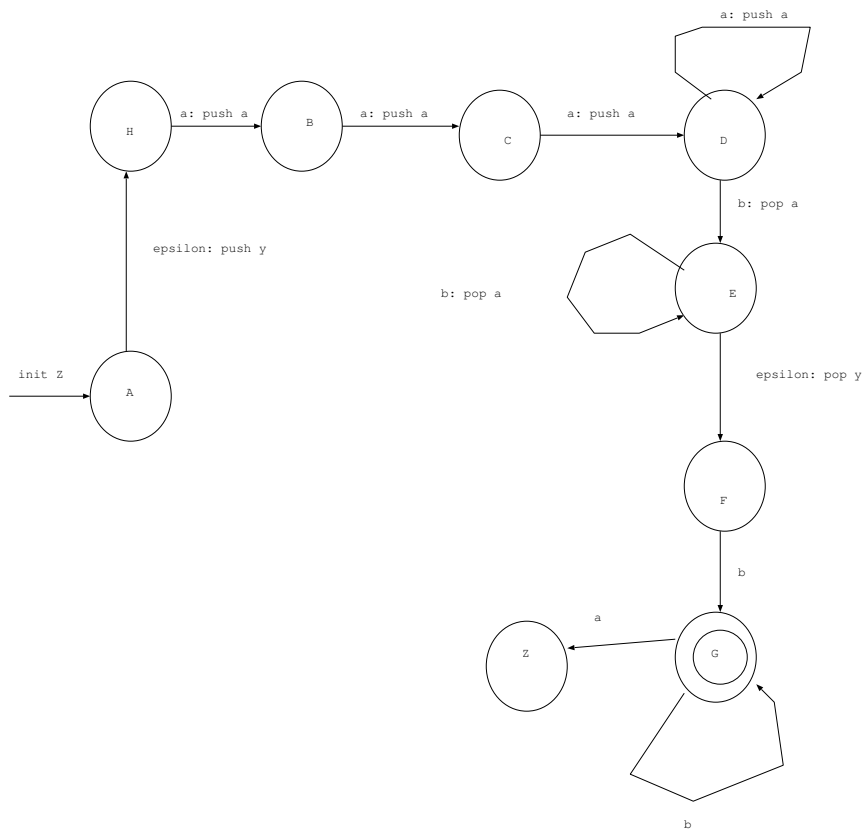


HW4

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1.



2.

By contradiction if we assume that this language is context-free. So there would be an m such that any string $w \in L$, $|w| \geq m$. We will choose $w = b^m a^{m+1} b^m$. w cannot be decomposed into $w = uv^k xy^k z$ where k is a natural number.

case 1: vxy are composed of all a's. If $k = 0$ then there will be at least one less a meaning that at least the longest run of a's and b's would be equal or the longest run of b's would be longer.

case 2: vxy are composed of all b's. If $k = 1$ then at least there would be one additional b. Meaning that at least the longest run of a's and b's would equal otherwise the longest run of b's would be longer.

case 3: vxy are composed of a's and b's. vxy cannot be greater than m so either the middle and left run or the middle and right run can be pumped at most. Also because of the restriction that v and y cannot be both epsilon we know that we will either pump just b's, just a's or both a's and b's. If we pump just a's or just b's $k = 1$ for just b's and $k = 0$ for just a's similar to case 1 and 2. Otherwise if we pump both a's and b's if $k = 0$ then there will be one less a or b in either of the runs. This will mean one of the runs of b's will now be equal or longer than the runs of a's.

3. Using proof by contradiction we will assume that the given language is context free. Therefore it can be pumped using pumping lemma. First we will pick a string in the language $z \in L$. $z = 1^m 0^{m+1} 2 1^{m+1} 0^m$ where $z \geq m$ and that z is divided up between $uvxyz$ where $m \leq |v^k xy^k|$. There are three cases to this problem.

Case 1: If the left side of the 2 is only pumped and k is greater than 1. In this case because both sides are equal in length the alpha value will be greater than the beta so it cannot be in the string.

Case 2: If the right side of the 2's is pumped then if we underpumped it a similar scenario to case would happen. If we underpump it then alpha will have more bits and will be greater in value.

Case 3: If the uvx split up between both sides of the 2. In this case if v or the y are not equal in the number of bits to each other then this is similar to case 1 and 2. If more bits from alpha are in v then there are bits from beta in y then you pump it making alpha a larger value. This is symmetric if y has a larger cardinality than v . Otherwise if $|v| = |y|$ then if we underpump v and y the string goes from $1^m 0^{m+1} 2 1^{m+1} 0^m$ to $1^m 0^m 2 1^m 0^m$. The resulting string after pumping the alpha will either be equal to or greater than the beta. Therefore by proof of contradiction L is not context free.

This answer was inspired by

<https://cs.stackexchange.com/questions/82637/prove-or-disprove-that-the-following-language-is-context-free>