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wolensing: A Python package to compute wave optics amplification factor for gravitational wave

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Software

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Summary

The wolensing Python package offers a solution for gravitational wave lensing computations within the full wave-optics regime. This tool is primarily designed to calculate the gravitational lensing amplification factor including diffractive effects, an essential component for generating accurate lensed gravitational wave waveforms. These waveforms are integral to astrophysical and cosmological studies related to gravitational-wave lensing.

Integrating with lensingGW (Pagano et al., 2020), wolensing provides solutions for image positions in the high-frequency regime where wave and geometrical optics converge. This functionality allows the amplification factor to be applicable across a wider frequency range. Another key feature of wolensing is its ability to plot time delay contours on the lens plane, offering researchers a visual tool to better understand the relationship between the lens system and the amplification factor.

wolensing is compatible with various lens models in lenstronomy (Birrer et al., 2021). There are also built-in lens models including point mass, singular isothermal sphere (SIS), and nonsingular isothermal ellipsoid (NIE) with jax (Bradbury et al., 2018) supporting GPU computation. Users can accommodate different lens models in the code with jax.

wolensing is available as an open-source package on PyPI and can be installed via pip.

Statement of need

Gravitational wave lensing studies have traditionally concentrated on the strong lensing case, utilizing the geometrical optics approximation. This approach predicts images with varying time delays and magnifications while maintaining uniform frequency evolution across these images. However, for lens masses around or below 100 solar masses, the scale of the gravitational (Schwarzschild) radius becomes comparable to the wavelength of the gravitational wave that is within the LIGO/Virgo/Kagra sensitivity range (10–10^3 Hz). Example lenses that exist within the mass range include stars and low-mass compact object remnants. In the LISA sensitivty range (mHz–Hz), similar considerations apply to lenses with masses ranging from 10^5 to 10^8 solar masses. Example lenses that exist in this mass range include dark matter subhalos and larger compact structures. In this limit when the Schwarzschild radius is around or below the gravitational-wave wavelength, wave optics effects introduces diffraction effects, necessitating a shift from geometrical to wave optics for accurate modeling. Indeed, in this regime, the frequency evolution of gravitational waves is influenced by the amplification factor, which is determined using a diffraction integral, resulting in a marked increase in computational complexity and cost.

The `wolensing` package addresses this challenge by providing efficient computation of the amplification factor for general lenses. To optimize computational speed, it includes built-in simple lens models that leverage `jax` for enhanced performance. Furthermore, `wolensing` integrates geometrical optics for high-frequency scenarios, reducing the computational cost in that regime.

The core component of `wolensing` is a 2-dimensional integrator that estimates the area between neighboring contour lines of the lensing time delay function (Diego et al., 2019). The integration method implemented works well for general lens systems and fine tuning of the settings is not required when changing the lens model. Other than scenarios with a single lens, `wolensing` can also be used to study systems with multiple lenses. Notably, (Cheung et al., 2021) and (Yeung et al., 2023) employed the package to analyze microlensing effects on top of type-I and type-II images produced by a Singular Isothermal Sphere (SIS) galaxy.

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