The Conditional Bernoulli and its Application to Speech Recognition

Sean Robertson

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1 Motivations

A major challenge in speech recognition involves converting a variable number of speech frames $\{x_t\}_{t \in [1,T]}$ into a variable number of transcription tokens $\{c_\ell\}_{\ell \in [1,L]}$, where $L \ll T$. In hybrid architectures, c_ℓ are generated as a by-product of transitioning between states s_t in a weighted finite-state transducer. In end-to-end neural ASR, this process is commonly achieved either with Connectionist Temporal Classification (CTC) [8] or sequence-to-sequence (seq2seq) architectures [2]. The former introduces a special blank label; performs a one-to-many mapping $c_\ell \mapsto \tilde{c}_t^{(i)}$ by injecting blank tokens until the transcription matches length T in all possible configurations (i) during training; and removes all blank labels during testing. Seq2seq architectures first encode the speech frames x_t into some encoding h, then some separate recurrent neural network conditions on h to generate the token sequence c_t.

In 2017, Luo et al. developed a novel end-to-end speech recognizer. Given a prefix of acoustic feature frames including the current frame $\{x_{t'}\}_{t' \in [t,T]}$ and a prefix of Bernoulli samples excluding the current frame $\{b_{t'}\}_{t' \in [t+1,T]}$, the recognizer produces a Bernoulli sample for the current frame $B_t \sim P_B(b_t | x_{\leq t}, b_{<t})$, plus or minus some additional conditioned terms. Whenever $B_t = 1$, the model "emits" a token drawn from a class distribution conditioned on the same information $C_t \sim P_C(c_t | x_{\leq t}, b_{<t})$. The paper had two primary motivations. First, though it resembles a decoder in a *seq2seq* architecture [2], it does not need to encode the entire input sequence x_t before it can start making decisions about what was said, making it suitable to online recognition. Second, we can interpret the emission points, or "highs," of the Bernoulli sequence $B_t = 1$ as a form of hard alignment: the token output according to C_t is unaffected by speech $x_{>t}$[1].

Because of the stochasticity introduced by sampling B_t discretely, the network cannot determine the exact gradient for parameterizations of B_t. Thus,

[1] This is not necessarily a synchronous alignment. $B_t = 1$ may occur well after whatever caused the emission. The last high $\arg\max_{t' < t} B_{t'} = 1$ cannot be assumed to bound the event to times after t' for the same reason. Finite t and vanishing gradients will force some synchronicity, however.

the authors rely on an estimate of the REINFORCE gradient [13]:

$$\frac{\partial R}{\partial \theta} = \mathbb{E}_b \left[\sum_{t=1}^{T} \left(\frac{\partial R_t}{\partial \theta} + \left(\sum_{t' \geq t} R_{t'} \frac{\partial}{\partial \theta} \log P(b_{t'} | b_{<t'}, c_{<\ell_{t'}}) \right) \right) \right] \tag{1}$$

Where

$$R_t = \begin{cases} \log P_C(C_t = c_{\sum_{t'<t} b_{t'}} | x_{\leq t}, b_{<t}, c_{\sum_{t'<t-1} b_{t'}}) & \text{if } B_t = 1 \\ 0 & \text{if } B_t = 0 \end{cases} \tag{2}$$

The reward (eq. (2)) is the log probability of the k-th class label, where k is the number of high Bernoulli values up to and including time t whenever $B_t = 1$. The return for time step t accumulates the instantaneous rewards for all non-past time steps $t' \geq t$.

In practice, using eq. (1) is very slow to train and yields mixed results. The authors found it was necessary to add a baseline function and an entropy function in order to converge. In a later publication [9], a bidirectional model[2] used Variational Inference to speed up convergence, though this failed to improve the overall performance of the model on the TIMIT corpus. The mixed performance and convergence of these models was blamed on the high-variance gradient estimate of eq. (1) [9].

We believe that the performance and convergence issues of these models are not due, at least in whole, to a high-variance estimate. Instead, we propose that the training objective has two other critical issues.

First, under the current regime, there is no natural choice of reward for when $B_t = 0$. Equation (1) accumulates future rewards to mitigate this, but the choice to do so biases the system to emit as soon as possible to reduce the number of total frames accumulating negative rewards. This bias could explain why, without an additional "entropy penalty," the model would learn to emit entirely at the beginning of the utterance [10].

Second, in order to ensure the total number of high Bernoulli values matched the total number of labels L during training, i.e. $\sum_t b_t = L$, the authors would force later samples to some specific value. This implies that $B_t \nsim P_B(b_t | \ldots)$, making the Monte Carlo estimate of eq. (1) biased. This bias could interact with the previous issue to produce a model that learns to immediately and repeatedly emit without stopping, since $b_{\geq L} = 0$, pushing $P_B(b_t | \ldots) \to 1$.

To solve both of these problems, we propose replacing the T i.i.d. Bernoulli random variables B_t sampled during training with a single sample B from the Conditional Bernoulli (CB) distribution, discussed in section 2. We will show that the switch elegantly supports the same inference procedure. Further, since the CB uses parameters generated at all time steps, rather than just the current time step, the rewards from when $B_t = 1$ will induce an error signal in the parameters for $B_{t'} = 0$. In addition to modifying eq. (1), we will also show how the CB can be applied to other gradient estimation methods, such as Straight-Through Estimators (STEs) [3] or RELAX-like estimators [11, 7].

[2] Forgoing the motivation for online speech recognition.

2 The Conditional Bernoulli

The Conditional Bernoulli distribution [6, 1, 4], sometimes called the Conditional Poisson distribution, is defined as

$$P\left(b \middle| \sum_t b_t = k; w\right) = \frac{\prod_t w_t^{b_t}}{\sum_{\left\{b':\sum_t b'_t = k\right\}} \prod_t w_t^{b'_t}} \tag{3}$$

Where $w_t = p_t/(1-p_t)$ are the odds/weights of a Bernoulli random variable $B_t \sim P(b_t; w_t) = p_t^{b_t}(1-p_t)^{(1-b_t)} = w_t^{b_t}(1-p_t)$. Equation (3) reads as "what is the probability that Bernoulli random variables $B = \{B_t\}_{t\in[1,T]}$ have values $\{b_t\}_t$, given that exactly k of them are high ($\sum_t b_t = k$)?" Letting $K = \sum_t B_t$, K is a random variable that counts the total number of "highs" in a series of Bernoulli trials. K is distributed according to the Poisson-Binomial (PB) distribution, a generalization of the Binomial distribution for when $p_i \neq p_j$. It is defined as

$$\begin{aligned} P(K = k; w) &= \sum_{\left\{b:\sum_t b_t = k\right\}} P(b; w) \\ &= \left(\prod_{t=1}^{T} 1 - p_t\right) \sum_{\left\{b:\sum_t b_t = k\right\}} \prod_{t=1}^{T} w_t^{b_t} \end{aligned} \tag{4}$$

If we use eq. (4) to marginalize out K from eq. (3), we recover the independent Bernoulli probabilities:

$$\begin{aligned} P(b; w) &= \sum_{k=0}^{T} P(b, k; w) = \sum_{k=0}^{T} P(b|k; w)P(k; w) \\ &= P(b|k'; w)P(k'; w) \text{ for some } k' = \sum_t b_t \\ &= \left(\prod_t 1 - p_t\right) \frac{\prod_{t=1}^{T} w_t^{b_t}}{\sum_{\left\{b':\sum_t b'_t = k'\right\}} \prod_{t=1}^{T} w_t^{b'_t}} \left(\sum_{\left\{b':\sum_t b'_t = k'\right\}} \prod_{t=1}^{T} w_t^{b'_t} \right) \\ &= \prod_{t=1}^{T} (1 - p_t) w_t^{b_t} \end{aligned} \tag{5}$$

Which is to say that, if we do not have knowledge of the number of highs *a priori*, assuming a Poisson-Binomial prior, we can sample B as a sequence of independent Bernoulli trials.

Direct calculation of equation eq. (3) involves summing over n-choose-k products of k odds, making it infeasible for large n and k. To combat this, Chen and Liu [5] propose a number of alternative algorithms where the sample B is constructed by iteratively deciding on the individual values of B_i. We will not only use these algorithms for efficiency: we will also use them to factor the CB distribution into more useful forms for different objectives.

To better describe these algorithms, we define the set of indices $t \in [1, T] = I$ s.t. $B = \{B_i\}_{i \in I}$. The set $A \subseteq I$ maps to some sample B such that all the high Bernoulli variables' indices can be found in A, i.e. $B_i = 1 \Leftrightarrow i \in A$. Then the CB can be restated as

$$P(A|k; w) = \frac{\prod_{a \in A} w_a}{R(k, I; w)} \tag{6}$$

where

$$R(v, S; w) = \sum_{\{A' \subseteq S : |A'| = v\}} \prod_{a \in A'} w_a \tag{7}$$

normalizes over all possible k-tuples of w_i in some set S. Equation (7) can be considered a generalization of n-choose-k: n-choose-k can be recovered by setting all $w_i = 1$. If we identify the product of weights from a set A as a weight indexed by A (i.e. $\prod_{a \in A} w_a \mapsto w'_A$), we can interpret eq. (6) as a categorical distribution.

The Draft Sampling procedure [5] recursively builds A by choosing a new weight to add to an ordered set. We use $j \in [1, |I|]$ to index elements of I in the order in which they are drafted into A: $I = \{i_j\}_j$, $A_j = (i_1, i_2, \ldots i_j)$, and $A_j^c = I \setminus A_j = \{i_{j+1}, i_{j+2}, \ldots, i_{|I|}\}$. Then the probability that some $i \in A_{j-1}^c$ is the j-th sample to be drafted into A is defined as

$$P(i \in A_j | A_{j-1}, k; w) = \frac{w_i R(k-j, A_{j-1}^c \setminus \{i\}; w)}{(k-j+1) R(k-j+1, A_{j-1}^c; w)} \tag{8}$$

Terms in both the numerator and denominator of eq. (8) sum over suffix sets of length $k-j+1$ that could be appended to A_{j-1} to get a k-tuple A. The numerator is the sum of products of odds including w_i. The conditional probability is conditioned on the remaining ("future") odds with respect to j, as well as whatever samples i_j were chosen in the past. The total probability of a drafted sample is

$$\begin{aligned}
P(A_k|k; w) &= \prod_{j=1}^{k} P(i_j \in A_j | A_{j-1}, k; w) \\
&= \prod_{j=1}^{k} \frac{w_{i_j} R(k-j, A_j^c, k)}{(k-j+1) R(k-j+1, A_{j-1}^c)} \\
&= \left(\prod_{j=1}^{k} w_{i_j} \right) \frac{R(0, A_k^c)}{k! R(k, I)} \\
&= \frac{1}{k!} P(A|k, w)
\end{aligned} \tag{9}$$

Section 2 produces almost the same probability as the Conditional Bernoulli, except for the factorial term. The factorial term accounts for the fact that samples are drafted into A_k in some fixed order. Summing over the probabilities of the $k!$ possible permutations of A_k yields the Conditional Bernoulli. We will call the distribution defined in the Draft Bernoulli (DB). Though the DB is not the same distribution as the CB, an expected value over the DB will be the same as that over the CB as long as the order of samples in A_k is ignored by the value function.

The ID-Checking Sampling procedure [5] is another useful treatment of the CB. This procedure builds A by iterating over Bernoulli trials and making binary decisions whether to include the trial in A. First, choose and fix an order j in which samples I will potentially be added to A. Let $A_{r_j,j} \subseteq A_j = (t_1, t_2, \ldots, t_j)$ be the subset of r_j samples ($|A_{r,j}| = r_j$) that have been added to A. At every step j, we choose to either add t_j to $A_{r_{j-1},j-1}$ and recurse on $A_{r_j,j} = A_{r_{j-1},j-1} \cup \{t_j\}$ or exclude t_j and recurse on $A_{r_j,j} = A_{r_{j-1},j-1}$. The probability of including t_j is

$$P(t_j \in A_{r_j,j} | A_{r_{j-1},j-1}, k; w) = \frac{w_{t_j} R(k - r_{j-1} - 1, A_j^c; w)}{R(k - r_{j-1}, A_j^c; w)} \tag{10}$$

From the perspective of Bernoulli trials, $P(t_j \in A_{r_j,j} | \ldots) = P(B_{t_j} = 1 | k - r_j; w)$. Equation (10) can be interpreted as the probability that B_{t_j} is high, given that $k - r_j$ remaining trials must be high. Like in eq. (8), the numerator and denominator of eq. (10) consist of products of weights of possible suffixes. The numerator only includes suffixes where w_{t_j} is a multiplicand.

The joint probability of a prefix of Bernoulli trials $b_{t_{\leq j}} = (b_{t_1}, b_{t_2}, \ldots, b_{t_j})$ using eq. (10) equals

$$\begin{aligned} P(b_{t_{\leq j}} | k - r_j; w) &= \prod_{j'=1}^{j} P(b_{t_{j'}} | k - r_{j'}; w) \\ &= \prod_{j'=1}^{j} \frac{w_{t_{j'}}^{b_{t_{j'}}} R(k - r_{j'}, A_j^c; w)}{R(k - r_{j'-1}, A_{j-1}^c; w)} \end{aligned} \tag{11}$$

The dependence on prior trials is implicit in the $r_{j'}$ term. We will call the family of distributions over different prefixes the ID-checking Bernoulli (IDB). When the prefix is the length of the entire sequence $j = T$, $P(b_{t_{\leq T}} | k - r_T; w) = P(b; k, w)$ and the IDB distribution matches the CB distribution.

Outside of statistics, Swersky et al. [12] linked the CB distribution with the goal of choosing a subset of k items from a set of n alternatives. In this case, the n alternatives are class labels, where one or more class labels may be active at a time. Models could be trained in a Maximum-Likelihood setting using the CB distribution: $B_i = 1$ implies class i is present and the probability of the data can be estimated via eq. (3). The authors note that it was insufficient to rely on the implicit prior induced by training via eq. (3) and had to explicitly learn and condition on it.

2.1 REINFORCE Objective

From section 1, we are interested in sampling T Bernoulli random variables such that the total number of emissions/highs matches the number of tokens L during training. We will start by considering the probability of a token sequence $c = \{c_\ell\}_{\ell \in [1,L]}$ under a model and work our way to a REINFORCE objective. For brevity, we supress conditioning on the acoustic data $\{x_t\}_{t\in[0,T]}$ and model parameters.

$$\begin{aligned}
P(c) &= P(c, L) \\
&= \sum_b P(c, b, L) \\
&= P(L) \sum_b P(b|L) P(c|b, L) \\
&= P(L) \mathbb{E}_{b|L} \left[P(c|b, L) \right]
\end{aligned}$$

Where $P(c) = P(c, L)$ follows from the fact that L is a deterministic function of c. Note that the expectation conditioning on L requires that the individual samples B_t are not entirely independent[3]. Taking the log, we get

$$\begin{aligned}
\log P(c) &= \log P(L) + \log \mathbb{E}_{b|L} \left[P(c|b, L) \right] \\
&\geq \log P(L) + \mathbb{E}_{b|L} \left[\log P(c|b, L) \right]
\end{aligned}$$

Where we have used Jensen's Inequality to establish a lower bound. Calling the bound R and differentiating with respect to some parameter θ, we get

$$\frac{\partial R}{\partial \theta} = \frac{\partial \log P(L)}{\partial \theta} + \frac{\partial}{\partial \theta} \mathbb{E}_{b|L} \left[\log P(c|b, L) \right] \tag{12}$$

We have yet to make any assumptions about the distributions of any $P(\cdot)$, except to say that $|c| = L$. To recover the REINFORCE objective of eq. (1), we remove all mention of L (including $P(L)$) and factor the conditional probability of the class labels as [9]:

$$P(c, b) = \prod_{t=1}^{T} P(c_{\ell_t} | b_{\leq t}, c_{<\ell_t})^{b_t} P(b_t | b_{\leq t}, c_{<\ell_t}) \tag{13}$$

where $\ell_t = \sum_{t'=0}^{t} b'_t$.

Under these assumptions, the rightmost expectation in eq. (12) decomposes

[3] Except the pathological case where exactly $P(B_t = 1) = 1$ for exactly L of T variables, and 0 otherwise.

into[4]

$$
\begin{aligned}
\frac{\partial}{\partial\theta}\mathbb{E}_b\left[\log P(c|b)\right] &= \frac{\partial}{\partial\theta}\mathbb{E}_b\left[\sum_{t=1}^{T} b_t \log P(c_{\ell_t}|b_{\leq t}, c_{<\ell_t})\right] \\
&= \sum_{t=1}^{T}\frac{\partial}{\partial\theta}\mathbb{E}_b\left[R_t\right] \text{ from eq. (2)} \\
&= \sum_{t=1}^{T}\frac{\partial}{\partial\theta}\mathbb{E}_{b_{\leq t}}\left[R_t\right] \text{ since } R_t \text{ not based on } b_{>t} \\
&= \sum_{t=1}^{T}\mathbb{E}_{b_{\leq t}}\left[\frac{\partial R_t}{\partial\theta} + R_t\frac{\partial}{\partial\theta}\log P(b_{\leq t}|c_{<\ell_t})\right] \\
&= \sum_{t=1}^{T}\mathbb{E}_{b_{\leq t}}\left[\frac{\partial R_t}{\partial\theta} + R_t\sum_{t'\leq t}\frac{\partial}{\partial\theta}\log P(b_{t'}|b_{t'-1}, c_{<\ell_{t'}})\right] \\
&= \mathbb{E}_b\left[\sum_{t=1}^{T}\left(\frac{\partial R_t}{\partial\theta} + \left(\sum_{t'\geq t}R_{t'}\frac{\partial}{\partial\theta}\log P(b_{t'}|b_{<t'}, c_{<\ell_{t'}})\right)\right)\right]
\end{aligned}
$$

We can see the two issues with the above REINFORCE objective discussed in section 1 by observing eq. (13). First, c_{ℓ_t} is undefined when ℓ_t exceeds L. Second, $P(c, b)$ is maximized whenever $B_t = 0$ for all t. The second problem may be solved by skipping the factorization of class label probabilities. However, in this case, L is still ignored and the first problem is still a problem. Furthermore, we would lose the ability to attribute credit to the t-th frame for classifying label c_{ℓ_t}.

The primary concerns above may be addressed by assuming $P(L)$ is PB-distributed and $P(b|L)$ is CB-distributed. Letting t_ℓ be the inverse mapping of ℓ_t, namely: $t_\ell = Sort(\{t : B_t = 1\})_\ell$. We define

$$
P(c, b|L) = P(c|b, L)P(b|L) = \left(\prod_{\ell=1}^{L} P(c_\ell|b_{t_{\leq\ell}})\right) P(b|L) \tag{14}
$$

and plug the conditional probability $P(c|b, L)$ into the expectation in eq. (12) to get the "global" CB REINFORCE gradient:

$$
\frac{\partial R}{\partial\theta} = \frac{\partial \log P(L)}{\partial\theta} + \mathbb{E}_{b|L}\left[\sum_{\ell=1}^{L}\left(\frac{\partial R_\ell}{\partial\theta} + R_\ell\frac{\partial}{\partial\theta}\log P(b|L)\right)\right] \tag{15}
$$

Where $R_\ell = \log P(c_\ell|b_{t_{\leq\ell}})$. While R_ℓ only depends on the high Bernoullis up to and including time t_ℓ, t_ℓ can only be determined by viewing the entire Bernoulli sample B.

[4]Thanks to Dieterich Lawson for this derivation.

Since all samples $B \sim P(b|L)$ will have exactly L highs, $\sum_t B_t = L$, the decomposition of the class label sequence probability is well-defined. The pathological case where reward is maximized when B_t is no longer a problem because we have switched to a global reward rather than a per-frame reward.

There are, however, two new issues introduced by eq. (15). The first is the same as if we stopped using a per-frame reward in eq. (1): we can no longer use the error signal for a specific c_ℓ to optimize a subset of B. Global estimates will tend to have higher variance than per-frame gradient estimates (TODO: Rao-Blackwell). The second problem is that $P(b|L)$ can no longer be auto-regressive. Equation (3) uses the entire set of odds from all frames. While there are ways to decompose eq. (3) into a fixed-order series of binary decisions [5], the current trial B_t would still be distributed according to the log-odds of non-past trials $w_{\geq t}$.

In Section 2, we noted that an expectation over a DB variable will yield the same expected value as the same expectation over a CB variable assuming that the value function in the expectation is not conditioned on the order in which samples are drafted. The total reward in eq. (15) satisfies this criterion. Thus the global DB REINFORCE objective maximizes the same expectation as the CB REINFORCE objective:

$$\frac{\partial R}{\partial \theta} = \frac{\partial \log P(L)}{\partial \theta} + \mathbb{E}_{A_L|L}\left[\sum_{\ell=1}^{L}\left(\frac{\partial R_\ell}{\partial \theta} + R_\ell \frac{\partial}{\partial \theta}\log P(A_L|L)\right)\right] \tag{16}$$

The advantage of the DB REINFORCE objective over the CB REINFORCE objective is it can leverage the relaxation of the DB, discussed in section 2.2.

Our final REINFORCE objective is frame-wise, courtesy of the IDB decomposition of the CB from eq. (11). Though a given trial sample B_{t_j} is conditioned on non-past weights $w_{t_{\geq j}}$, it is only conditioned on samples from the past $b_{t_{<j}}$. Setting $t_j = j$, we decompose the joint probability of the class label sequence and the CB sample as

$$P(c, b|L) = P(c|b, L)P(B|L) = \prod_{t=1}^{T} P(c_{\ell_t}|b_{\leq t})^{b_t} P(b_t|L - r_t) \tag{17}$$

Equation (17) is very similar to eq. (13), except the conditioning on the number of class labels L forces ℓ_t to be well-defined whenever $B_t = 1$. The derivation of the IDB REINFORCE gradient is almost identical to that for eq. (1), yielding

$$\frac{\partial R}{\partial \theta} = \frac{\partial \log P(L)}{\partial \theta} + \mathbb{E}_{b|L}\left[\sum_{t=1}^{T}\left(\frac{\partial R_t}{\partial \theta} + \left(\sum_{t' \geq t} R_{t'} \frac{\partial}{\partial \theta}\log P(b_{t'}|L - r_t)\right)\right)\right] \tag{18}$$

where $R_t = b_t \log P(c_{\ell_t}|b_{\leq t})$.

The IDB REINFORCE gradient solves the problem of ill-defined ℓ_t, provides a frame-wise gradient update, and avoids the pathological case of maximum probability when $\forall t.B_t = 0$. However, were we to use eqs. (17) and (18) on their own, the model will likely still learn to emit early so as to minimize future accumulated (negative) rewards.

To mitigate this tendency, we can leverage the fact that the IDB factors the CB in a fixed but arbitrary order of trials t_j. Denoting eq. (17) as the forward IDB joint probability, we define the backward IDB joint probability as

$$P(c, b|L) = \prod_{t=1}^{T} P(c_{L-\ell_t}|b_{>t})^{b_t} P(b_t|L - r_{T-t}) \tag{19}$$

2.2 Continuous relaxations

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