

HW0

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This is an inline equation $x + y = 3$

This is a displayed equation:

$$x + \frac{y}{z - \sqrt{3}} = 2$$

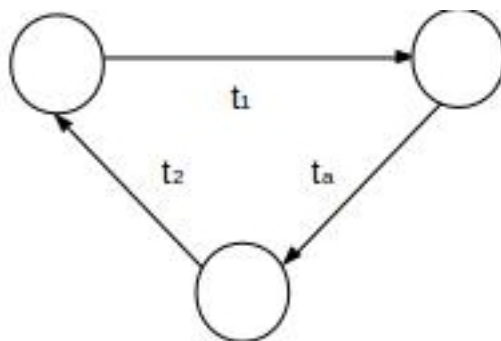
This is how you define a piece-wise linear function:

$$f(x) = \begin{cases} 3x + 2 & \text{if } x < 0 \\ 7x + 2 & \text{if } x \geq 0 \text{ and } x < 10 \\ 5x + 22 & \text{otherwise.} \end{cases}$$

This is a matrix:

9	9	9	9
6	6	6	
3		3	3

This is a figure incorporated in a LaTeX file



2. Show that \mathbb{N} (natural numbers) and \mathbb{Z} (integer numbers) are equinumerous.

To show that the set of natural numbers and integer numbers are equinumerous we have to show the cardinality of \mathbb{N} and \mathbb{Z} are the same. If there is a function $f: \mathbb{N} \rightarrow \mathbb{Z}$ that is bijective then the cardinalities of sets \mathbb{N} and \mathbb{Z} would be the same.

Suppose we have a function where $x \in \mathbb{N}$ and $f(x) \in \mathbb{Z}$

$$f(x) = \begin{cases} 0 & \text{if } x \text{ equals } 0 \\ \frac{x}{2} * -1 & \text{if } x \bmod 2 = 0 \\ \frac{(x-1)}{2} + 1 & \text{otherwise.} \end{cases}$$

If f is onto then $\forall y \in \mathbb{Z} \exists x \in \mathbb{N}$ where $f(x) = y$. Using existential instantiation and universal instantiation either $\frac{x}{2} * -1 = y$ or $\frac{(x-1)}{2} + 1 = y$. If $\frac{x}{2} * -1 = y$. For $\frac{x}{2} * -1 = y$ is equal to $x = -2y$ and $\frac{(x-1)}{2} + 1 = y$ equals $2y - 1 = x$. If y is negative then for $-2y = x$, x will equal a natural number. If y is positive then $2y - 1 = x$, x will equal a natural number because it cannot be negative and y can't be 0 because x will be 0 as well. So therefore for all values of y a natural number of x can be reached so therefore function f is onto.

Using proof by contradiction assume f is not one to one so $\exists x \exists y (f(x) = f(y) \rightarrow x \neq y \wedge x, y \in \mathbb{N} \wedge x, y)$. If x is odd and y is even or vice versa then $\frac{y}{2} * -1 = \frac{(x-1)}{2} + 1$. This can be reduced to $y = -x - 1$. This is a contradiction because since x and y are natural numbers they cannot be negative so $0 > -x - 1$ and $y > 0$ so therefore they cannot be equal. If x and y are both even then $\frac{y}{2} = \frac{x}{2}$ which reduces to $x = y$ which is a contradiction because x and y cannot equal each other. If x and y are both odd then $\frac{(y-1)}{2} + 1 = \frac{(x-1)}{2} + 1$ which reduces down to $x = y$ and for the same reason is a contradiction because x and y equal each other.

therefore the function f is one to one and onto so it is bijunction and therefore natural numbers have the same cardinality as integers and are equinumerous.

3. Let $f: S \rightarrow S$ be a total function. Prove that, if S is infinite, f can be

one-to-one without being onto, and onto without being one-to-one.

For f to be one-to-one but not onto for $f(x) = y$ where $x \in S$ and $y \in S$ there is at least one value of y and x $f(x) \neq y$.

$$f(x) = \begin{cases} x + 1 & \text{if } x \geq 0 \\ x - 1 & \text{otherwise.} \end{cases}$$

Function f cannot be 0. If x is greater or equal to 0 then $0 = x + 1$. The only value for x where this could be true is $x = -1$ but x cannot be a negative number. Otherwise for $0 = x - 1$ x would need to be equal to 1 but x cannot equal or greater than 0 so therefore for either case 0 is an element of S but $f(x) \neq 0$.

To prove that f is one to one we will use proof by contradiction and assume that function f is not one to one so there is some value $\exists x \exists y \in S$ where $x \neq y \wedge f(x) = f(y)$ so if x is equal to 0 or greater and y is less than 0 or vice versa so $x + 1 = y - 1$ which can be reduced to $x + 2 = y$. This is a contradiction because y in this case is less than 0 and x can either be 0 or greater so adding 2 to 0 or greater cannot result in a negative number. If x and y are greater or equal to 0 then $x + 1 = y + 1$ or if they are both less than 0 so $x - 1 = y - 1$ but these both can be reduced to $x = y$ and this is a contradiction because $x \neq y$ so therefore the function f is one to one.

A function f can be onto and not one-to-one

$$x \in S$$

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ x + 1 & \text{if } x < 0 \\ x - 1 & \text{if } x > 0 \end{cases}$$

f is not one-to-one because $f(0) = f(-1) = f(1) = 0$. Function f is onto. If x is greater than 0 then $y \in S$ so $y = x - 1$. This can be reduced to $1 + y$

$= x$. The value y can be any value in S and correspond to a value x which is also in S . Otherwise if x is 0 y is 0 and if x is less than 0 then $y = x + 1$. This can be reduced to $y - 1 = x$. For any value $y \in S$ $x \in S$ every value of y corresponds to a value x . Therefore the function f is onto.

4. Show that the relation R defined by

$$\forall_{m,n} \in N, (m, n) \in R \Leftrightarrow (m - n) \bmod 3 = 0$$

is an equivalence relation, and describe its equivalence classes.

The relation R is reflexive because for any real number x , $(x - x) \bmod 3 = 0$ because $x - x$ will always be 0.

The relation R is symmetric because for any real numbers x and y this say $x - y = z$ so $y - x = -z$. So by swapping the order of x and y the relation is $z \bmod 3 = 0$ or $-z \bmod 3 = 0$. If z is negative or not makes no difference for mod so therefore R is symmetric.

The relation R is transitive. Suppose x, y, z are all real numbers. Suppose R_y