Correspondence Analysis

In practice

Stéphane Dray

2021-12-07

Data

We consider the relationship between hair and eyes colors measured on 592 students in Delaware University.

```
mydata <- read.table("http://pbil.univ-lyon1.fr/R/donnees/snee74.txt",
    header = TRUE, stringsAsFactors = TRUE)
names(mydata)
```

```
## [1] "cheveux" "yeux"    "sexe"
```

```
head(mydata)
```

```
##   cheveux   yeux    sexe
## 1    Noir Marron    Male
## 2   Blond   Bleu Femelle
## 3    Noir   Bleu    Male
## 4  Marron Marron Femelle
## 5    Roux Marron    Male
## 6  Marron   Bleu    Male
```

Factors in R

A qualitative variables is stored as `factor` with different categories (`levels`)

```
is.factor(mydata$yeux)
```

```
## [1] TRUE
```

```
levels(mydata$yeux)
```

```
## [1] "Bleu"     "Marron"   "Noisette" "Vert"
```

Univariate analysis

- Compute the vector with the number of individuals for each eye color category and display the results on a plot.

Contingency table

- Build the contingency table crossing hair and eyes colors. See `?table`
- Compute row and columns marginal relative frequencies

Bivariate analysis

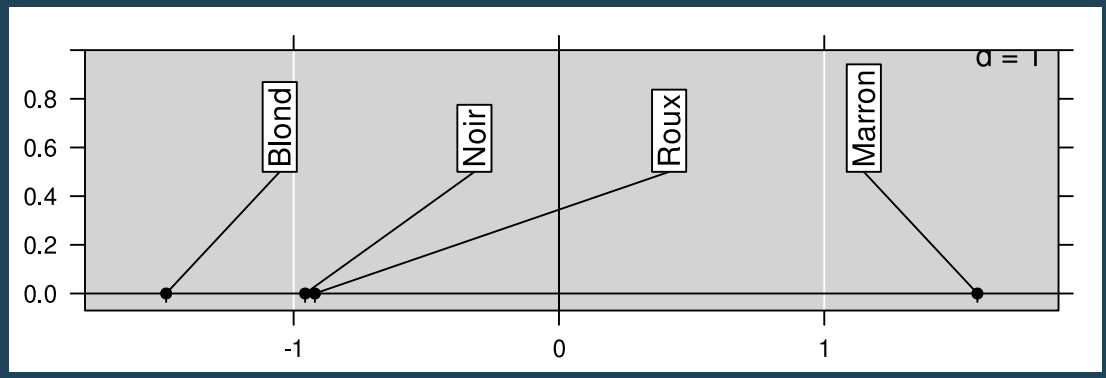
- Compute and interpret the results of a χ^2 test

Bivariate analysis

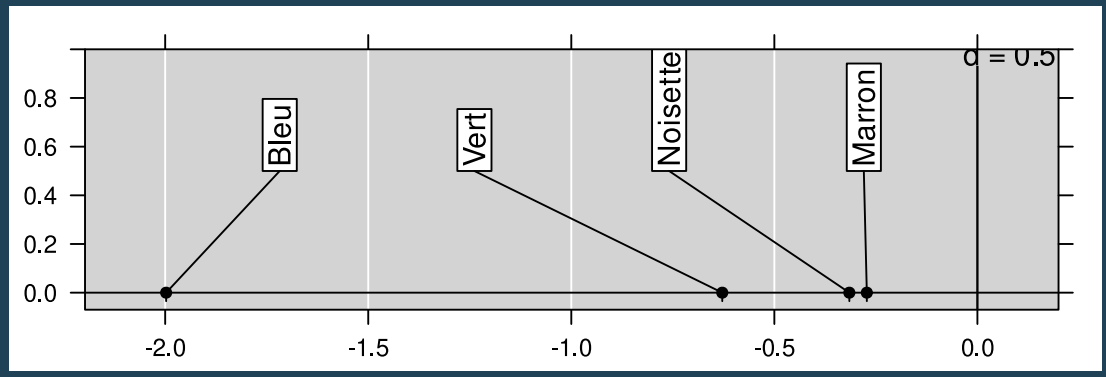
- Display and interpret the associations between categories with `mosaicplot`

Scoring

Suppose that we can display hair colors by a score



what would be the best way to display eyes colors? (given the distribution of students)



Weighted averaging

- Assign a random score to hair colors
- Compute the position for the eye color 'Vert'

Compute scores for all eye colors

By repeating the same formulas

```
score.e <- sapply(1:4, function(x) sum(score.h * mytab[,
    x]/sum(mytab[, x])))
```

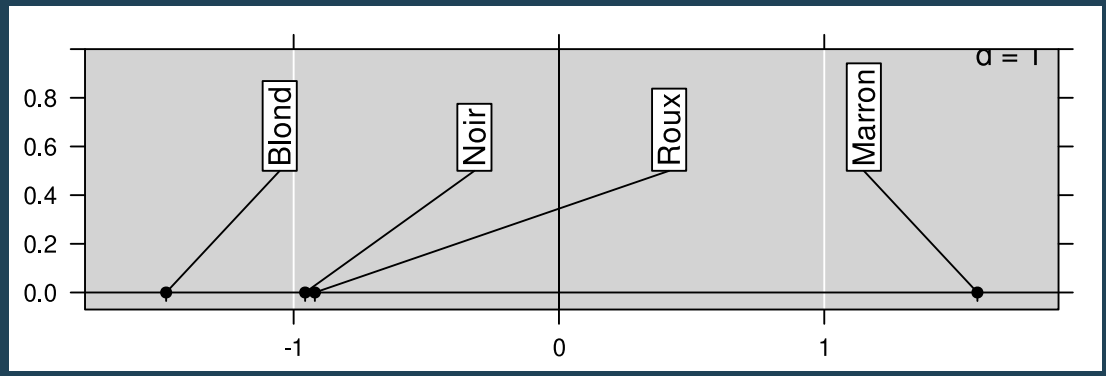
or using matrix algebra

```
t(prop.table(mytab, 2)) %*% score.h
```

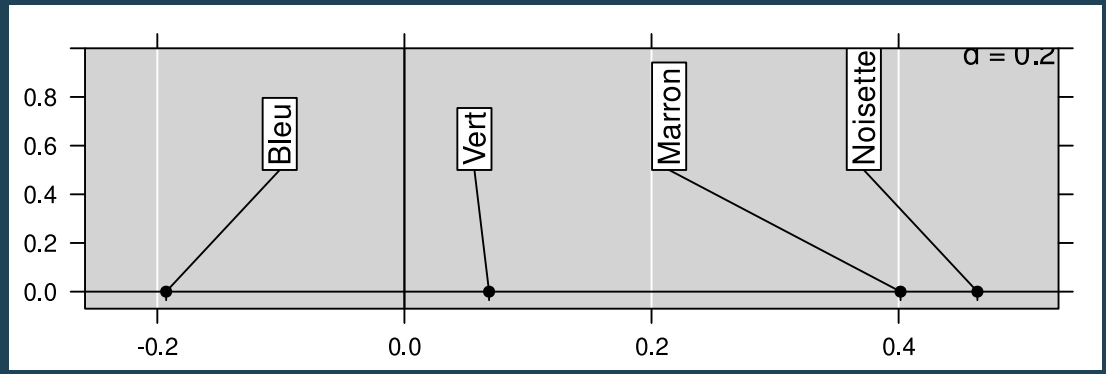
```
##
##                  [,1]
##   Bleu     -0.19286557
##   Marron    0.40154742
##   Noisette  0.46376597
##   Vert      0.06851621
```

Best representation

if



then



Reciprocal representation

- Start from a random score for eyes colors to position hair colors by averaging

Reciprocal averaging

1. Set a random score for columns
2. Use the column score to compute a score for rows by weighted averaging
3. Compute a new score for columns by weighted averaging of row score
4. Center and scale the two scores using row and column weights
5. Repeat steps 2-4 until convergence

Correspondence analysis

The solution of the iterative algorithm can also be obtained by the diagonalization procedure provided by the `dudi.coa` function

```
library(ade4)
coa1 <- dudi.coa(unclass(mytab), scannf = FALSE)
```

- Check the link between χ^2 statistic and total inertia

Interpretation

- Compute a row score with unit norm (`$l1`) to obtain a column score by weighted averaging (`$co`) with maximal variance (`$eig`)
- Compute a row score with unit norm (`$c1`) to obtain a column score by weighted averaging (`$li`) with maximal variance (`$eig`)
- Check both results

Graphical representation

- Represent and interpret the results using the `scatter` function (check the `method` argument of `scatter.coa`)

Inertia statistics

- Use the `inertia.dudi` function to compute inertia statistics for rows and columns
- Represent absolute contributions using the `plot` function (see `?plot.inertia`)