Makov Models

- Markov chain (MC)

$(\mathcal{S}, P)$

$\mathcal{S}$: state space, set of all possible states

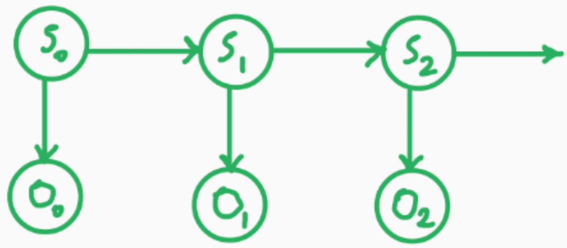
P: transition function (matrix); $P : \mathcal{S} \longrightarrow \Delta(\mathcal{S})$

$$P(s'|s) = \mathbb{P}\left[s_{t+1} = s' \mid s_t = s\right]$$ → probability of transitioning from s to s'

$$\underset{\in \mathcal{S}}{s_0} \xrightarrow{P(s_1|s_0)} \underset{\in \mathcal{S}}{s_1} \xrightarrow{P(s_2|s_1)} s_2 \quad \cdots$$

- Hidden Markov Model (HMM)

$(\mathcal{S}, P, \mathcal{O}, \mathcal{Z})$



$\mathcal{S}$: state space

P: transition function (matrix); $P : \mathcal{S} \longrightarrow \Delta(\mathcal{S})$

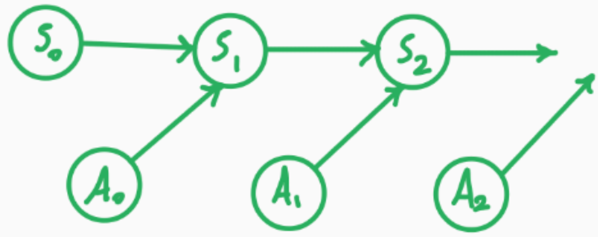
$\mathcal{O}$: observation space, set of all possible observations

$\mathcal{Z}$: observation function (emission probabilities); $\mathcal{Z} : \mathcal{S} \longrightarrow \Delta(\mathcal{O})$

$$Z(o|s) = \mathbb{P}\left[o_t = o \mid s_t = s\right]$$ → probability of observing o at s

$$\begin{tikzcd}
s_0 \arrow[r, "Z(o_0|s_0)"] \arrow[rr, bend right, "P(s_1|s_0)"'] & o_0 & s_1 \arrow[r, "Z(o_1|s_1)"] \arrow[rr, bend right, "P(s_2|s_1)"'] & o_1 & s_2 & \cdots
\end{tikzcd}$$

- Markov Decision Process (MDP)



(S, A, P)

S : state space

A : action space, set of all possible actions

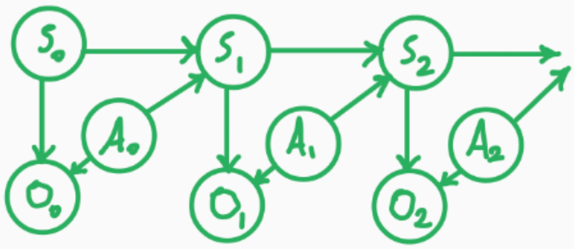
P : transition function

$$P(s' \mid s, a) = \mathbb{P}[S_{t+1} = s' \mid S_t = s, A_t = a]$$

$\downarrow P(S_1 \mid S_0, A_0) \quad \downarrow P(S_2 \mid S_1, A_1)$

$S_0 \quad A_0 \longrightarrow S_1 \quad A_1 \longrightarrow S_2$

- Partially Observable Markov Decision Process (POMDP)



(S, A, P, O, Z)

S : state space

A : action space

P : transition function

O : observation space

Z : observation probabilities

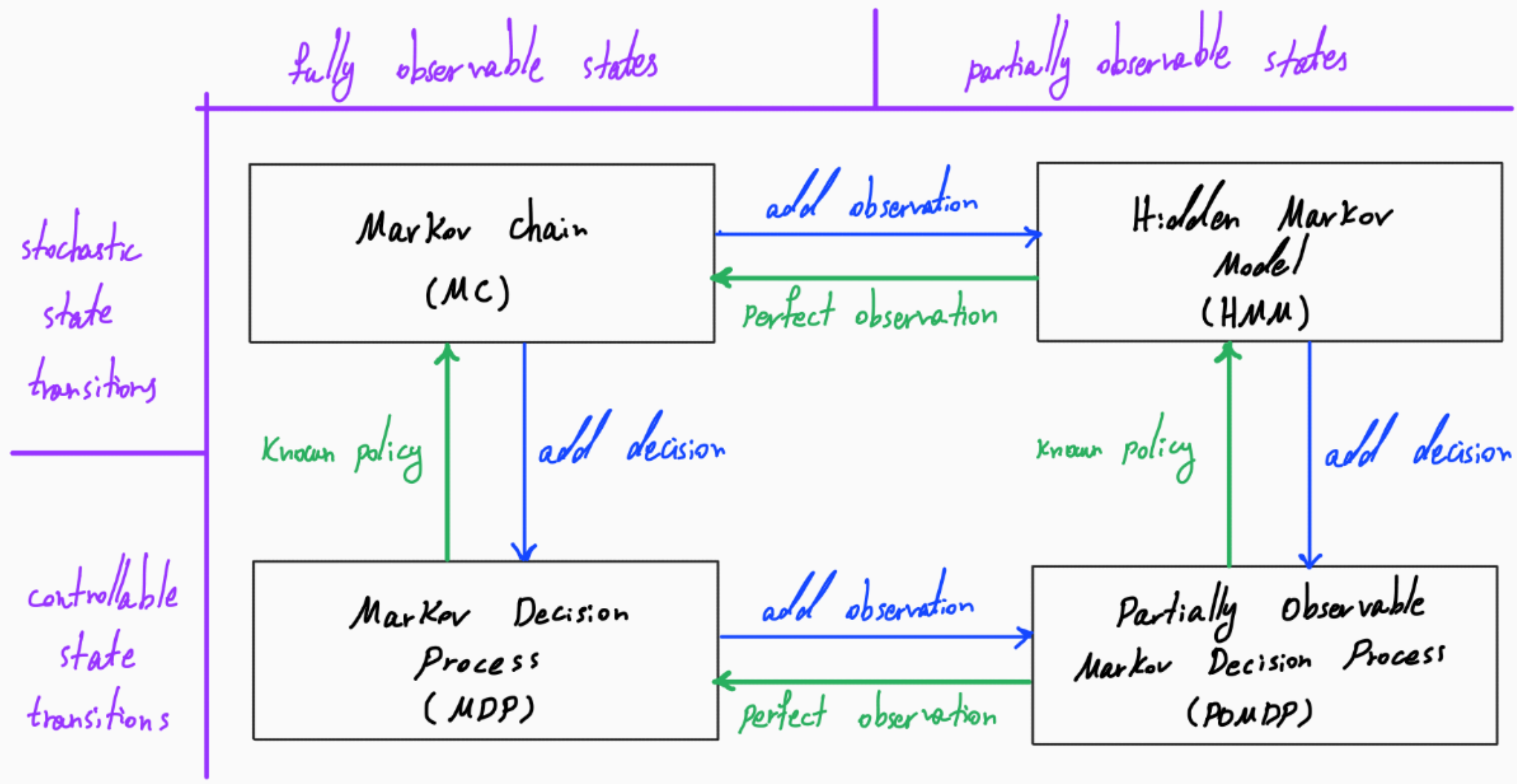
$Z(o \mid s, a) \rightsquigarrow$ probability of observing o at s taking a

$\downarrow Z(O_0 \mid S_0, A_0) \quad \downarrow Z(O_1 \mid S_1, A_1)$

$S_0 \quad A_0 \longrightarrow O_0 \quad S_1 \quad A_1 \longrightarrow O_1$

$P(S_1 \mid S_0, A_0)$

Overview of Markov Models

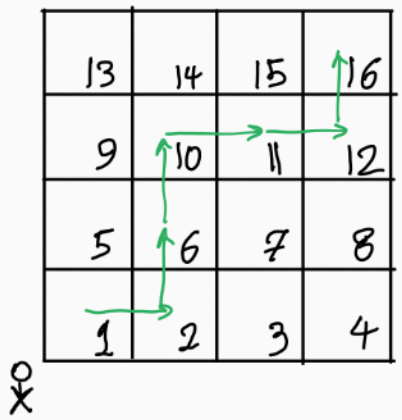


Example:

MC

{ goes right 0.6
{ goes up 0.4

$S = \{1, 2, \dots, 16\}$

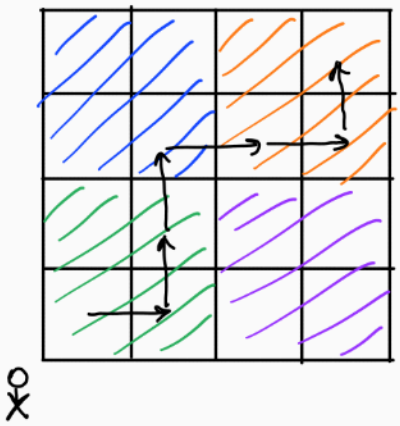


HMM

{ goes right 0.6
{ goes up 0.4

$S = \{1, 2, \dots, 16\}$

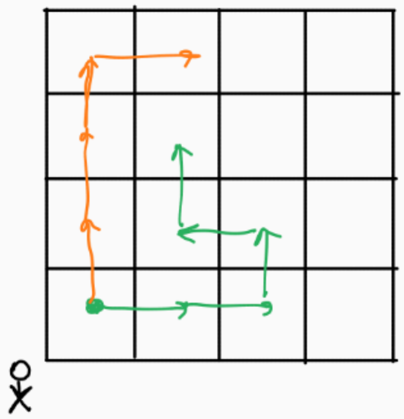
$O = \{\bullet, \bullet, \bullet, \bullet\}$



MDP

$S = \{1, 2, \dots, 16\}$

$A = \{r, l, u, d, s\}$

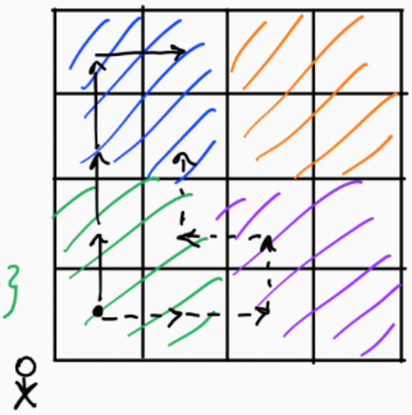


POMDP

$S = \{1, 2, \dots, 16\}$

$A = \{r, l, u, d, s\}$

$O = \{\bullet, \bullet, \bullet, \bullet\}$



Example: Entertainment system

states of human user $S = \{ E , N \}$

system's observations of human $O = \{\text{smile}, \text{frown}, \dots\}$

system's actions $A = \{\text{content } 1, \text{content } 2, \dots, \text{content } K\}$