

HW11

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1.

$L = \{ p(M)p(w) : M \text{ uses a finite number of tape cells when running on input } w \}$

This language is acceptable but not decidable. It is not decidable because the size of the tape is unknown. We can create a turing machine that simply gives M the input w and L and check that it used a finite number of tape cells. Essentially $p(M)p(w)$ is the same as the problem $\{ P(M)p(w) : M \searrow w \}$ which we know is not decidable.

$L = \{ p(M)p(w)01^n0 : M \text{ uses at most } n \text{ tape cells when running on input } w \}$

This language is decidable because we know the number n of tape cells that you can check. If it loops we can check if the head goes beyond n cells. If it loops over the same cells we can count the number of times it goes over the cells.

2.

$\{ p(M) : |L(M)| \leq 10 \}$

This language is not turing acceptable. The turing machine might go forever to confirm that there are 10 strings or less in the machine.

$$\{ p(M) : |L(M)| \geq 10 \}$$

This language is acceptable because if there is at least 10 words in the language it can be accepted. This is not decidable because its complement is not turing acceptable.

$$\{ p(M)p(w) : M \searrow w \text{ in 10 steps or less } \}$$

This is turing decidable because it has a finite number of steps so we know for sure it halts. If in 10 steps it has not printed a Y then it would print a N by the last step.

$$R = \{ p(M)p(w) : M \searrow w \text{ in 10 steps or more } \}$$

This language halts so we know it is acceptable. We will use reduction to prove it is not decidable. We know from the class notes that $\{ p(M)p(w) : M \searrow w \}$. If we build a machine M' . This machine first takes input w and checks that it is at least 10 steps. If not it adds more steps to the tape. This tape is then given to R . If R returns a Y or a N then M' returns a Y or a N otherwise it loops. So therefore R reduces to $\{ p(M)p(w) : M \searrow w \}$ so the language is only accepting by not decidable.

3. Use reduction to prove that the language is not decidable.

$$L = \{ p(M_1)p(M_2) : L(M_1) \subseteq L(M_2) \}$$

We know that it is not turing decidable for a turing machine to determine if two sets are equal. Using reduction we will take a turing machine L that checks if two sets are equal. This machine L needs to check that $L(M_1) \subseteq L(M_2)$ and $L(M_2) \subseteq L(M_1)$. We know that L can be reduced to $L(M_1) \subseteq L(M_2)$ and we know this cannot be decidable. If it could be decidable then it would mean L would be decidable and that would be a

contradiction. Therefore $L = \{ p(M_1)p(M_2) : L(M_1) \subseteq L(M_2) \}$ is not decidable.