HW4

Shane Drafahl

18 September ,2017

1. Let $|x|_a$ be the number of occurrences of the symbol a in the string x.

Define a context-free grammar for the language $L = \{\ w \in \{\ 1, 0\}\ ^* : |w|_0 = |w|_1\}$

G = $\{\ (S), (1, 0), S, P\}$
$P = \{\ \ S \rightarrow 01|10|1S0|0S1|SS|\epsilon\ \}$

Using a inductive proof I will give a formal proof that the grammar genderate L

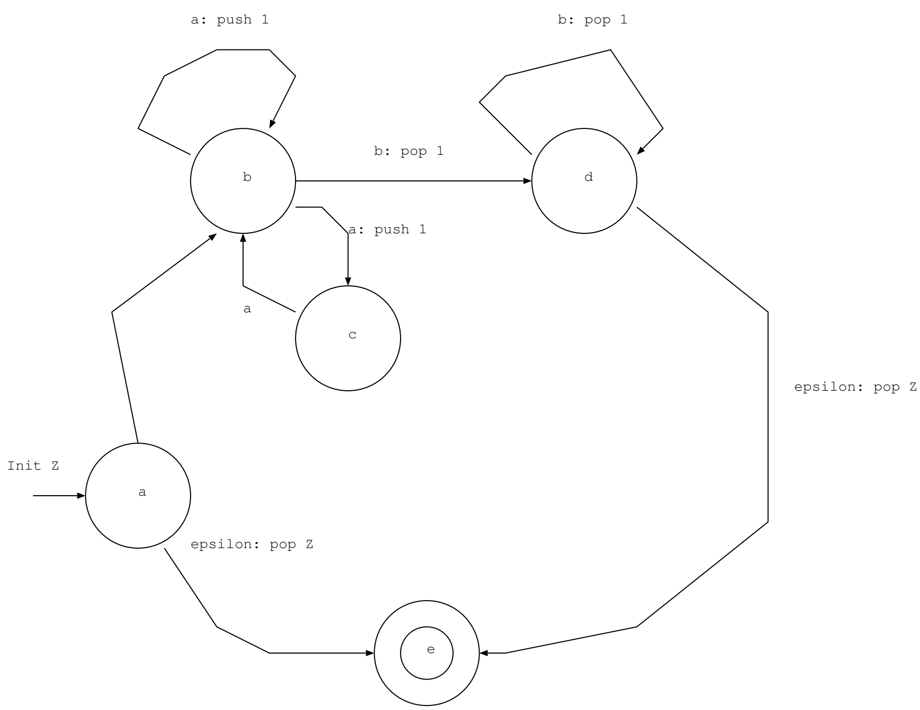
Basis: $01 \in \{\ 1, 0\}\ ^*, 10 \in \{\ 1, 0\}\ ^*$ and $\epsilon \in \{\ 1, 0\}\ ^*$ and for all cases there are either one 1 and one 0 or zero 0 and zero 1 so they are in the language L and can be generated by G.

IH: Suppose that A,B can be generated from G and that $A, B \in L$

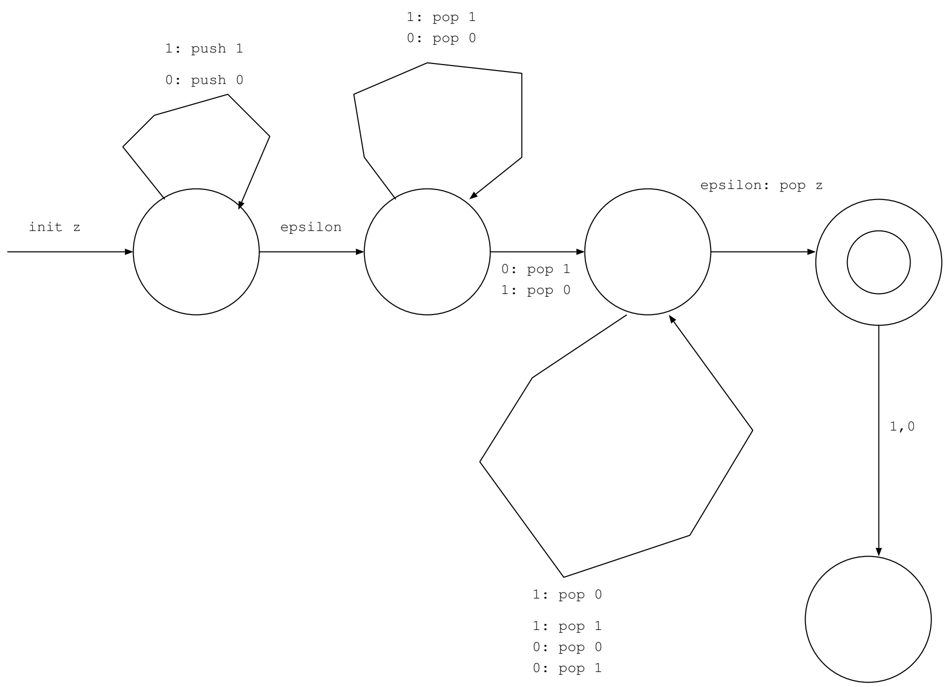
Structural: Suppose that there are strings A,B and $A, B \in L$ by the IH and they are all generated from G. Suppose that they are concatenated AB or BA. All strings have the same number of 1's and 0's so therefore the resulting string AB or BA would also have have the same number of 1's and 0's so $AB, BA \in L$. One of the transitions in the grammar is $S \rightarrow SS$ so $S \rightarrow AB$ or $S \rightarrow BA$ since we know from the IH that $A, B \in L$ so therefore

$S \to A, B$. Therefore AB and BA can be created by the grammar G and they are in the language L. Alternatively there is the transition 1S0 or 0S1. So suppose 1A0 or 0A1. These transitions would also create stirngs in the language L because A has a equal number of 1's and 0's so adding a 1 and a 0 to either side would maintain that property. So therefore grammar G produces all strings in language L.

2. Define a NPDA for the language $L = \{\ a^n b^m : m, n \in N, m \leq n \leq 2m\}$



3.


1: push 1
0: push 0
1: pop 1
0: pop 0
init z
epsilon
0: pop 1
1: pop 0
epsilon: pop z
1,0
1: pop 0
1: pop 1
0: pop 0
0: pop 1