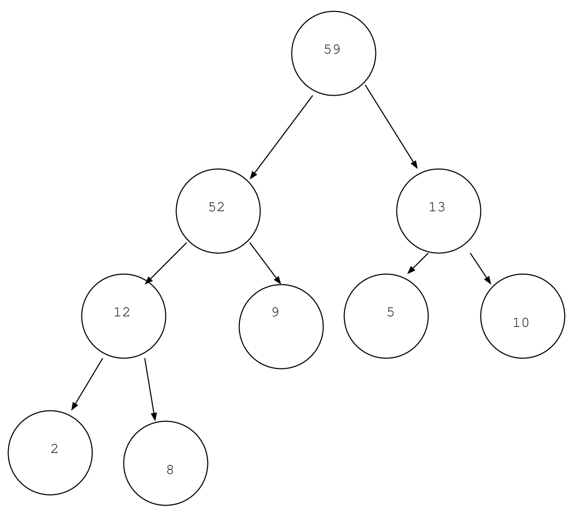
HW3

Shane Drafahl

20 October,2017

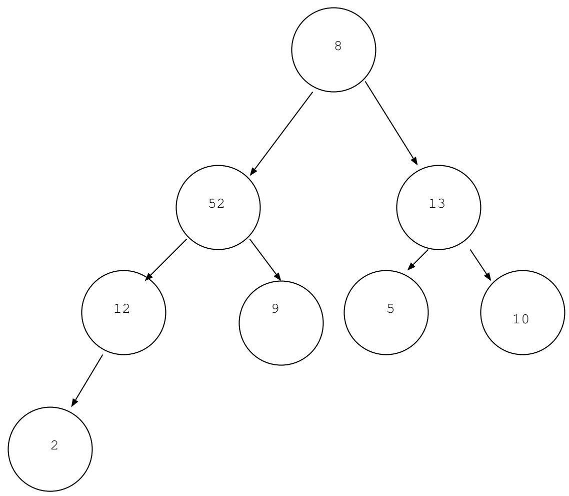
1. (a)

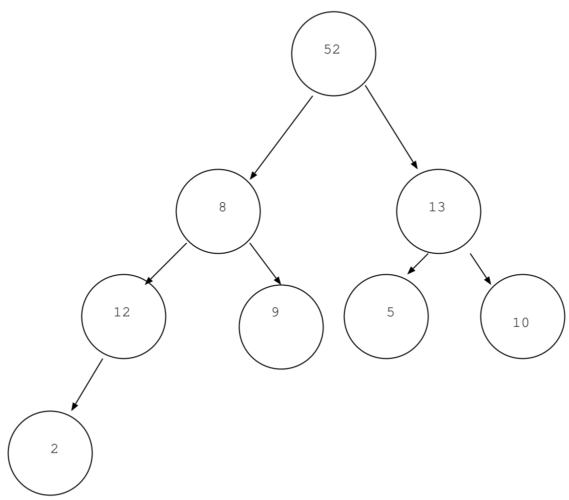


First we will replace the removed element with the last element in the tree

(b):

Then we must heapify the data structure

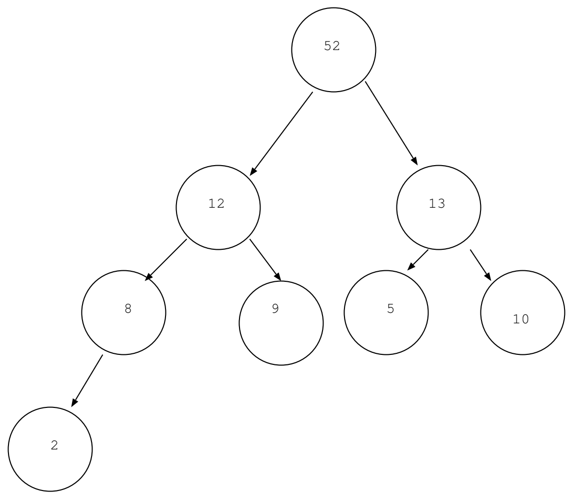




The max heap is now been heapified.

2. Solve the following recurrences. You can not use master theorem to solve them. You must show the steps in your derivation.

(a): If we draw out a 3 iterations then we would get



$[cn]_1 + [\frac{cn}{3} + \frac{2cn}{3}]_2]_2 + [\frac{cn}{9} + \frac{2cn}{9} + \frac{2cn}{9} + \frac{4cn}{9}]_3$
where the brackets represent the layers this can be reduced to $[cn]_1 + [cn]_2 + [cn]_3 + ...$ for all iterations. So now we just need to find the number of iterations. Every iteration divides n by 3 or by $\frac{2}{3}$. So there should be logarithmic number of iterations. So $T(n) = cnlog(n)$.
(b): First we will draw out a few iterations to find what the infinite series would be.
The sum of the the function would be $cn + \frac{cn}{5} + \frac{cn}{25} + ...$
Notice that the denominator 5^i where i is the iteration number. So we can write this as an infinite series.
$\sum_{i=1}^{\infty}(\frac{cn}{5})^i$.
From this we can determine that the total sum is $\frac{5nc}{4}$. This means that this function is O(n).
(c): We will draw out the first few iterations again. $[n^{log_5(7)}]_0 + [\frac{n^{log_5(7)}}{2} + \frac{n^{log_5(7)}}{2}]_1 + [\frac{n^{log_5(7)}}{4} + \frac{n^{log_5(7)}}{2}]_4 + \frac{n^{log_5(7)}}{2}]_4 + \frac{n^{log_5(7)}}{2}]_4]_2 + ...$
As we can tell this can ben reduced to $n^{log_5(7)} + n^{log_5(7)} + n^{log_5(7)} + ...$ so that means we just have to find the height of the tree.
Notice the denominator's value is 2^i where i is value of the iteration. So we know the last iteration is when $\frac{n}{2^i} = 1$ so there will be $log_2(n)$ iterations. So the sum is $log_2(n) * n^{log_5(7)}$