Dynamic Strangle as a Lossless Hedge

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Purchasing a call and a put contract on the same underlying generally produces a "well-shaped" profit-and-loss curve with a loss at the center and profit emerging on either side beyond two breakeven points. For fixed premiums, one could in principle reposition the strikes to invert these breakeven points, raising the we well and eliminating loss. This brief note examines how realistic, variable premiums affect such an approach, showing that a truly lossless configuration generally requires the two contracts to be acquired at different times. For sufficiently large differences in the underlying prices at the times of purchase, this mechanism can provide a lossless hedge for a position that has developed into deep in-the-money status.

I. OVERVIEW

This short note explores the risk associated with a novel dynamic construction a position resembling the familiar strangle. The aim is to find conditions for an arbitrage-like lossless position. To fix notation we begin with a reminder of the profit/loss curves for a long put, a long call, and their combination in a strangle.

A **long put** with strike price K_P and premium P has profit/loss φ_P as a function of underlying asset price S

$$\varphi_P(S) = \max(K_P - S, 0) - P , \qquad (1)$$

sketched in Fig. 1.

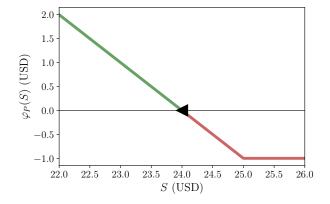


FIG. 1. Profit and loss curves for a long put with representative values of $K_P = 25$ USD and P = 1 USD. The breakeven point $S_- = K_P - P$, is shown as a left-pointing triangle.

If $S > K_P$, at the time of exercising, the option finishes worthless, and the net loss is the premium P. If instead $S < K_P$, the profit increases linearly as S decreases, bounded by $\varphi_P(0) = S_-$, where S_- is also the breakeven point for the put

$$S_{-} = K_P - P . (2)$$

A long call with strike price K_C and premium C has profit/loss

$$\varphi_C(S) = \max(S - K_C, 0) - C , \qquad (3)$$

sketched in Fig. 2.

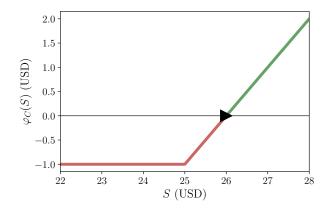


FIG. 2. Profit and loss curves for a long call with representative values of $K_C = 25$ USD and C = 1 USD. The breakeven point $S_+ = K_C + C$, is shown as a right-pointing triangle.

If $S < K_C$ at the time of exercising, the call is worthless and the net loss is the premium C. If instead $S > K_C$, the net profit increases linearly with S, without bound. The breakeven point for the call occurs at underlying price

$$S_+ = K_C + C . (4)$$

A strangle combines a long put and a long call giving a total profit/loss $\varphi(S) = \varphi_P(S) + \varphi_C(S)$,

$$\varphi(S) = \max(K_P - S, 0) + \max(S - K_C, 0) - \Pi$$
, (5)

with total premium $\Pi = P + C$. A generic strangle with $K_P < K_C$, is sketched in Fig. 3. This produces a profit when S moves significantly outside the strike region (K_P, K_C) , but has a loss in the middle. The two breakeven points for this strangle are

$$S_{-} = K_{P} - \Pi , \quad S_{+} = K_{C} + \Pi .$$
 (6)

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If $K_P < K_C$, then $S_- < S_+$. When $K_P > K_C$ however, it is possible to have inverted breakeven points $S_- > S_+$.

This note explores the feasibility of a strangle position whose profit/loss curve is raised in such a way that the loss well is entirely eliminated, $\varphi(S) \geq 0$ for all S.

II. FIXED PREMIUM MODEL

We begin by assuming, unrealistically, that the strikes can be adjusted without changing the total premium

$$\Pi = P + C \ . \tag{7}$$

The sequence shown in Fig. 4 demonstrates the effect of shifting the strikes K_C and/or K_P , in such a way that its breakeven points are inverted $S_- \geq S_+$, and $\varphi(S) \geq 0$, for all S.

III. VARIABLE PREMIUM MODEL

The final two strangles in the sequence 4 would amount to an arbitrage in that they are completely lossless. Unsurprisingly, this is not an achievable position when premiums are allowed to vary realistically. To see this, note that the arbitrage-like scenarios have minimum profit

$$\varphi_{\min} = K_P - K_C - \Pi \ge 0 . \tag{8}$$

However, if the put and call are both purchased simultaneously, they will have premiums bounded by

$$P \ge \max(0, K_P - S) \ge K_P - S , \qquad (9a)$$

$$C \ge \max(0, S - K_C) \ge S - K_C , \qquad (9b)$$

so that the total premium is bounded by $\Pi \geq K_P - K_C$, which can be restated as the contradiction

$$\varphi_{\min} \le 0$$
, (10)

Under these circumstances, the arbitrage can only be achieved if one or both contracts has been mis-priced in violation of the simple bounds (9).

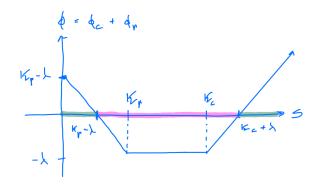


FIG. 3. Profit and loss curves for a generic strangle.

A. Dynamic Strangle

Now consider the case where the put is purchased when the underlying asset has price S_P , and the call is purchased at some other time when the underlying asset has price S_C . In that case, our simple bounds (9) become

$$P \ge \max(0, K_P - S_P) \ge K_P - S_P$$
, (11a)

$$C \ge \max(0, S_C - K_C) \ge S_C - K_C$$
, (11b)

so that the total premium is bounded by

$$\Pi \ge (K_P - S_P) + (S_C - K_C) , \qquad (12)$$

which can be restated as

$$\varphi_{\min} \le S_P - S_C \ . \tag{13}$$

So if one purchased a put or a call, and then that position develops a deep in-the-money status, one could purchase a corresponding call or put such that $S_P\gg S_C$. This could cause $\varphi_{\min}\geq 0$, such that the second option contract acted as a lossless hedge insuring at least some profit even if S were to experience a high volatility event such that the deep in-the-money contract completely collapsed. Thus, for a deep in-the-money contract, one may find it possible to dynamically construct a strangle with inverted breakeven points that locks out loss. This dynamic strangle hedge would of course come at a cost, unappealing to someone holding a deep in-the-money option contract. However that cost would only reduce the overall profit, and could not convert it to a loss.

IV. SUMMARY

This note analyzes the feasibility of a lossless strangle. We show that such a position is not realistic, but that a deep in-the-money option contract can be dynamically reconfigured into a lossless position. This lossless position resembles a strangle with inverted breakeven points, amounting to an arbitrage. Although this hedge reduces the upside potential, it cannot force the overall strategy into a net loss—hence "lossless." The main ingredient for this arbitrage is purchasing the two contracts at sufficiently different market conditions, rather than relying on static mispricing.

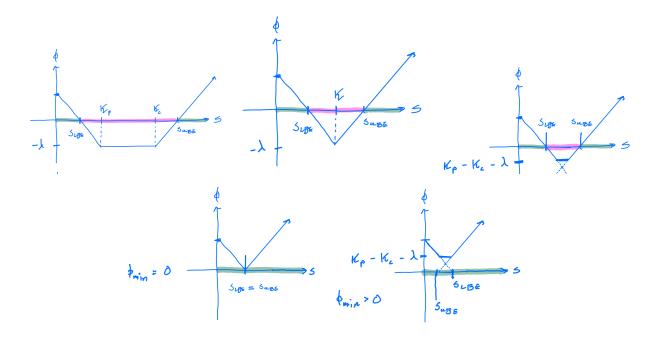


FIG. 4. A sequence of profit and loss curves as strikes K_C and/or K_P are repositioned while the combined premium $\Pi = C + P$, is unrealistically held fixed. The strike positions are as follows: a generic strangle with $K_P < K_C$ (top left), a straddle with $K_P = K_C$ (top middle), an inverted strangle with $0 < (K_P - K_C) < \Pi$ (top right), a critically inverted lossless strangle where the breakeven points are identical with $0 < (K_P - K_C) = \Pi$ (bottom left), and lastly a generic inverted lossless strangle where the breakeven points are inverted with $\Pi < (K_P - K_C)$ (bottom right).