the Hough transform is not only slow, but if implemented without forethought can easily overrun the available memory. Even if the programming environment allows the allocation of an array larger than the available memory space through virtual memory, the number of page swaps required for this will be very demanding because the accumulator array is used in a randomly accessed fashion, rarely stopping in contiguous memory as it skips from index to index.

Consider the task of finding ellipses in an 800x600 image. Assuming that the radii of the ellipses are oriented along principal axes, the parameter space is four-dimensional. (x,y) defines the center of the ellipse, and a and b denote the two radii. Allowing the center to be anywhere in the image, adds the constraint 0<x<800 and 0<y<600. If the radii are given the same values as constraints, what is left is a sparsely filled accumulator array of more than 230 billion values.

A program thus conceived is unlikely to be allowed to allocate sufficient memory. This doesn't mean that the problem can't be solved, but only that new ways to constrain the size of the accumulator array are to be found, which makes it feasible. For instance:

1. If it is reasonable to assume that the ellipses are each contained entirely within the image, the range of the radii can be reduced. The largest the radii can be is if the center of the ellipse is in the center of the image, allowing the edges of the ellipse to stretch to the edges. In this extreme case, the radii can only each be half the magnitude of the image size oriented in the same direction. Reducing the range of a and b in this fashion reduces the accumulator array to 57 billion values.
2. Trade accuracy for space in the estimation of the center: If the center is predicted to be off by 3 on both the x and y axis this reduces the size of the accumulator array to about 6 billion values.
3. Trade accuracy for space in the estimation of the radii: If the radii are estimated to each be off by 5 further reduction of the size of the accumulator array occurs, by about 256 million values.
4. Crop the image to areas of interest. This is image dependent, and therefore unpredictable, but imagine a case where all of the edges of interest in an image are in the upper left quadrant of that image. The accumulator array can be reduced even further in this case by constraining all 4 parameters by a factor of 2, for a total reduction factor of 16.

By applying just the first three of these constraints to the example stated about, the size of the accumulator array is reduced by almost a factor of 1000, bringing it down to a size that is much more likely to fit within a modern computer's memory.

**Efficient ellipse detection algorithm**[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=14" \o "Edit section: Efficient ellipse detection algorithm)]

Yonghong Xie and Qiang Ji give an efficient way of implementing the Hough transform for ellipse detection by overcoming the memory issues.[[17]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-XieJi2002-17) As discussed in the algorithm (on page 2 of the paper), this approach uses only a one-dimensional accumulator (for the minor axis) in order to detect ellipses in the image. The complexity is O(N3) in the number of non-zero points in the image.

Limitations[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=15" \o "Edit section: Limitations)]

The Hough transform is only efficient if a high number of votes fall in the right bin, so that the bin can be easily detected amid the background noise. This means that the bin must not be too small, or else some votes will fall in the neighboring bins, thus reducing the visibility of the main bin.[[18]](https://en.wikipedia.org/wiki/Hough_transform#cite_note-18)

Also, when the number of parameters is large (that is, when we are using the Hough transform with typically more than three parameters), the average number of votes cast in a single bin is very low, and those bins corresponding to a real figure in the image do not necessarily appear to have a much higher number of votes than their neighbors. The complexity increases at a rate of  with each additional parameter, where  is the size of the image space and  is the number of parameters. (Shapiro and Stockman, 310) Thus, the Hough transform must be used with great care to detect anything other than lines or circles.

Finally, much of the efficiency of the Hough transform is dependent on the quality of the input data: the edges must be detected well for the Hough transform to be efficient. Use of the Hough transform on noisy images is a very delicate matter and generally, a denoising stage must be used before. In the case where the image is corrupted by speckle, as is the case in radar images, the [Radon transform](https://en.wikipedia.org/wiki/Radon_transform) is sometimes preferred to detect lines, because it attenuates the noise through summation.

See also[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=16" \o "Edit section: See also)]

* [Generalised Hough transform](https://en.wikipedia.org/wiki/Generalised_Hough_transform)
* [Randomized Hough transform](https://en.wikipedia.org/wiki/Randomized_Hough_transform)
* [Radon transform](https://en.wikipedia.org/wiki/Radon_transform)
* [Fourier transform](https://en.wikipedia.org/wiki/Fourier_transform)

References[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=17" \o "Edit section: References)]

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External links[[edit](https://en.wikipedia.org/w/index.php?title=Hough_transform&action=edit&section=18" \o "Edit section: External links)]

* [hough\_transform.cpp](http://cimg.sourceforge.net/screenshots.shtml)[[*dead link*](https://en.wikipedia.org/wiki/Wikipedia:Link_rot)] – C++ code – example of CImg library ([open source](https://en.wikipedia.org/wiki/Open_source_software) library, [C++](https://en.wikipedia.org/wiki/C%2B%2B) source code, [Grayscale](https://en.wikipedia.org/wiki/Grayscale) images)
* [Interactive Demonstration on the Basics of the Hough Transform](http://matlabtricks.com/post-39/understanding-the-hough-transform)
* <http://www.rob.cs.tu-bs.de/content/04-teaching/06-interactive/Hough.html> –[Java Applet](https://en.wikipedia.org/wiki/Java_Applet) + Source for learning the Hough transformation in slope-intercept form
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* <http://imaging.gmse.net/articledeskew.html> – Deskew images using Hough transform ([Visual Basic](https://en.wikipedia.org/wiki/Visual_Basic) source code)
* <http://www.mitov.com/products/visionlab> – [Delphi](https://en.wikipedia.org/wiki/Embarcadero_Delphi), [C++](https://en.wikipedia.org/wiki/C%2B%2B) and [.NET](https://en.wikipedia.org/wiki/.NET_Framework) free for educational purposes library containing Line, Circle and Line segment Hough transform components.
* [Tarsha-Kurdi, F., Landes, T., Grussenmeyer, P., 2007a. Hough-transform and extended RANSAC algorithms for automatic detection of 3d building roof planes from Lidar data.](http://www.isprs.org/proceedings/XXXVI/3-W52/final_papers/Tarsha-Kurdi_2007.pdf) ISPRS Proceedings. Workshop Laser scanning. Espoo, Finland, September 12–14, 2007.
* [Into](http://intopii.com/into/) contains open source implementations of linear and circular Hough transform in C++
* <http://www.vision.ime.usp.br/~edelgado/defesa/code/hough.html> Hough-transform for Ellipse detection, implemented in C.
* [scikit-image](http://scikit-image.org/docs/dev/api/skimage.transform.html) Hough-transform for line, circle and ellipse, implemented in Python.
* [[1]](http://www.mathworks.com/matlabcentral/fileexchange/40537-wavelet-based-circular-hough-transform) Hough transform based on wavelet filtering, to detect a circle of a particular radius. (Matlab code.)
* [Hough transform for lines using MATLAB](http://www.mathworks.com/help/images/detect-lines-in-images.html)
* [Hough transform for circles and ellipses in MATLAB](http://www.mathworks.com/help/images/ref/imfindcircles.html)
* [KHT](http://www2.ic.uff.br/~laffernandes/projects/kht/) – C++ source code.
* [3DKHT](http://www.inf.ufrgs.br/~oliveira/pubs_files/HT3D/HT3D_page.html) – C++ source code and datasets.