

HW4

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1. Let $|x|_a$ be the number of occurrences of the symbol a in the string x .

Define a context-free grammar for the language $L = \{ w \in \{ 1, 0 \}^* : |w|_0 = |w|_1 \}$

$$G = \{ (S), (1, 0), S, P \}$$
$$P = \{ S \rightarrow 01|10|1S0|0S1|SSS|\epsilon \}$$

Using a inductive proof I will give a formal proof that the grammar generate L

Basis: $01 \in \{ 1, 0 \}^*$, $10 \in \{ 1, 0 \}^*$ and $\epsilon \in \{ 1, 0 \}^*$ and for all cases there are either one 1 and one 0 or zero 0 and zero 1.

IH: Suppose that w can be generated from G and that $w \in L$

Structural: Suppose that there are strings A, B, C and $A, B, C \in L$ by the IH and they are all generated from G . Suppose that they are concatenated ABC or CBA . All strings have the same number of 1's and 0's so therefore the resulting string ABC or CBA would also have the same number of 1's and 0's so $ABC, CBA \in L$. One of the transitions in the grammar is $S \rightarrow SSS$ so $S \rightarrow ABC$ or $S \rightarrow CBA$ since we know from the IH that $A, B, C \in L$ so therefore $S \rightarrow A, B, C$. Therefore ABC and CBA can be created by the grammar G and they are in the language L .

2. Define a NPDA for the language $L = \{ a^n b^m : m, n \in N, m \leq n \leq 2m \}$

3.

