Paul Trapping of Atoms: Assignment 3

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Physics of Data
Quantum Information with Atoms and Photons

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Free particle in a 1D time-dependent harmonic potential

- **Time dependent hamiltonian**:

$$\hat{H}(t) = \hat{H}_0 + \hat{W}(t) = \frac{p^2}{2m} + \frac{1}{2} m\omega_0^2 \hat{x}^2 cos(\omega t) \tag{1}$$

- **Effective Hamiltonian**: Considering the high-frequency approximation with $\omega \gg \omega_0$ and $\omega \gg \|E_0\|$, the expansion formula yields the following effective hamiltonian:

$$\hat{H}_{eff} = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2}{4} \left(\frac{\omega_0}{\omega}\right)^2 \hat{x}^2 \tag{2}$$

which indeed describes a particle under a harmonic potential.

Optical Lattice in the Tight-Binding Regime

- **Time dependent hamiltonian**:

$$\hat{H}(t) = -J\sum_i \left(|i+1\rangle\langle i| + |i\rangle\langle i+1| \right) + V cos(\omega t)\sum_i i^2 |i\rangle\langle i| \tag{3}$$

- **Effective Hamiltonian**: Considering the high-frequency approximation with $\omega \gg V$ and $\omega \gg J$, the expansion formula yields the following effective hamiltonian:

$$\hat{H}_{eff} = \sum_i -J\left(1 - \frac{V^2}{4\omega^2}(2i+1)^2\right)\left(|i+1\rangle\langle i| + |i\rangle\langle i+1|\right) \tag{4}$$

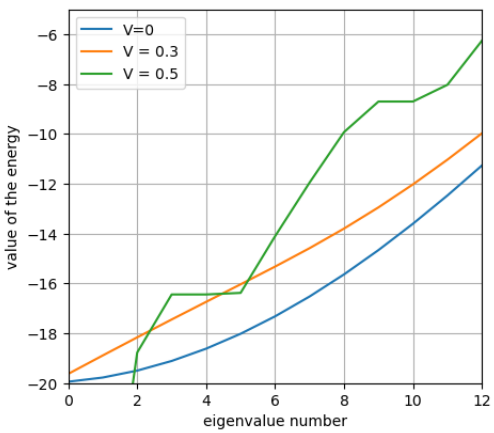
- No diagonal elements;
- Only kinetic components

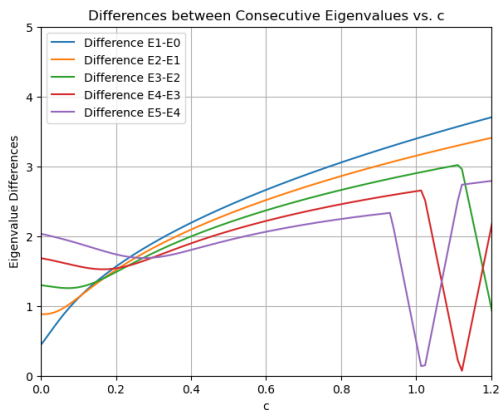
Transition to the Continuum Case

- Parameter c which describes how fast does the coupling change over position:

$$\hat{H}_{eff} = -J \sum_i (1 + c\,(2i+1)^2)\,(|i+1\rangle\langle i| + |i\rangle\langle i+1|)$$

$$E(k) = -2J(i)\cos(k) \quad \text{and} \quad c = -\frac{V^2}{4\omega^2}$$





- Equal Spacing between consecutive eigenvalues with $V \simeq 0.3$
- Linear trend, typical of harmonic oscillator

Gaussian Wavepacket

Gaussians Wavepacket Parameters

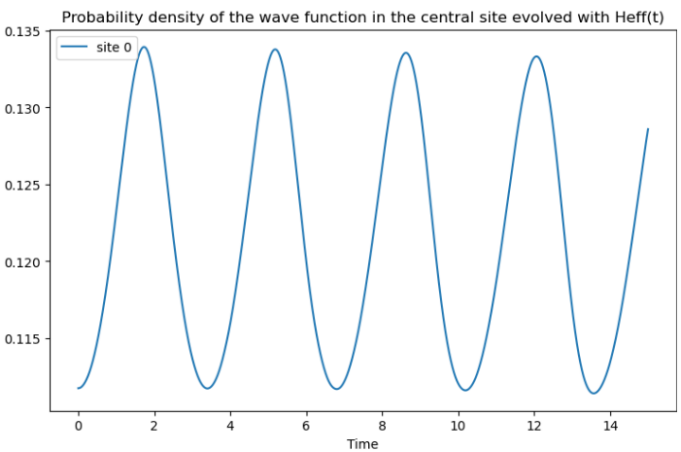
- $\mu = 0.5$
- $\sigma = 5$

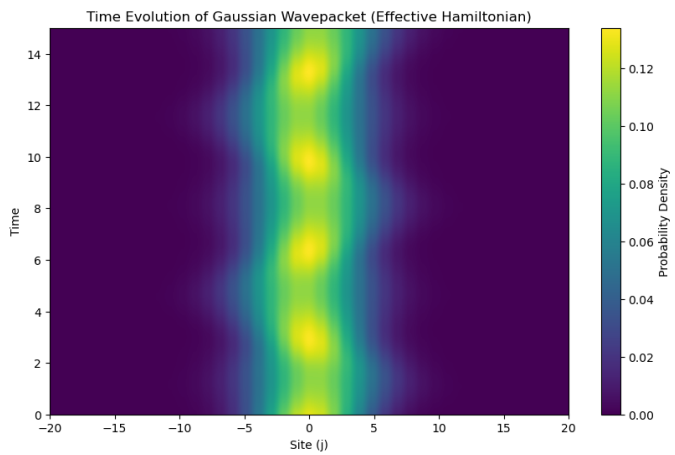
Lattice Parameters

- $N = 20$
- $J = 10$
- $V = 0.3$: best param
- $\omega = 9$: high-frequency

- **Effective Hamiltonian**: probability evolution in the center

  Frequency of oscillation $f = 0.292\text{Hz} \rightarrow T = 3.422\text{s}$





Gaussian Wavepacket

Gaussian Wavepacket Parameters

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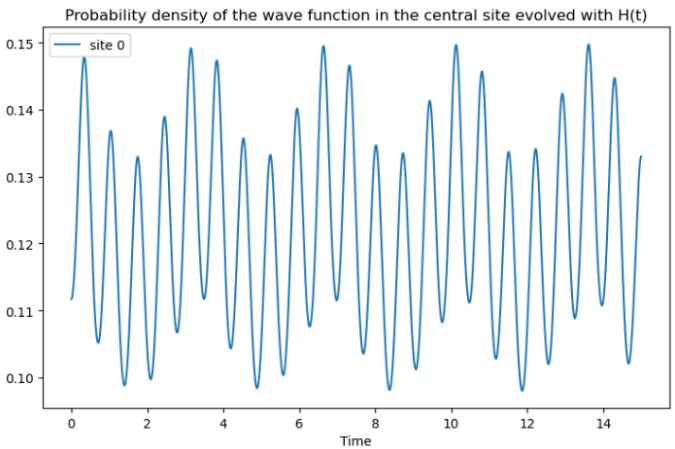
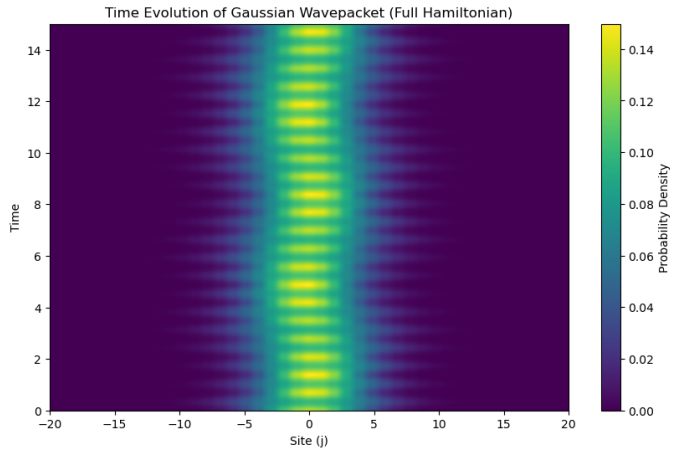
Lattice Parameters

- $N = 20$
- $J = 10$
- $V = 0.3$: best param
- $\omega = 9$: high-frequency

- **Time dependent Hamiltonian**: probability evolution in the center

  Frequency of oscillation $f = 0.291\text{Hz} \rightarrow T = 3.412\text{s}$



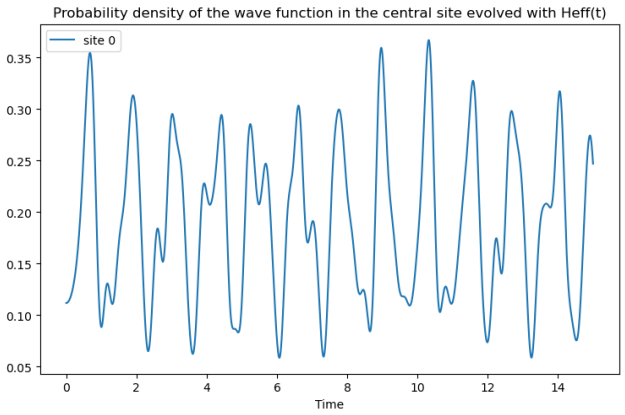


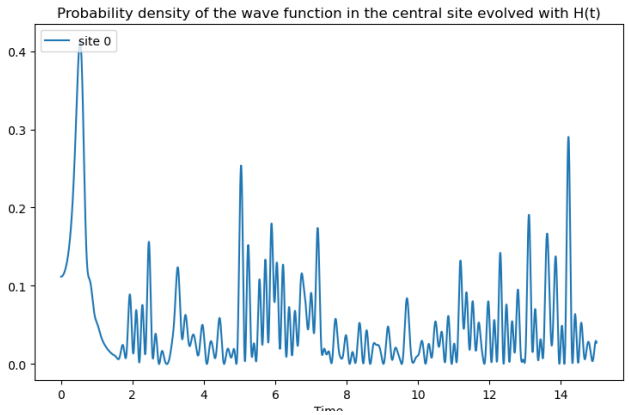
- **Agreement!** (some micromotion)

Approximation Failure

The high-frequency approximation breaks down for:

- **Small values of** ω: considering $\omega = 3$, $J = 10$ and $V = 0.3$





- **Big values of** V: considering $\omega = 9$, $J = 10$ and $V = 30$