HW4

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1. Let $|x|_a$ be the number of occurrences of the symbol a in the string x.

Define a context-free grammar for the language $L = \{\ w \in \{\ 1, 0\}\ ^* : |w|_0 = |w|_1\}$

G $= \{\ (S), (1, 0), S, P\}$
$P = \{\ \ S \rightarrow 01|10|1S0|0S1|SS|\epsilon\ \}$

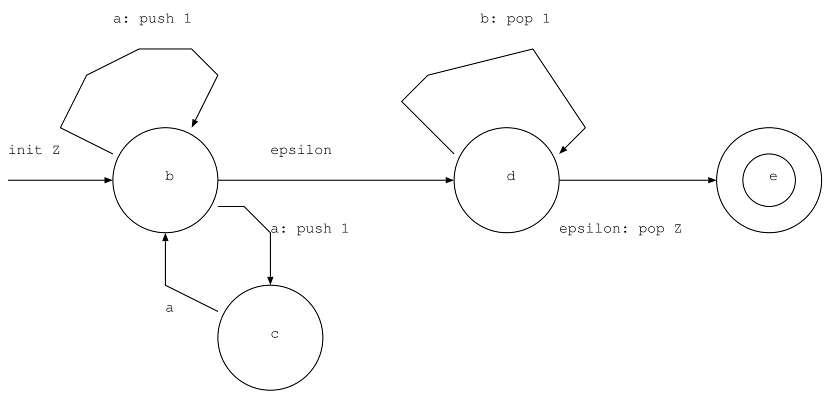
Using a inductive proof I will give a formal proof that the grammar genderate L

Basis: $01 \in \{\ 1, 0\}\ ^*, 10 \in \{\ 1, 0\}\ ^*$ and $\epsilon \in \{\ 1, 0\}\ ^*$ and for all cases there are either one 1 and one 0 or zero 0 and zero 1 so they are in the language L and can be generated by G.

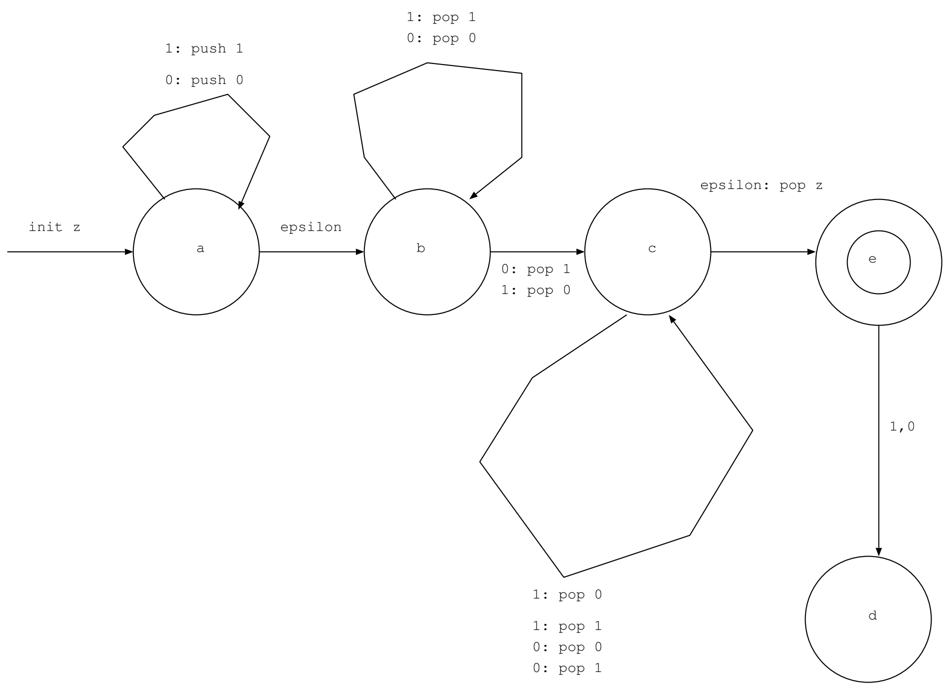
IH: Suppose that A,B can be generated from G and that $A, B \in L$

Structural: A,B can be created from transitions from S so me must prove fro 1S0, 0S1, and SS. For SS so for AB or BA if we concatenated the strings A and B the 1's and 0's would still be equal because they are equal for A and B. For the transition 0S1 and 1S0 or 0A1 and 1A0. Its assumed by the IH that S is in L and be generated from G so A is in L and is generated from G. If we add a 1 and a 0 the number of 1's and 0's still equal because they do for A and adding a 1 and a 0 is still equal.

2. Define a NPDA for the language $L = \{\ a^n b^m : m, n \in N, m \leq n \leq 2m\}$



3.


1: push 1
0: push 0
1: pop 1
0: pop 0
init z
a
epsilon
b
c
epsilon: pop z
e
0: pop 1
1: pop 0
1,0
1: pop 0
1: pop 1
0: pop 0
0: pop 1
d