HW3

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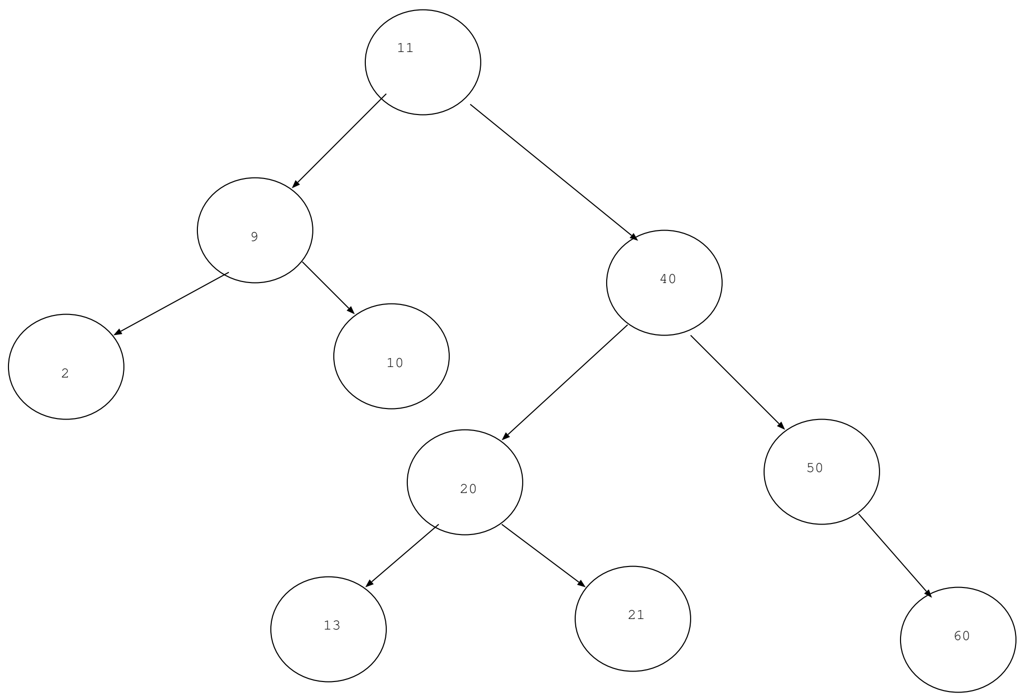
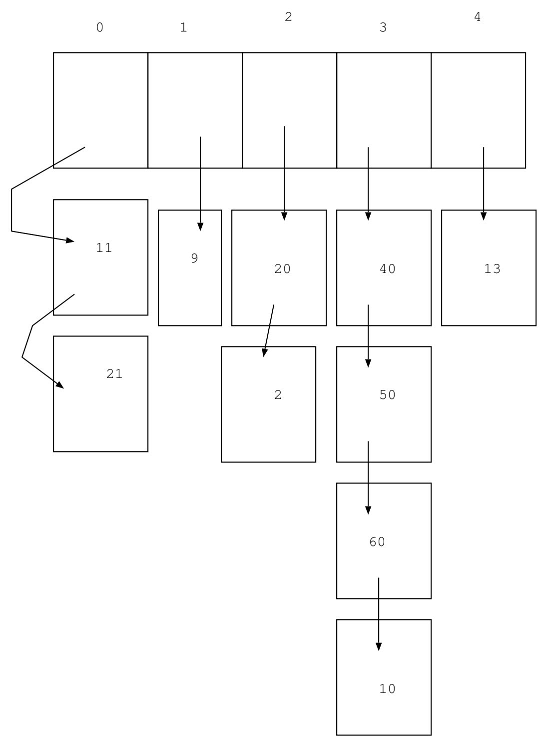
1. (a)



(b). Every node in this tree follows the requirements to be an AVL tree. Every node has a difference of height for it children that is either -1,0,1.

(c). $(2x+3) \bmod 5 \leq 4$ so we can assume the hashset only has a size of 4.



2. Consider that binary tree T is a perfectly balanced tree so each node must have 2 children or 0 children. The tree has $n = 2^{\ell} - 1$ distinct integers so the tree must have n nodes.

Lemma $n = 2^{h+1} - 1$ where h is the height of the binary tree.

Basis: Suppose a tree $T^{'}$ has only a single root node so h = 0. $1 = 2^1 - 1$.

Inductive Hypothesis: Suppose that $n = 2^{h+1} - 1$ is true for tree T_1, T_2.

Recursion:

Using structural induction for T_1, T_2 returns the number of node for each tree $n = 2^{h+1} - 1$ where h is the height for either tree. Both trees need to have the same height or else the new binary tree might not be perfectly balanced . If we combine T_1 and T_2 and for order it to be a perfectly balanced tree we will add a single node N that will be the new root node that is a parent with the roots from T_1 and T_2. The height of the new tree 1 + h the number of nodes. The number of nodes in the new tree will be $2^{h+1} - 1 + 2^{h+1} - 1 + 1$ or it can be reduced to $2^{h+2} - 2 + 1$ ... $n = 2^{h+2} - 1$. Since $1 + h = h^{'}$ the new tree will have $2^{h^{'}+1} - 1$ nodes. So therefore for all perfectly balanced trees there are $2^{h+1} - 1$ nodes for its height h. QED

$n = 2^{\ell} - 1$ is the number of nodes in the tree so therefore ℓ = h + 1 where h is the height of the tree.

The algorithm is

```
// T is a tree
// T.R is the root node
// T.R.L is the left child of the root
// T.R.R is the right child of the root

getElementSmallerThan(T) {
    return T.R.R
}
```

This algorithm is obviously O(1) this algorithm is correct because by the lemma there are $2^{h+1} - 1$ nodes in the tree and we want a value that is smaller than $2^{h-1} - 1$ nodes. If there are a nodes in the tree then we need to find the node smaller than $\frac{a+1}{4} - 1$ nodes. Notice that $\frac{a+1}{4} - 1$ equals the number of nodes in the subtree of the right child of the right child of the root node. This makes sense because it should be about a fourth of the nodes it needs to be smaller than. For a tree of three nodes the right child has no children so $\frac{3+1}{4} - 1 = 0$ so in that case the right child of the root would be the largest value in the tree being smaller than 0 values in the tree. For larger trees almost a quarter of the entire tree is the subtree of the right child of the right child of the root. Meaning that the right child of the root is smaller than $2^{\ell-2} - 1$ or $n = 2^{h-1} - 1$ elements in the tree. QED

(b) We start with a tree that looks like