HW2

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1. Prove or disprove the following

(a) $5n^2 - 2n + 26 \in O(n^2)$

We will prove this with the def of big oh. The def of $O(n^2)$ is there exists positive constants c and n_0 such that 0 ≤ f(n) ≤ c * g(n) for all $n_0 \leq n$. In this case f(n) = $5n^2 - 2n + 26$ and g(n) = n^2. We can divide both sides by n^2 and we can go from $0 \leq 5n^2 - 2n + 26 \leq c * n^2$ to $0 \leq \frac{24}{n} + 5 \leq c$. $\frac{24}{n=1} + 5 = 29$ and if $f_1(n) = \frac{24}{n} + 5$ then $f_1(n+1) \leq f_1(n)$ because as natural number n increases it increases the denominator. C could be 29 or greater and $0 \leq 5n^2 - 2n + 26 \leq c * n^2$ would be true so therefore $5n^2 - 2n + 26 \in O(n^2)$ because the property is true.

(b) $\forall_a \geq 1 : a^n \in O(n!)$

We will prove this with def of big oh. So the given statement is equivalent to $0 \leq a^n \leq c * n!$. Using induction we can prove it.

Basis: Starting at n = 1 because the performance of an algorithm with n = 0 is irrelevent. 0 $leqa \leq c$ is true because for any a c can be a constant of c = (a + 1). Inductive Hypothesis: Suppoose $0 \leq a^n \leq c * n!$ is true.

Inductive Step: We need to prove $0 \leq a^{n+1} \leq c * n! * (n+1)$. a^{n+1} increases by some a multiplied by $a * a^n$ from the IH. While the right side $c * n! * (n+1)$ from the IH is multiplied by (n + 1) for $c * n! * (n + 1)$. In this case of a, n increases meaning at some point it will increase by more when a < n. So the right side is increasing at a faster rate then the left side of the comparison. This means that there is a point where $n! \leq a^n$ for some a for a given range of n. We can just say $c = a^n = n!$ for the n where they equal. So for $0 \leq a^{n+1} \leq c * n! * (n + 1)$ if $a^n > n!$ then the constant c multiplied by $n! * c$ will be greater than or equal to a^n because c is equal to the the value at which n! overtakes a^n so if for n! n is beyond the point where n! overtakes than it will already overtake and be a greater value. The other case is that $a^n \leq n!$ for some n then it wont matter what c is because n! will be increasing at a greater rate. So $0 \leq a^{n+1} \leq c * n! * (n+1)$. a^{n+1} is true and therefore $\forall_a \geq 1 : a^n \in O(n!)$ is true.