HW13

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1. $G = (\{\ S, A, B\} \{\ 1, 0\}\ , S, P)$
$P = \{$
$S \rightarrow ASA|BSB|\epsilon$
$A \rightarrow 1$
$B \rightarrow 0$
$\}$

2. $L = \{\ p(M)p(w)$: M uses a finite number of tape cells when running on input w }
This language is turring acceptable. For $\forall_w \in L$ every w will only take finite number of tape cells so we know it will halt when the head of M reaches the last tape cell so $\forall_w \in L$, $M \searrow w$. So therefore L is acceptable. L is not a decidable language. We can prove this by reduction. We want to Reduce L to the halting problem. Suppose by contradiction we assume L is decidable. This means that there exists a machine M that will say yes to strings in the language. Othwerwise if the string is not in the language it will say no. If a string requires an infinte number of cells while running on the string then it cannot halt because it cannot process more cells after it halts. So if a string x is not in L then it will not halt. Therefore if a string is in L it will halt otherwise it will not halt if it is not in L. Our assumption was that there exists a machine that decides L but this is a contradiction because that would mean that a machine can decide the halting problem. Therefore L is not decidable.

Otherwise another way to prove this is the complement of L is a language where M uses an infinite number of tape cells. This is not acceptable because this machine can never say yes because it will be infinitely checking tape cells for a yes. Therefore becaues the complement is not turring acceptable it cannot be decidable.

$L = \{\ p(M)p(w)01^n0$: M uses at most n tape cells when running on input w }

This language is turring decidable. This language is acceptable because if it uses at most n tape cells it can give a Y. To prove its turring decidable we will look at all the cases.
Case 1: $M \searrow w$ in this case you just have to count the number of states that the head goes through. If it uses more than n then M says no and if it uses less or n then M says yes.

Case 2: $M \nearrow w$ there are two ways this can happen. It can either run forever over a finite amount of states or the head can transition over an infinite set of states. If it transitions over a finite number of states then M could count the number of times the head transitions into each state and put a limit on the number of times it can enter the same state multiple times. This prevents looping over a finite number of states forever. After it detects looping if it has gone through more than n transitions M can say no otherwise it can say yes. If M loops over an infiite number of transition then at some point it would reach N an pass it so as soon as it goes over N it can say no. So therefore this machine can always produce a yes or no so it is decidable.