HW4

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1. Use the pumping lemma to prove that L = $\{ ww : w \in \{ 0, 1\}^* \}$ is nonregular.

We will prove this by proof of contradition and assume that L is a regular language. So therefore for L, $\exists_n \in N$ such that $\forall_x \in L$, if $|v| \geq n$, then x can be decomposed into three strings $\exists_{u,v,w} \in \{ 0, 1\}^*$. Where x = uvw. $|uv| \leq n, |v| > 0$ such that $\forall_k \in N, uv^k w \in L$.

Because there are two w's we know that $|ww| = 2a$ where a are some natural number. Suppose that uvw = $a^p b^l a^p b^l$ where $p + l = n$, p and l are natural numbers. This implies that $v = b^i$ where some i is a natural number greater than 0. Suppose that $uv^2w = a^p b^{l+i} a^p b^l$. $|uv^2w| = p+l+i+p+l = 2(n)+i$. This is a contradiction because the cardinality is no longer a multiple of 2 and should be in the language but it is not so therefore L is not regular.

2. Prove that $L = \{ w \in \{ a, b\}^* : |w|_a \neq |w|_b \}$ is not regular.

Using the closure properties we can take the complement of language L and take its complement $L^{'}(M_1) = \Sigma \ L_M$. $L^{'}(M_1) = \{ w : \{ a, b\}^* |w|_a = |w|_b \}$. We can then concat $L^{''}(M_3) = L^{'}(M_1) * L^{'}(m_2)$ where $L^{''}(M_3) = \{ ww : \{ a, b\}^* |w|_a = |w|_b \}$. That last part we can use homomorphism where $a \rightarrow 0$ and $b \rightarrow 1$. To get $L^{'''}(M_4) = \{ ww : w \in \{ 0, 1\}^* \}$ we proved in question 1 that $\{ ww : w \in \{ 0, 1\}^* \}$ is not regular and because we produced a non-regular language from the closure of two regular languages that is a contradiction so therefore L is not a regular language.