# HW0

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23 August,2017

This is an inline equation $x + y = 3$
This is a displayed equation:

$$x + \frac{y}{z - \sqrt{3}} = 2$$

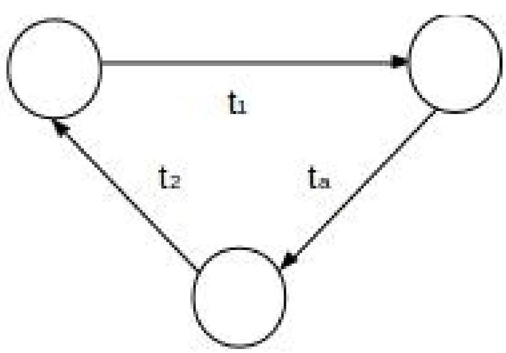
This is how you define a piece-wise linear function:

$$f(x) = \begin{cases} 3x + 2 & \text{if x} < 0 \\ 7x + 2 & \text{if x} \geq 0 \text{ and x} < 10 \\ 5x + 22 & \text{otherwise.} \end{cases}$$

This is a matrix:

| | | | |
|---|---|---|---|
| 9 | 9 | 9 | 9 |
| 6 | 6 | 6 | |
| 3 | | 3 | 3 |

This is a figure incorporated in a LaTeX file



2. Show that N (natural numbers) and Z (integer numbers) are equinumerous.
To show that the set of natural numbers and integer numbers are equinumerous we have to show the cardinality of N and Z are the same. If there is a function f: N $\rightarrow$ Z that is bijective then the cardinalities of sets N and Z would be the same.

Suppose we have a function where $x \in N$ and $f(x) \in Z$

$$f(x) = \begin{cases} 0 & \text{if x equals 0} \\ \frac{x}{2} * -1 & \text{if x mod 2 = 0} \\ \frac{(x-1)}{2} + 1 & \text{otherwise.} \end{cases}$$

If f is onto then $\forall_y \in$ Z $\exists_x \in N$ where $f(x) = y$. Using existential instantiation and universal instantiation either $\frac{x}{2} * -1 = y$ or $\frac{(x-1)}{2} + 1 = y$. If $\frac{x}{2} * -1 = y$. For $\frac{x}{2} * -1 = y$ is equal to x = -2y and $\frac{(x-1)}{2} + 1 = y$ equals 2y - 1 = x. If y is negative then for -2y = x, x will equal a natural number. If y is positive then 2y - 1 = x, x will equal a natural number because it cannot be negative and y cant be 0 because x will be 0 as well. So therefore for all values of y a natural number of x can be reached so therefor function f is onto.

Using proof by contradiction assume f is not one to one so $\exists_x \exists_y (f(x) = f(y) \rightarrow x \neq y \wedge x, y \in N \wedge x, y)$. If x is odd and y is even or vice versa then $\frac{y}{2} * -1 = \frac{(x-1)}{2} + 1$. This can be reduced to y = -x - 1. This is a contradiction because since x and y are natural numbers they cannot be negative so $0 > -x - 1$ and $y > 0$ so therefore they cannot be equal. If x and y are both even then $\frac{y}{-2} = \frac{x}{-2}$ which reduces to x = y which is a contradiction because x and y cannot equal each other. If x and y are both odd then $\frac{(y-1)}{2} + 1 = \frac{(x-1)}{2} + 1$ which reduces down to x = y and for the same reason is a contradiction because x and y equal each other.

therefore the function f is one to one and onto so it is bijunction and therefore natural numbers have the same cardinality as integers and are are equinumerous.

3. Let f : S $\rightarrow$ S be a total function. Prove that, if S is infinite, f can be

one-to-one without being onto, and onto without being one-to-one.

For f to be one-to-one but not onto for f(x) = y where $x \in S$ and $y \in S$ there is at least one value of y and x $f(x) \neq y$.

$$f(x) = \begin{cases} x+1 & \text{if x} \geq 0 \\ x-1 & \text{otherwise.} \end{cases}$$

Function f cannot be 0. If x is greater or equal to 0 then 0 = x + 1. The only value for x where this could be true is x = -1 but x cannot be a negative number. Otherwise for 0 = x - 1 x would need to be equal to 1 but x cannot equal or greater than 0 so therefore for either case 0 is an element of S but $f(x) \neq 0$.