HW0

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This is an inline equation $x + y = 3$
This is a displayed equation:

$$x + \frac{y}{z - \sqrt{3}} = 2$$

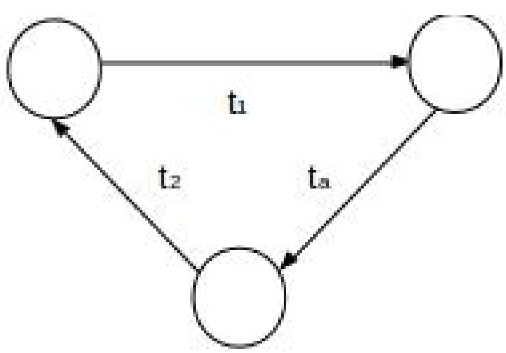
This is how you define a piece-wise linear function:

$$f(x) = \begin{cases} 3x + 2 & \text{if x} < 0 \\ 7x + 2 & \text{if x} \geq 0 \text{ and x} < 10 \\ 5x + 22 & \text{otherwise.} \end{cases}$$

This is a matrix:

9	9	9	9
6	6	6	
3		3	3

This is a figure incorporated in a LaTeX file



2. Show that N (natural numbers) and Z (integer numbers) are equinumerous.
To show that the set of natural numbers and integer numbers are equinumerous we have to show the cardinality of N and Z are the same. If there is a function f: N $\rightarrow$ Z that is bijective then the cardinalities of sets N and Z would be the same.

Suppose we have a function where $x \in N$ and $f(x) \in Z$

$$f(x) = \begin{cases} 0 & \text{if x equals 0} \\ \frac{x}{2} * -1 & \text{if x mod 2 = 0} \\ \frac{(x-1)}{2} + 1 & \text{otherwise.} \end{cases}$$

If f is onto then $\forall_y \in$ Z $\exists_x \in N$ where $f(x) = y$. Using existential instantiation and universal instantiation either $\frac{x}{2} * -1 = y$ or $\frac{(x-1)}{2} + 1 = y$. If $\frac{x}{2} * -1 = y$. For $\frac{x}{2} * -1 = y$ is equal to x = -2y and $\frac{(x-1)}{2} + 1 = y$ equals 2y - 1 = x. If y is negative then for -2y = x, x will equal a natural number. If y is positive then 2y - 1 = x, x will equal a natural number because it cannot be negative and y cant be 0 because x will be 0 as well. So therefore for all values of y a natural number of x can be reached so therefor function f is onto.

Using proof by contradiction assume f is not one to one so $\exists_x \exists_y (f(x) = f(y) \rightarrow x \neq y \wedge x, y \in N \wedge x, y)$. If x is odd and y is even or vice versa then $\frac{y}{2} * -1 = \frac{(x-1)}{2} + 1$. This can be reduced to y = -x - 1. This is a contradiction because since x and y are natural numbers they cannot be negative so $0 > -x - 1$ and $y > 0$ so therefore they cannot be equal. If x and y are both even then $\frac{y}{-2} = \frac{x}{-2}$ which reduces to x = y which is a contradiction because x and y cannot equal each other. If x and y are both odd then $\frac{(y-1)}{2} + 1 = \frac{(x-1)}{2} + 1$ which reduces down to x = y and for the same reason is a contradiction because x and y equal each other.

therefore the function f is one to one and onto so it is bijunction and therefore natural numbers have the same cardinality as integers and are are equinumerous.

3. Let f : S $\rightarrow$ S be a total function. Prove that, if S is infinite, f can be one-to-one without being onto, and onto without being one-to-one.

For f to be one-to-one but not onto for f(x) = y where $x \in S$ and $y \in S$ there is at least one value of y and x $f(x) \neq y$.

$$f(x) = \begin{cases} x+1 & \text{if x} \geq 0 \\ x-1 & \text{otherwise.} \end{cases}$$

Function f cannot be 0. If x is greater or equal to 0 then 0 = x + 1. The only value for x where this could be true is x = -1 but x cannot be a negative number. Otherwise for 0 = x - 1 x would need to be equal to 1 but x cannot equal or greater than 0 so therefore for either case 0 is an element of S but $f(x) \neq 0$.

To prove that f is one to one we will use proof by contradiction and assume that function f is not one to one so there is some value $\exists_x \exists_y \in S$ where $x \neq y \wedge f(x) = f(y)$ so if x is equal to 0 or greater and y is less than 0 or vice versa so x + 1 = y - 1 which can be reduced to x + 2 = y. This is a contradiction because y in this case is less than 0 and x can either be 0 or greater so adding 2 to 0 or greater cannot result in a negative number. If x and y are greater or equal to 0 then x + 1 = y + 1 or if they are both less than 0 so x - 1 = y - 1 but these both can be reduced to x = y and this is a contradiction because $x \neq y$ so therefore the function f is one to one.

A function f can be onto and not one-to-one

$x \in S$

$$f(x) = \begin{cases} 0 & if \text{ x} = 0 \\ x+1 & if \text{ x} < 0 \\ x-1 & if \text{ x} > 0 \end{cases}$$

f is is not one-to-one because f(0) = f(-1) = f(1) = 0. Function f is onto. If x is greater than 0 then $y \in S$ so y = x - 1. This can be reduced to 1 + y = x. The value y can be any value in S and correspond to a value x which is also in S. Otherwise if x is 0 y is 0 and if x is less than 0 then y = x + 1.

This can be reduced to y - 1 = x. For any value $y \in S$ $x \in S$ every value of y corresponds to a value x. Therefore the function f is onto.

4. Show that the relation R defined by

$\forall_{m,n} \in N$, (m, n) $\in R \Leftrightarrow$ (m - n) mod 3 = 0

is an equivalence relation, and describe its equivalence classes.

The relation R is reflexive because for any real number x, (x - x) mod 3 = 0 because x - x will always be 0.

The relation R is symmetric because for any real numbers x and y this say x - y = z so y - x = -z. So by swapping the order of x and y the relation is z mod 3 = 0 or -z mod 3 = 0. If z is negative or not makes no difference for mod so therefore R is symmetric.

The relation R is transitive. Suppose x,y,z are all real numbers. Suppose R(x, y) is true so (x - y) mod 3 = 0. R(y, z) is also true so (y - z) mod 3 = 0. Since we know that (x,y) is true we can assume that (x-y) = 3p or this can be reduced to x = 3p + y and the same for (y-z) = 3f which can be reduced to z = y - 3f. So for R(x, z) would be (3p + y) - (y - 3f) = 3f + 3p or 3(f + p). 3(f + p) mod 3 will always be 0 so therefore it is true and relation R is transitive.

Relation R is reflexive, symmetric, and transitive so therefore is a equivalence relation. The equivalence class for this relation R $[x]_R = \{\ y \mid (x, y) \in R\}$ where y is equal to any multiple of 3.

5. Show that $\sum_{i=1}^{n} i^2$ = (2n + 1)(n + 1)n/6

Using weak induction on the natural number

Basis: $\sum_{i=1}^{1} i^2$ = 1 and (2 + 1)(1 + 1)1/6 = 1

Inductive Hypothesis: For $\sum_{i=1}^{n} i^2$ is equal to (2n + 1)(n + 1)n/6 and true for n.

Inductive Step: To prove for n+1 this get the n + 1. (2(n + 1) + 1)(n +

2)($\frac{n+1}{6}$). This reduces to $\frac{2n^3+7n^2+6n}{6} + \frac{2n^2+7n+6}{6} = \frac{2n^3+9n^2+13n+6}{6}$. Now we will find $\sum_{i=1}^{n+1} i^2 =$ (2n + 1)(n + 1)$\frac{n}{6}$ + $(n+1)^2$ this reduces to $\frac{(2n^3+3n^2+n)}{6}$ + $\frac{(6n^2+12n+6)}{6} = \frac{2n^3+9n^2+13n+6}{6}$. So therefore n + 1 is correct.