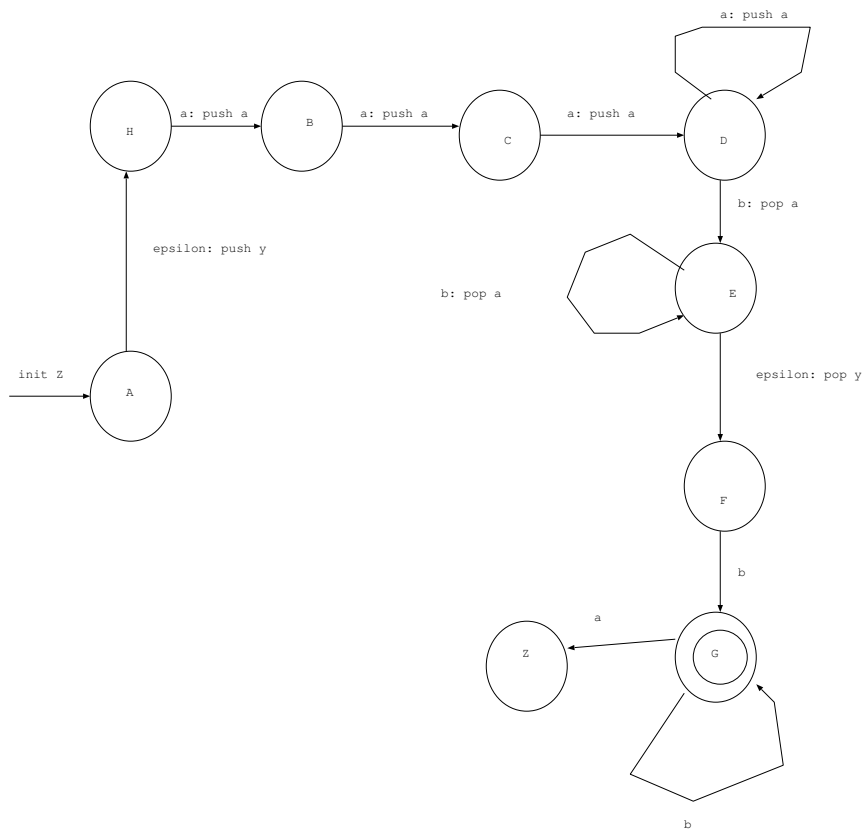


HW4

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1.



2.

By contradiction if we assume that this language is context-free. So there would be an m such that any string $w \in L$, $|w| \geq m$. We will choose $w = b^m a^{m+1} b^m$. w cannot be decomposed into $w = uv^k xy^k z$ where k is a natural number.

case 1: vxy are composed of all a's. If $k = 0$ then there will be at least one less a meaning that at least the longest run of a's and b's would be equal or the longest run of b's would be longer.

case 2: vxy are composed of all b's. If $k = 1$ then at least there would be one additional b. Meaning that at least the longest run of a's and b's would equal otherwise the longest run of b's would be longer.

case 3: vxy are composed of a's and b's. vxy cannot be greater than m so either the middle and left run or the middle and right run can be pumped at most. Also because of the restriction that v and y cannot be both epsilon we know that we will either pump just b's, just a's or both a's and b's. If we pump just a's or just b's $k = 1$ for just b's and $k = 0$ for just a's similar to case 1 and 2. Otherwise if we pump both a's and b's if $k = 0$ then there will be one less a or b in either of the runs. This will mean one of the runs of b's will now be equal or longer than the runs of a's.

3. With proof by contradiction we will assume L is context-free. Therefore we can use closure properties such as reversal to get $L' = L^R$. We will then concatenate $L'' = LL' = a2bb^R2a^R$. We will then do a pumping lemma on L'' . So there would be an m such that any string $w \in L$, $|w| \geq m$. we will choose $w = 1^m 210^m 0^m 121^m$. If we split this up with $uv^k xy^k z$ where k is a natural number. The only way this can be pumped to produce a symmetric string is if $v^k xy^k = 0^k 0^k$ in the middle of the string. If it is pumped anywhere else it will not be symmetric meaning it will not be the reversal of one string. We cannot pump in the middle because the 1001 in the middle are the b's or bb. 10 is greater than 1 by 1 in binary. So if we underpump by setting $k = 0$. and keeping it symmetrical so both the b's in the middle are reduced by at least 1 or more. This would mean that either it is not symmetric or $b \leq a$. Therefore L'' is not context free so by contradiction L' or L would not be context free QED.