HW1

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1. Use mathematical induction to prove the following statements

(a)
$\forall_n \geq 1$, $\sum_{i=1}^{n} i^3 = (\sum_{i=1}^{n} i)^2$

lemma: $\sum_{i=1}^{n} i = \frac{n^2}{2} + \frac{n}{2}$ where $n \in N$
Base Case: $\sum_{i=1}^{1} i = 1 = \frac{1^2}{2} + \frac{1}{2}$
Inductive Hypothesis: $\sum_{i=1}^{n} i = \frac{n^2}{2} + \frac{n}{2}$
Inductive Steps:
Using induction on n
$\frac{(n+1)^2}{2} + \frac{n+1}{2} = \frac{n^2}{2} + \frac{3n}{2} + 1 = \frac{n^2}{2} + \frac{n}{2} + (n+1)$

So therefore the lemma is true for all natural numbers n greater than 1.

With the lemma we can solve the proof easier

Base Case: For n = 1 would be $\sum_{i=1}^{1} i^3 = 1^3 = 1$. $(\sum_{i=1}^{1} i)^2 = 1$.

Inductive Hypothesis: From the base n is true for $\forall_n \geq 1$, $\sum_{i=1}^{n} i^3 = (\sum_{i=1}^{n} i)^2$

Inductive Step: Using lemma we can re-write this to $(\frac{n^2}{2} + \frac{n}{2})^2 = \sum_{i=1}^{n} i^3$ We will first add n+1, $(\frac{(n+1)^2}{2} + \frac{(n+1)}{2})^2 = \frac{n^4}{4} + \frac{3n^3}{2} + \frac{13n^2}{4} + 3\text{n} + 1$. Now we will do $(\frac{n^2}{2} + \frac{n}{2})^2 + (n+1)^3 = \frac{n^4}{4} + \frac{3n^3}{2} + \frac{13n^2}{4} + 3\text{n} + 1$. So therfore $\forall_n \geq 1$, $\sum_{i=1}^{n} i^3 = (\sum_{i=1}^{n} i)^2$ is true for all natural numbers for n greater than or equal to 1.

(b) $\forall_n \geq 4$, $2^n < n!$