HW6

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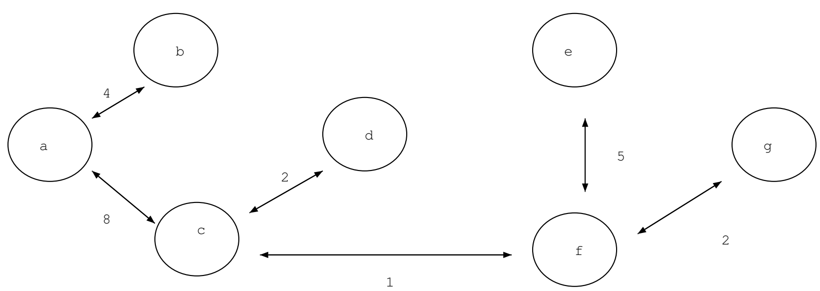
5 December,2017

1.

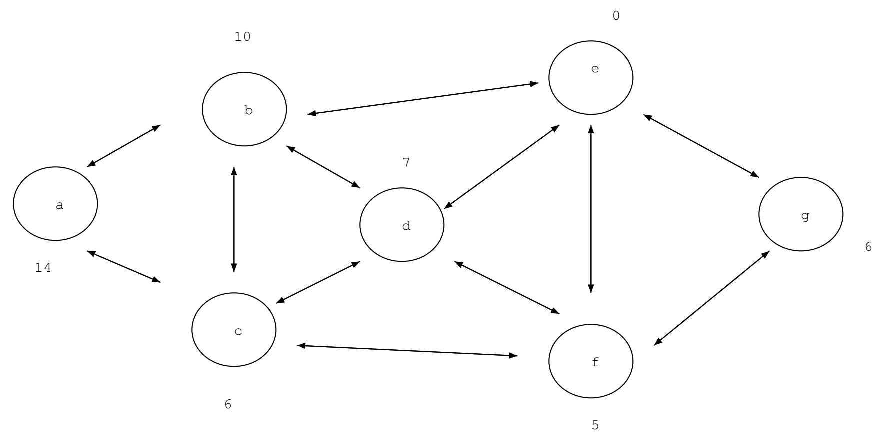
a.



b. Each vertice has the shortest distance from e listed next to each vertice.

2. This algorithm produces a minimum spanning tree. I will prove this by first writing a proof to show it will produce a minum spanning tree.

This algorithm starts from the edge with the greatest weight and removes the edge if it does not disconect the edge. If we assume by proof of contradiction that this algorithm does not produce a tree so it must produce



cycles in a graph. That means it it cannot remove an edge because it would disconnect the graph. This is a contradition because a cycle has more than one path so it should be able to remove the edge. Therefore this algorithm will always produce a tree.

Using induction we can prove that the algorithm will produce the minium spanning tree.
Basis: Suppose that there is a set F of edges currently in the graph. The minimum spanning tree must be a subset of F in the beginning because F contains all edges at this point.
Inductive Hyptothesis: The minimum spanning tree T is a subset of F.
Induction: We want to show that any edge the algorithm removes by the end of an iteration T, the minimum spanding tree, is a subset of F. When the algorithm removes and edge c we will consider the cases.
case 1: $c \notin T$, for this case T is still a subset of F.
case 2: $c \in T$, for this case removing c would disconnect T because T is a tree. The graph must be connected after removing c so there must exist a cycle before removing c from F. We will call this other cycle not in T f. The algorithm removes the greatest to the smallest weights so we can say that the minimum spaning tree is $T^{''} = T - e + f$ which is a supbset of F. $T^{'}$ does is a spanning tree because e is removed so the cycle with e and f does not exist. The weight of e equals f because if e has a greater weight this would be a contradition because T is a miniumum spanding tree. If the opposite were true and f was greater than e the algorithm goes throug decending algorithm so its impossible for the algorithm to reach two edges that have a

cycle with each other and process the one with the minium weight. So therefore the weights must equal and thus $weight(T^{'}) = weight(T)$. Therefore by each iteration the minimum spanding tree is a subset of the remaining edges.

We have proved that each iteration of the algorithm the minimum spanding tree is a subset of the the set of edges after each iteration. The minium spanding tree is a subset, so when the algorithm visits all the edges and the loop ends the final set of edges should still have the minium spanding tree as a subset. Therefore the final set of edges is the minimum spanding tree. Therefore the algorithm produces the minimum spanding tree.

3. In a directed graph, if a vertex can reach all other vertices, then we will call it a dominating vertex. Design an algorithm to find the dominating vertices in a graph. Prove its correctness and explain the time complexity of your algorithm.

My approach to this problem will be to first convert the graph problem into a spanning tree via prims algorithm. This will prevent cycles in the graph. Once this is done I can search via DFS from every node in the graph to find how many nodes it can find a path to. It will store the max depth it finds in each direction for efficiency.

```
G {
     nodes[] // array of Nodes
}

N {
     neighbors[] // adjacent nodes
     depth := 1
     searched := false
}

algorithm(Graph g) {
     g = primsAlgorithm(g) // creates instance of minimum spanning tree and se
```

```
        foreach (Node n in g.nodes) { // iterates over all nodes in the graph
            if(!n.visited) {
                depth := findDepth(n)
                if(depth == length(g.nodes)) {
                    return g
                }
            }
        }
    }

    findDepth(Node n) {
        if(n.visited) {
            return n.distance
        }
        n.visited = true
        foreach (Node n1 in n.neighbors) {
            n.depth += findDepth(n1)
        }
        return n.depth
    }
```

First I will prove findDepth. findDepth is a DFS based algorithm. For each node it is called uppon changes its value to visited so it cannot visit nodes it has visited before. It recursivly finds the depth of each adjacent node and sums them together to find the depth. If it has no adjacent nodes then it should return one because that is the default. So when the algorithm is called on a node that has adjacent nodes that are all leaves they would return 1 each meaning its depth would be the number of leaf nodes plus one. The plus one being itself. The node that called this node then would add that value to its own and its neighbors as well. This algorithm cannot get stuck in a cycle either where its depth would be in a cycle dependency because prims algorithm is used on the graph resulting in a spanning tree that would eliminate any cycles in the graph. findDepth is used in the algorithm. The aglorithm visits each node that has not been visited and uses findDepth or in other words uses DFS on every node to find its depth. If the depth of a single nodes equals the number of nodes or the length of the array of nodes in the graph that would mean there is a path between that node and every other node in the graph.

This algorithm first uses prims algorithm which is $O(V + E)$. After this this is uses a DFS based algorithm on all Nodes that are not visited. I would argue this algorithm runs in $O(V + E)$. Each vertice is only visited once in the algorithm. Every Node that is visited by findDepth is marked with a visited boolean so it does not need to be processed again. So therefore the algorithm grow linearly with its input size.

4. To solve this problem I will create a 2D array that will hold such for that arr[x][y] x is the sum for the corresponding subset $A_1, ...A_j$ such that arr[x][y] will hold a true or false if there is a possibility for the specified sum. I can then create the 2D array and build it from the ground up.

We will say hasSubSet(A, n) is a recurance that determins if there is a subset with half the sum of A as its sum . The n is the range of the subset, for example if n = 1 then its asking if there if the first element in A equals half the sum of A. There are two cases or two sub problems. The first is if the subset hasSubSet(A, n-1) is true then hasSubSet(A, n) would also be true. The other case is when in consideration of the n'th element of the subset. For this case we need to check that if we take val = sum(A)/2 - A[n] we need to determine if val can be summed up. Also if hasSubSet(A, n-1) is true like in the first case then we can determine that hasSubset(A, n) is true when considering the last element. If either case is true then hasSubSet(A, n) is true. The runtime of such recurance is $O(2^n)$ worst case.

```
function algorithm(A[]) { // given an array of numbers
     length := length(A) // number of elements
     sum:= 0

     foreach(int x in A) {
         sum += x // calculating the sum
     }
```

```
        // Since A is assumed to only have natural numbers
        // there cannot be a set of half the value if it is odd
        if(sum % 2 == 0) {

            sums[n+1][sum/2+1] // boolean array for possible sums

            // obviously the null set is a subset of all sets so there would alwa
            for(x:=0;x<n+1;x++) {
                sums[x][0] = true
            }

            // Process the top row except for (0,0)
            sums[0][A[0]] = true

            for(x:=1;x<n+1;x++) {
                for(y:=1;y<m/2+1;y++) {
                    val:= A[x] // corresponding value
                    if(y < val) {
                        sums[x][y] = sums[x-1][y]
                    } else {
                        sums[x][y] = sums[x-1][y-val] || sums[x-1][y]
                    }
                }
            }

            return sums[n][sum/2]

        } else {
            return false
        }
    }
```

This algorithm at first requires linear computations such as the sum of numbers in A. Each index in the 2D boolean array is a sub problem. It takes constant time to process each sub problem. The 2D array is the size $(sum(A)/2) * length(A))$ or if we say there are n elements and $half :=$ $sum(A)/2$ then it runs in $O(n * half)$.