HW3

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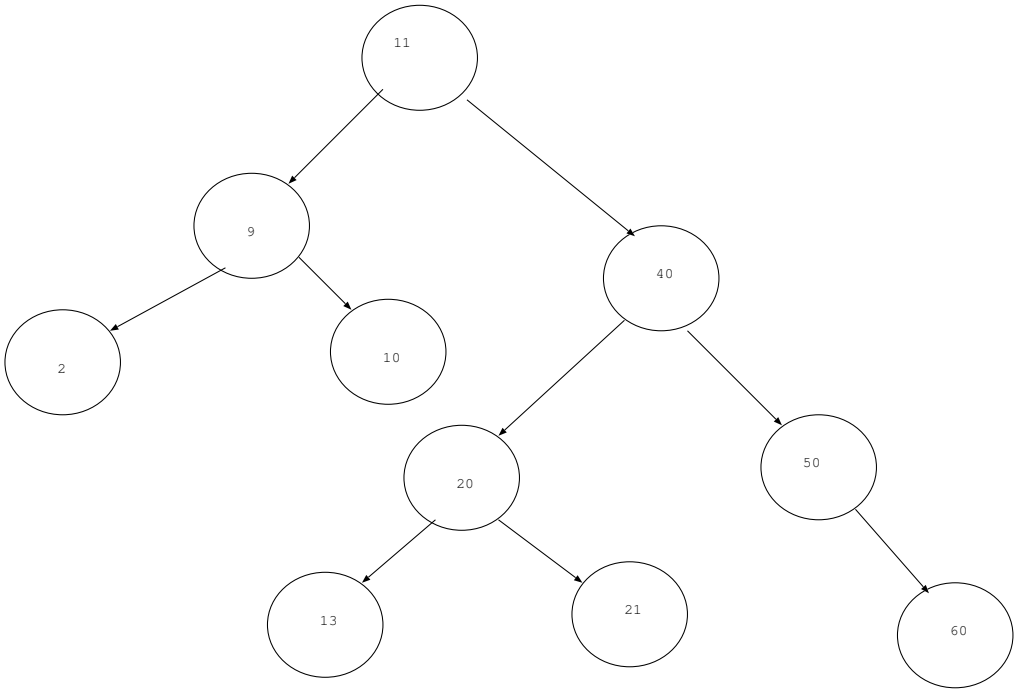
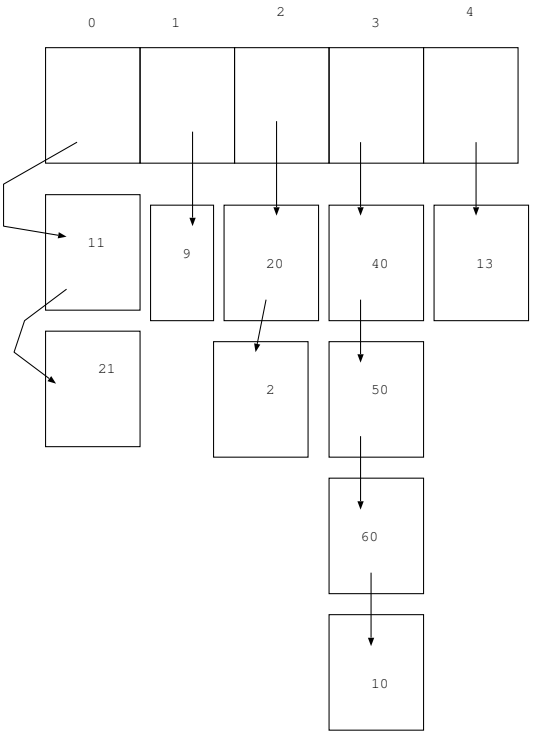
1. (a)



(b). Every node in this tree follows the requirements to be an AVL tree. Every node has a difference of height for it children that is either -1,0,1.

(c). $(2x + 3) \bmod 5 \leq 4$ so we can assume the hashset only has a size of 4.



2. Consider that binary tree T is a perfectly balanced tree so each node must have 2 children or 0 children. The tree has $n = 2^{\ell} - 1$ distinct integers so the tree must have n nodes.

Lemma $n = 2^{h+1} - 1$ where h is the height of the binary tree.

Basis: Suppose a tree T' has only a single root node so h = 0. $1 = 2^1 - 1$.

Inductive Hypothesis: Suppose that $n = 2^{h+1} - 1$ is true for tree T_1, T_2.

Recursion:

Using structural induction for T_1, T_2 returns the number of node for each tree $n = 2^{h+1} - 1$ where h is the height for either tree. Both trees need to have the same height or else the new binary tree might not be perfectly balanced . If we combine T_1 and T_2 and for order it to be a perfectly balanced tree we will add a single node N that will be the new root node that is a parent with the roots from T_1 and T_2. The height of the new tree 1 + h the number of nodes. The number of nodes in the new tree will be $2^{h+1} - 1 + 2^{h+1} - 1 + 1$ or it can be reduced to $2^{h+2} - 2 + 1$ ... $n = 2^{h+2} - 1$. Since $1 + h = h^{'}$ the new tree will have $2^{h^{'}+1} - 1$ nodes. So therefore for all perfectly balanced trees there are $2^{h+1} - 1$ nodes for its height h. QED

$n = 2^{\ell} - 1$ is the number of nodes in the tree so therefore ℓ = h + 1 where h is the height of the tree.

The algorithm is

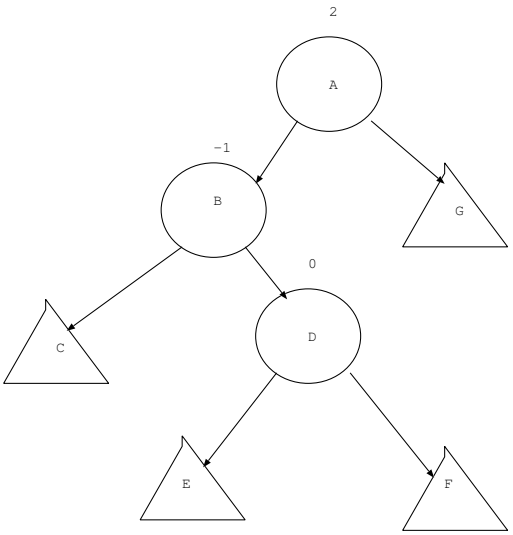
```
// T is a tree
// T.R is the root node
// T.R.L is the left child of the root
// T.R.R is the right child of the root

getElementSmallerThan(T) {
    return T.R.R
}
```

This algorithm is obviously O(1) this algorithm is correct because by the lemma there are $2^{h+1} - 1$ nodes in the tree and we want a value that is smaller than $2^{h-1} - 1$ nodes. If there are a nodes in the tree then we need

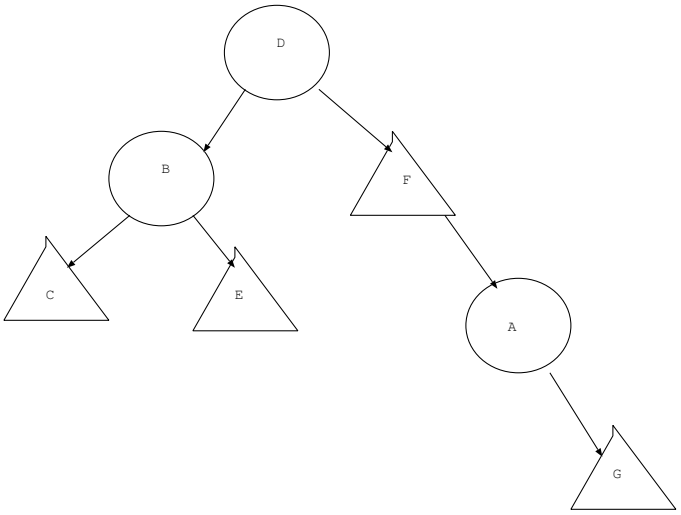
to find the node smaller than $\frac{a+1}{4} - 1$ nodes. Notice that $\frac{a+1}{4} - 1$ equals the number of nodes in the subtree of the right child of the right child of the root node. This makes sense because it should be about a fourth of the nodes it needs to be smaller than. For a tree of three nodes the right child has no children so $\frac{3+1}{4} - 1 = 0$ so in that case the right child of the root would be the largest value in the tree being smaller than 0 values in the tree. For larger trees almost a quarter of the entire tree is the subtree of the right child of the right child of the root. Meaning that the right child of the root is smaller than $2^{\ell-2} - 1$ or $n = 2^{h-1} - 1$ elements in the tree. QED

(b) We start with a tree that looks like

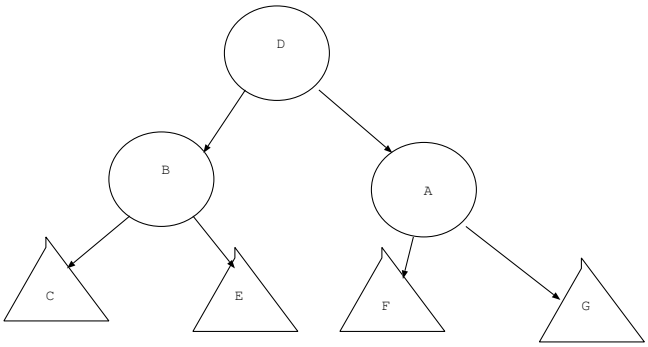


Suppose that tree C has a height of $h^{'}$, E and F has a height of h and G has a height of $h^{''}$. B has a balance factor of -1 so $(h^{'} + 1) - (h + 2) = -1$ so $h^{'} = h$. A has a balance factor of 2 so that means that $(h+3) - (h^{''} + 1) = 3$ so $h = h^{''}$.

So because the D is less than a and greater than B we will rotate the D.



This tree is still not balanced. So we will rotate a subtree to get.



The balance factor for D,B,A are 0 each. It is a perfectly balanced tree.

3. (a) In order to do this can just simply sort each element with merge sort and then insert them into the tree. This would mean the root would be the smallest value and the leaf node would be the largest value. Every number would be greater than the next so the node that holds a_n a_{n+1} would always be the right child for a natural number n that is less than the length of numbers in the list. The run time of the algorithm would be be

the for loop plus the merge sort and we know that merge sort has a time of $O(nlog(n))$ so the runtime is $nlog(n) + n = O(nlog(n))$.

```
function createSpecialTree(List list) {
    list = mergeSort(list)
    Tree t
    for(i = 0; i<list.length;i++) {
        element = list.get(i)
        t.add(element)
    }
}


function mergeSort(List m) {
    if (length of m  1 then) {
        return m
    }

    right = []
    left = []

    for(i = 0; i < m.length; i++) {
        x = m.get(i)
        if i >= (m.length)/2 then
            right.add(x)
        else
            left.add(x)
    }
    right = merge_sort(right)
    left = merge_sort(left)
    return merge(left, right)

}

function merge(left, right) {
    mergedList = []

    while(!left.isEmpty() && !right.isEmpty()) {
        if (first(left)  first(right)) {
            append first(left) to mergedList
```

```
            left = rest(left)
        } else {
            mergedList.append(first(right))
            right = rest(right)
        }
    }

    while(!left.isEmpty()) {
        mergedList.append(first(left))
        left = rest(left)
    }
    while(!right.isEmpty()) {
        mergedList.append(first(right))
        right = rest(right)
    }

    return mergedList
    }
}
```