Lab 2: Frequency Estimation

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Learning Objectives

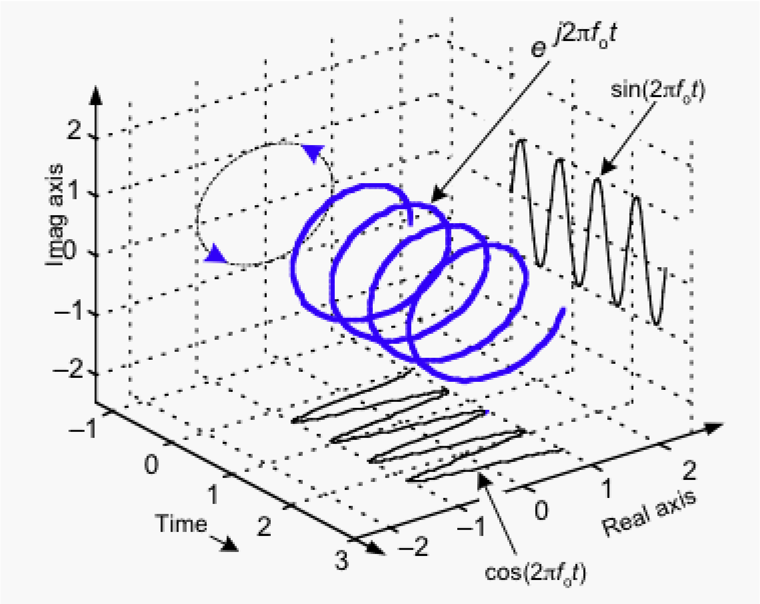
- Send a complex exponential signal through the SDR
- Estimate the complex gain and frequency of the RX complex exponential
  - Correlation method,
  - FFT method with oversampling
  - Gradient descent (advanced)
- Estimate the carrier frequency offset

Complex Exponential

- Continuous-time complex exponential signal:

$$x(t) = A\exp(2\pi i\, ft) = A\exp(i\omega t)$$

  - $A =$ complex magnitude
  - $f =$ frequency in Hz, $\omega = 2\pi f$ = angular frequency in rad/s
- Basic signal for Fourier analysis of linear systems
- Good initial signal to send and receive in SDRs
  - Easy to visualize response
  - Verifies that SDR is operational
  - Can measure carrier frequency offset



Transmitting a Complex Exponential

- ❑ Create the signal
- ❑ Create a TX object
- ❑ Load the buffer
- ❑ Will repeat continuously

```
x = A*exp(2*pi*1i*nu*(0:n-1)');

[tx, rx] = plutoCreateTxRx(...
    nsampsFrame = n, sampleRate = sampleRate);

tx.release();
tx.transmitRepeat(x);
```

Estimating the Frequency
Correlation Method

- Suppose $r[n] = A\, e^{2\pi i\, \nu n} + w[n]$
- Take one-step correlation:

$$z = \frac{1}{N-1} \sum_{n=0}^{N-2} r[n+1]\, r^*[n]$$

- When there is no noise:

$$z = \frac{1}{N-1} \sum_{n=0}^{N-2} A e^{2\pi i\, \nu(n+1)} A^* e^{-2\pi i\, \nu n} = |A|^2 e^{2\pi i \nu}$$

- Hence, we take estimate: $\hat{\nu} = \frac{1}{2\pi} angle(\, z\,)$
- Method is simple
- But tends to work well only when noise is small

Estimating the Frequency with DTFT

- Suppose $r[n] = A\, e^{2\pi i\, \nu_0 n}, n = 0, \ldots, N-1$
- Take DTFT with normalized frequency:

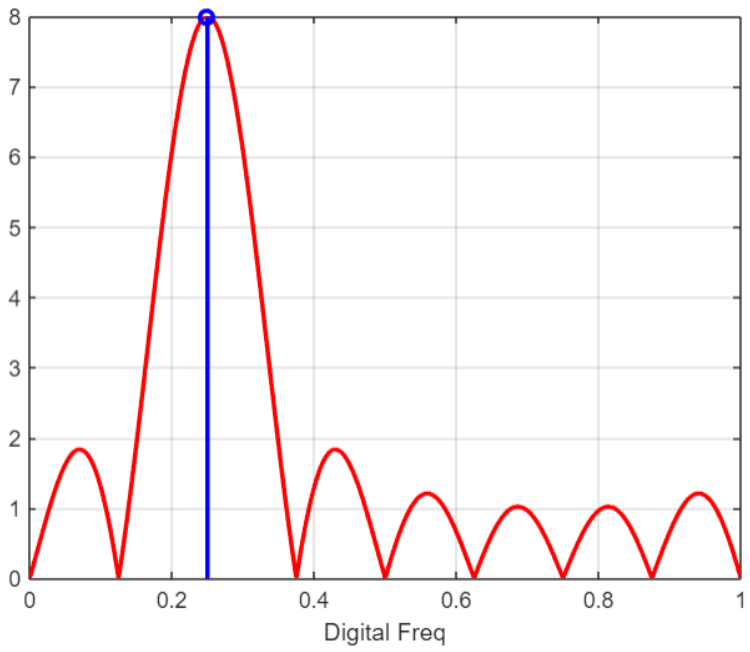
$$R(\nu) \coloneqq \sum_{n=0}^{N-1} r[n] e^{-2\pi i \nu n}$$

- DTFT of a complex exponential is a discrete sinc:

$$R(\nu) = \frac{\sin(\pi N(\nu - \nu_0))}{\sin(\pi(\nu - \nu_0))}$$

- Then $|R(\nu)|$ is maximized at $\nu = \nu_0$
- Ideal estimate for frequency:

$$\hat{\nu} = \arg\max_{\nu} |R(\nu)|$$



ν_0

Estimating the Frequency with FFT

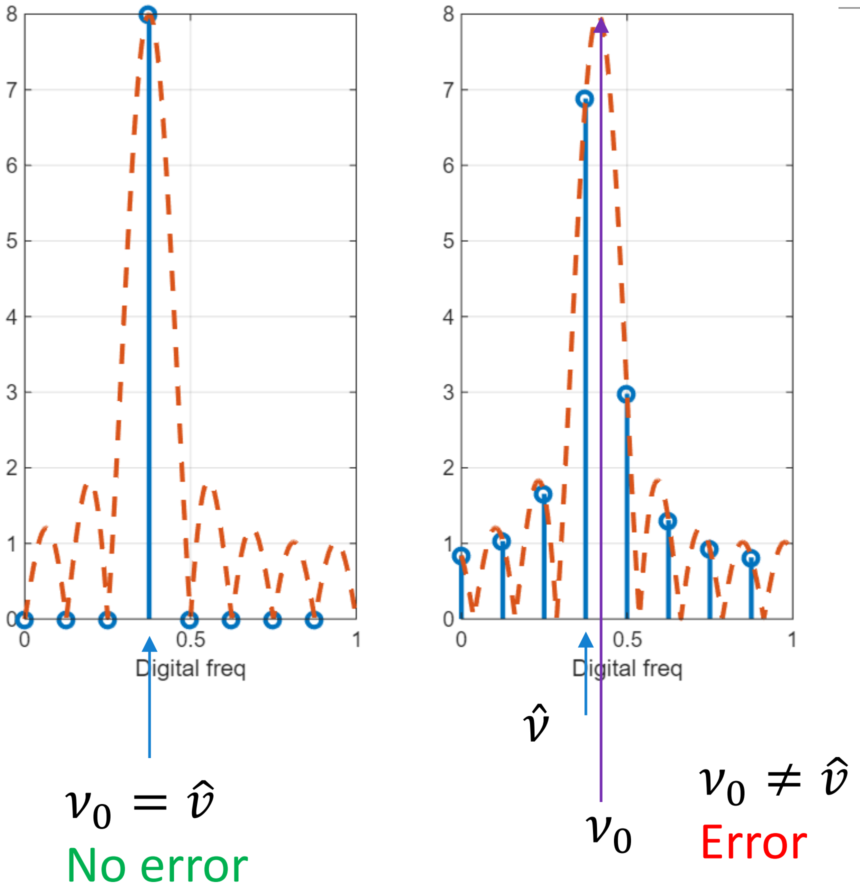
- DTFT method from prev. slide

$$\hat{\nu} = \arg\max_{\nu} |R(\nu)|$$

- Generally computed with FFT
- If $R_N[k] = FFT(r)$ with N points:

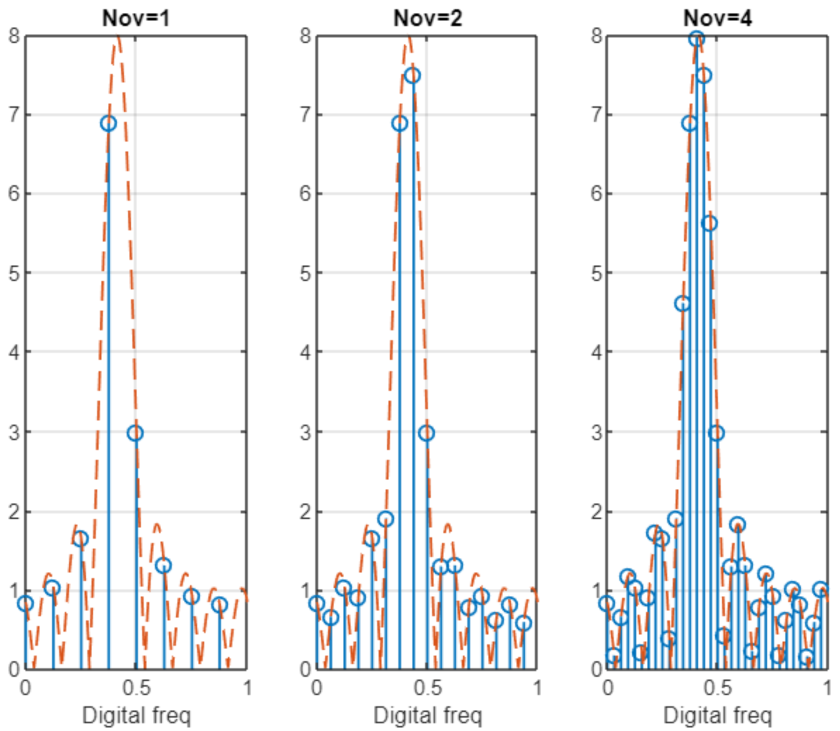
$$R_N[k] = R\left(\frac{k}{N}\right)$$

- May lead to rounding error



Using Oversampling

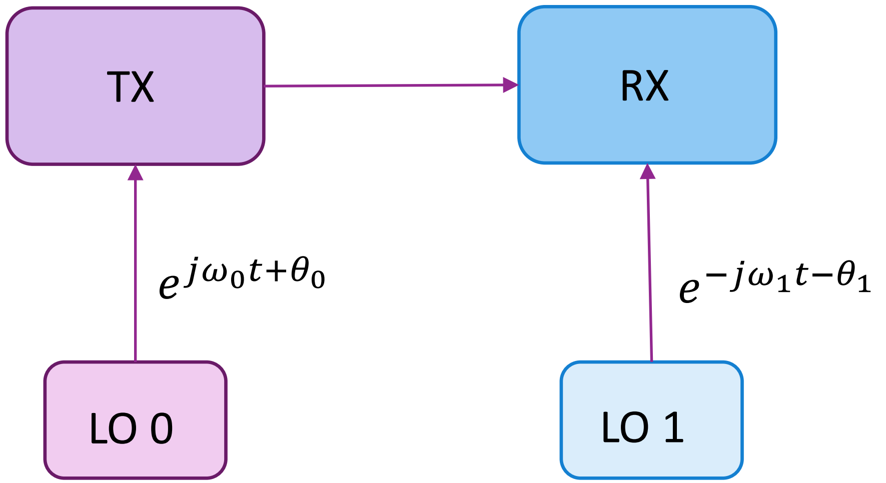
- Use oversampling to reduce rounding error
- Oversampling by N_{OV} (oversampling ratio):
  - Let $M = N \times N_{OV}$
  - Zero pad r with $M - N$ zeros
  - Take M point FFT
- Obtains frequency within $\frac{1}{M} = \frac{1}{NN_{OV}}$

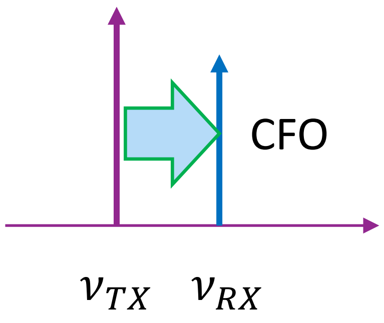


Lower frequency error

Carrier Frequency Offset

- Recall: Mixer at TX and RX driven by a local oscillator (LO)
- LO generally derived from a crystal with resonant frequency
- LO frequencies may be slightly mismatched
  - Physical differences in crystal and temperatures
- Causes a carrier frequency offset (CFO)
- Lab: Use frequency estimation to estimate CFO





Estimating Gain with Least Squares

- Signal is $r[n] = c\, e^{2\pi i \nu n} + w[n]$
- Suppose we have estimate $\hat{\nu}$ for ν (e.g., via correlation or FFT method)
- How do we estimate complex gain c ?
- Can estimate complex gain via least squares (see ML class):

$$\hat{c} = \arg\min \sum |r[n] - c\, u[n]|^2, \qquad u[n] = e^{2\pi i \nu n}$$

- Solution given by LS formula: $\hat{c} = \boldsymbol{u}^*\boldsymbol{r}/\boldsymbol{r}^*\boldsymbol{r}$

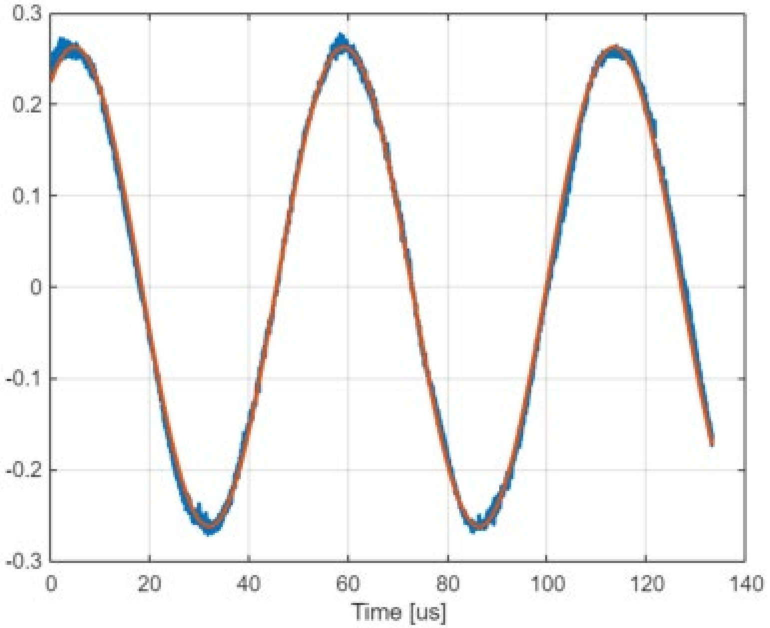
Lab 2

Frequency Estimation and Carrier Frequency Offset

Table of Contents

Complex exponentials are the most fundamental signals for all frequency domain analysis of linear system. In this lab you will learn to:

- Send a complex exponential or continuous-wave (CW) signal through the SDR
- Estimate the complex gain and frequency of a sinusoid via (1) correlation method, (2) FFT method
- Estimate the carrier frequency offset
- Save data for files for offline processing



- Lab 2 in the [SDR lab github]