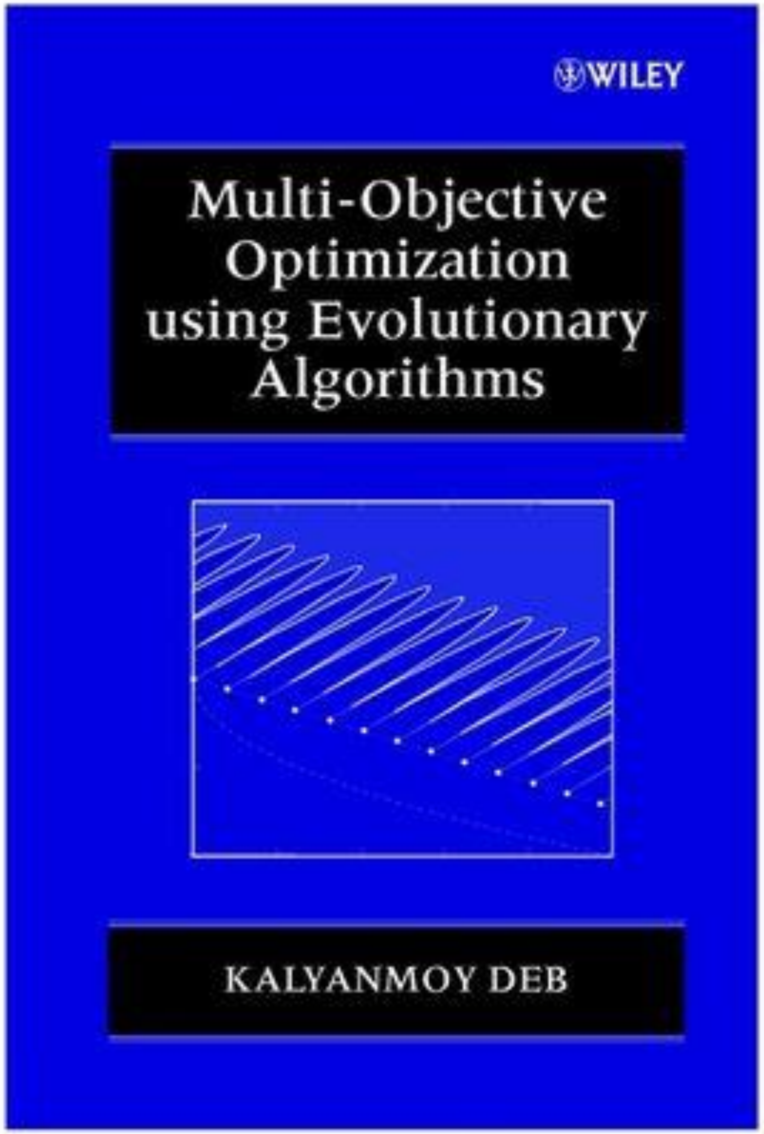
Intro to
Multi-Objective Optimisation



- Textbook
- Next slides: A selection of slides from the author

Multi-Objective Evolutionary Algorithms

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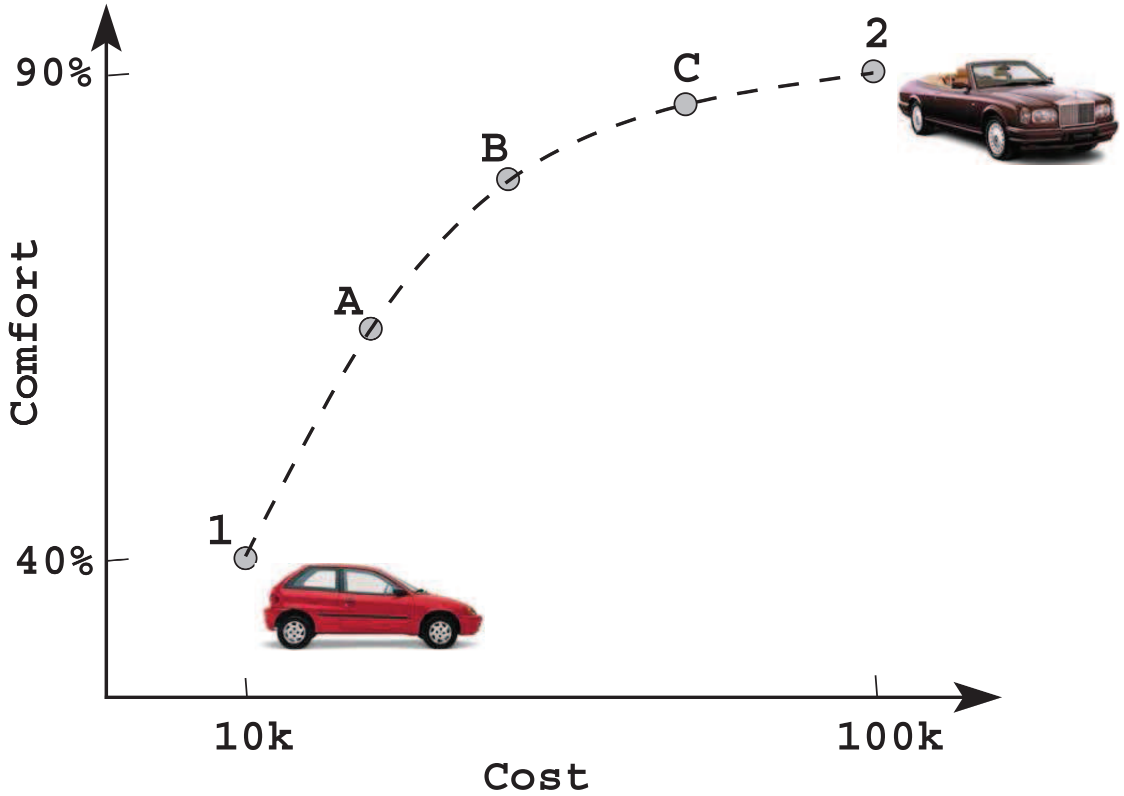
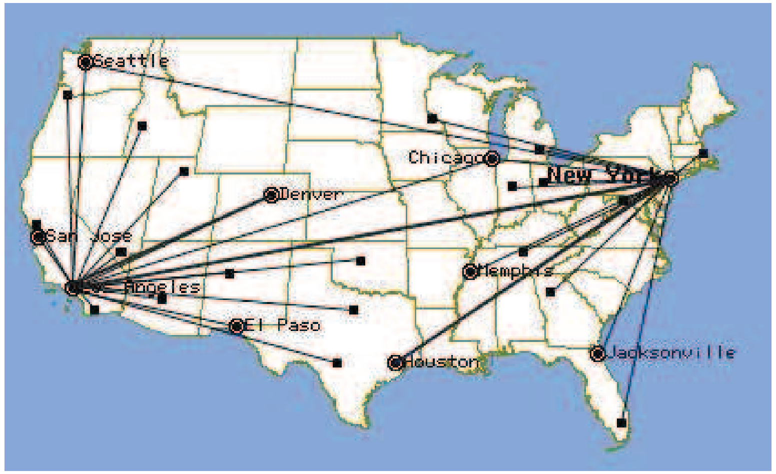
Multi-Objective Optimization

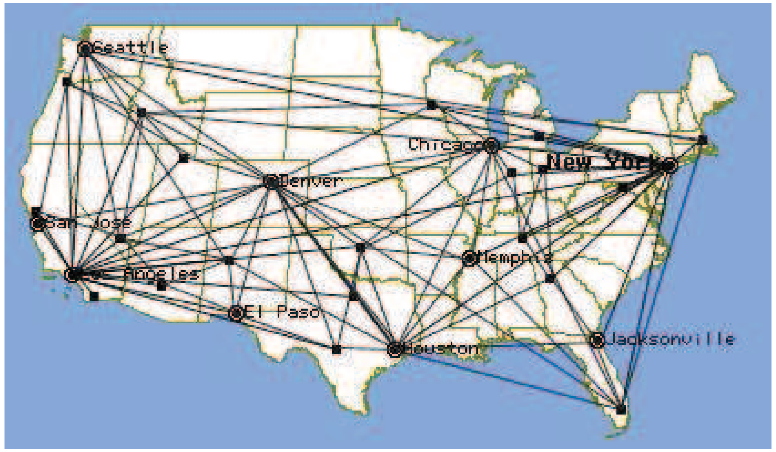
- We often face them



More Examples



A cheaper but inconvenient flight



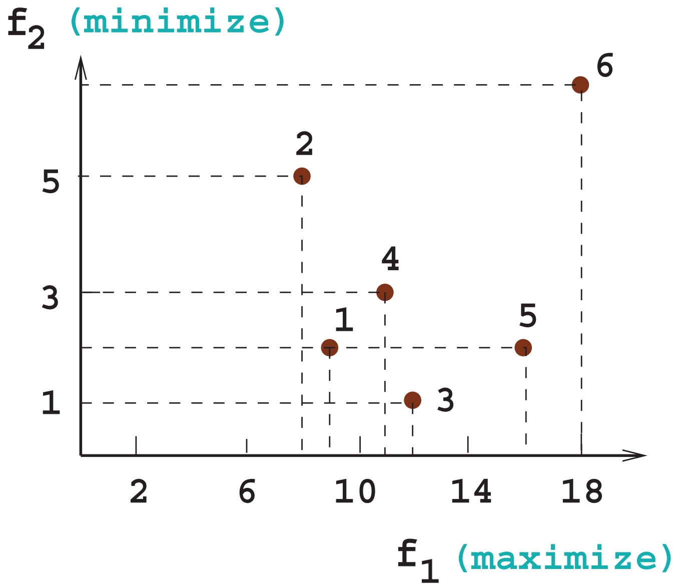
A convenient but expensive flight

Which Solutions are Optimal?

Domination:

$\mathbf{x}^{(1)}$ dominates $\mathbf{x}^{(2)}$ if

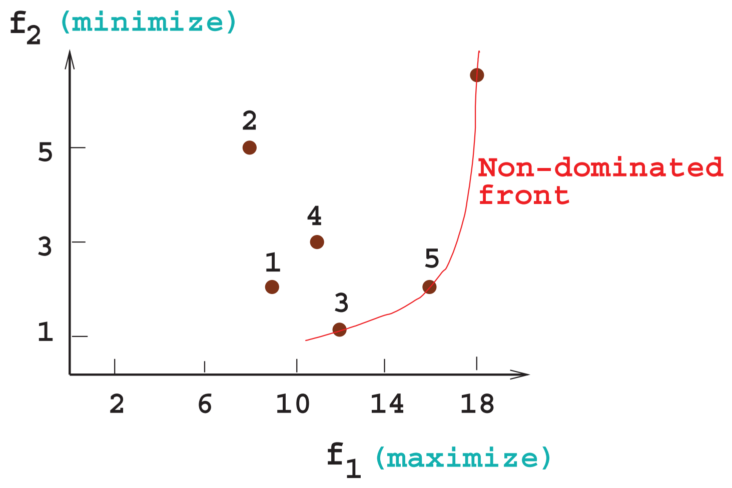
1. $\mathbf{x}^{(1)}$ is no worse than $\mathbf{x}^{(2)}$ in all objectives
2. $\mathbf{x}^{(1)}$ is strictly better than $\mathbf{x}^{(2)}$ in at least one objective



Pareto-Optimal Solutions

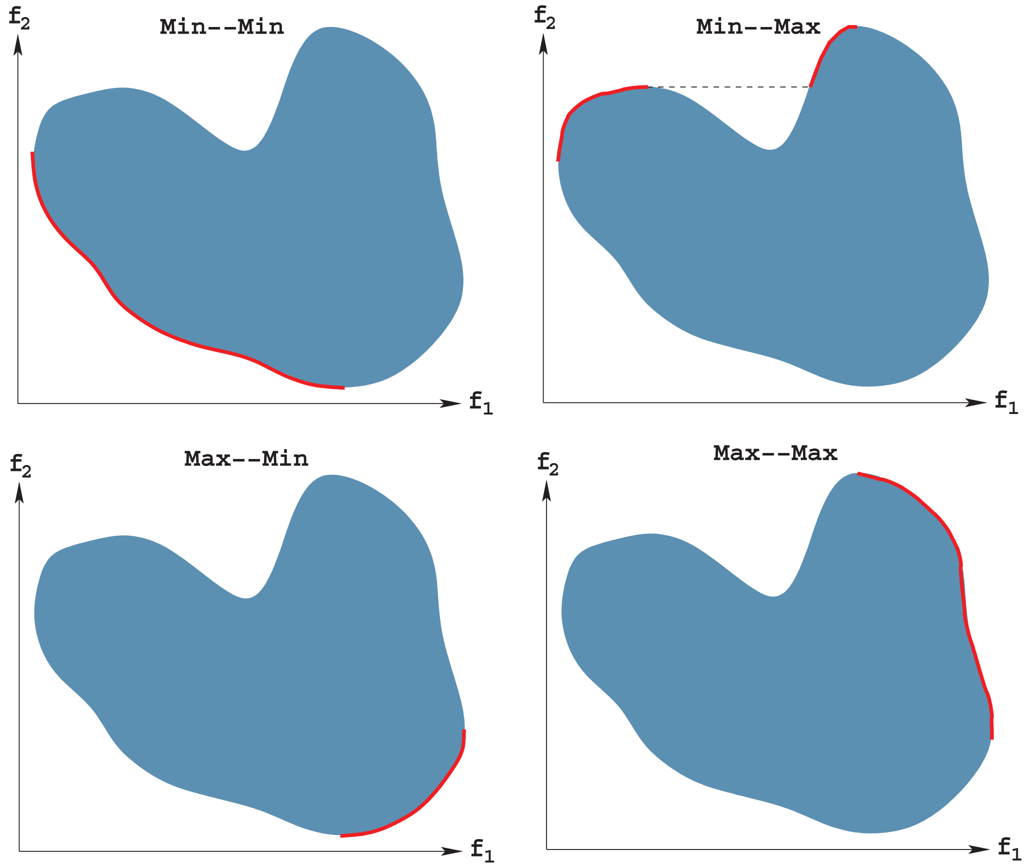
Non-dominated solutions: Among a set of solutions P, the non-dominated set of solutions P' are those that are not dominated by any member of the set P. $O(MN^2)$ algorithms exist.

Pareto-Optimal solutions: When $P = \mathcal{S}$, the resulting P' is Pareto-optimal set

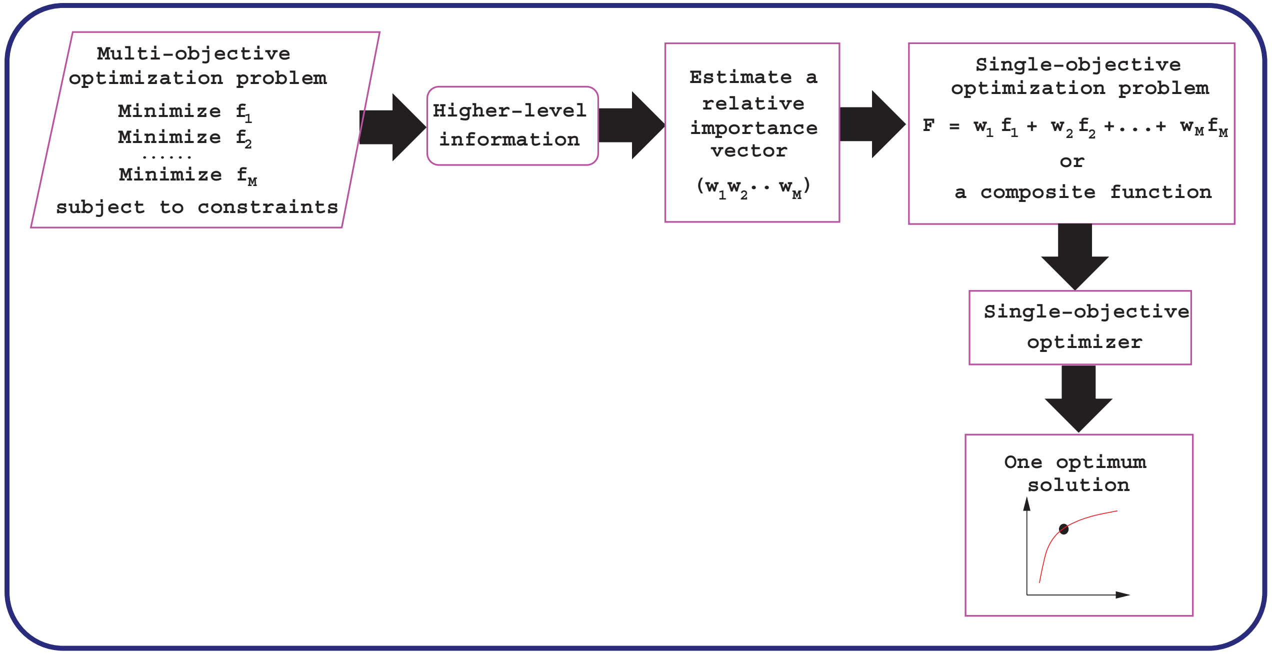


A number of solutions are optimal

Pareto-Optimal Fronts



Preference-Based Approach



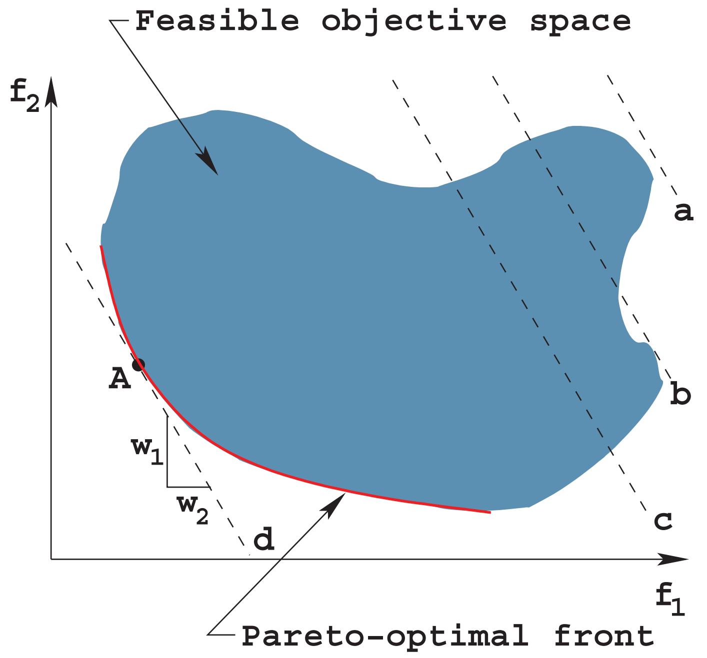
- Classical approaches follow it

Weighted Sum Method

- Construct a weighted sum of objectives and optimize

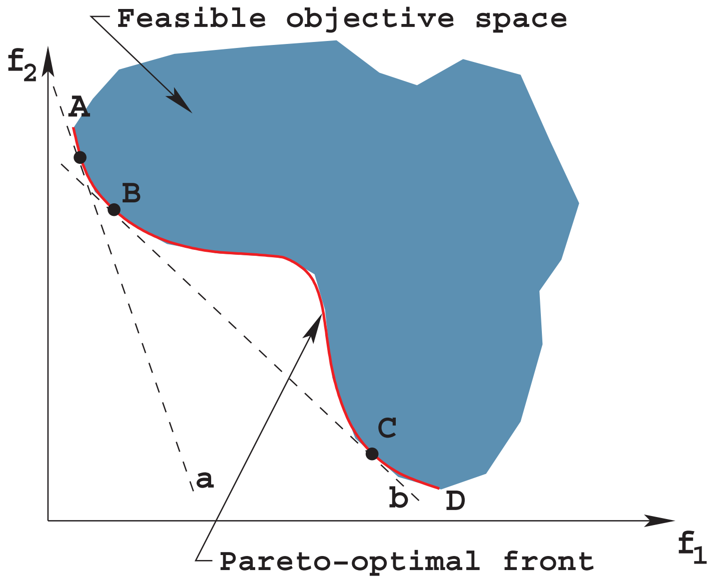
$$F(\mathbf{x}) = \sum_{m=1}^{M} w_m f_m(\mathbf{x}).$$

- User supplies weight vector $\mathbf{w}$

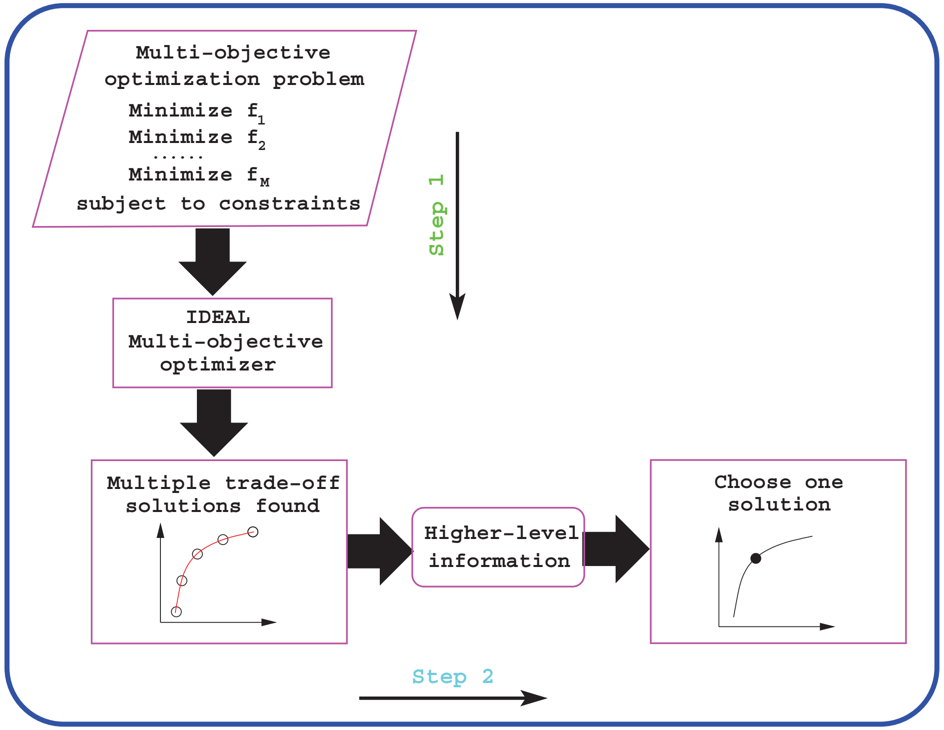


Difficulties with Weighted Sum Method

- Need to know **w**
- Non-uniformity in Pareto-optimal solutions
- Inability to find some Pareto-optimal solutions



Ideal Multi-Objective Optimization

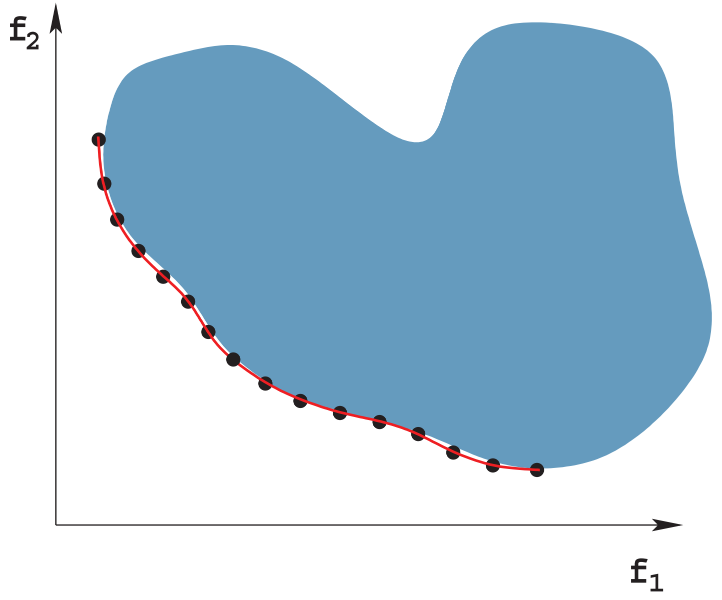


Step 1 Find a set of Pareto-optimal solutions

Step 2 Choose one from the set

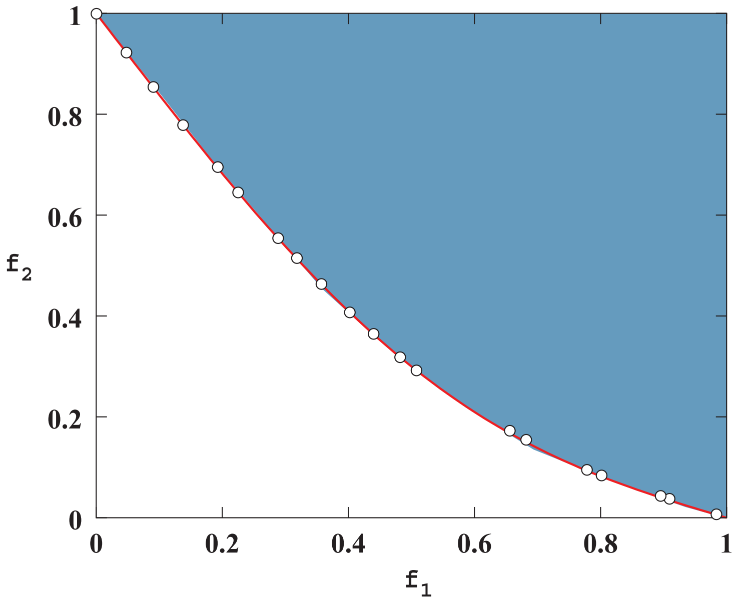
Two Goals in Ideal Multi-Objective Optimization

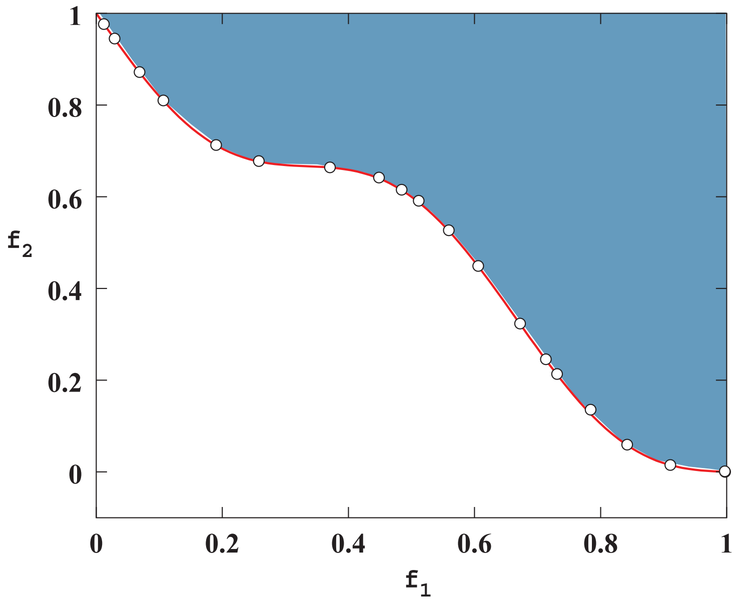
1. Converge on the Pareto-optimal front
2. Maintain as diverse a distribution as possible



Why Evolutionary?

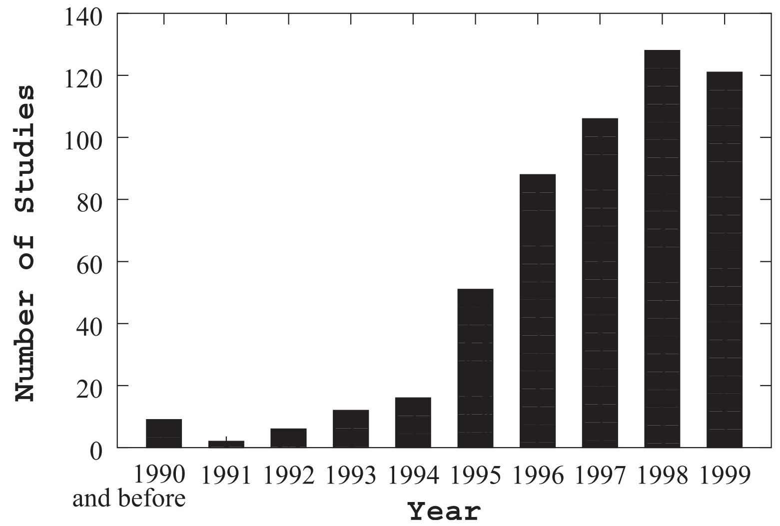
- Population approach suits well to find multiple solutions
- Niche-preservation methods can be exploited to find diverse solutions





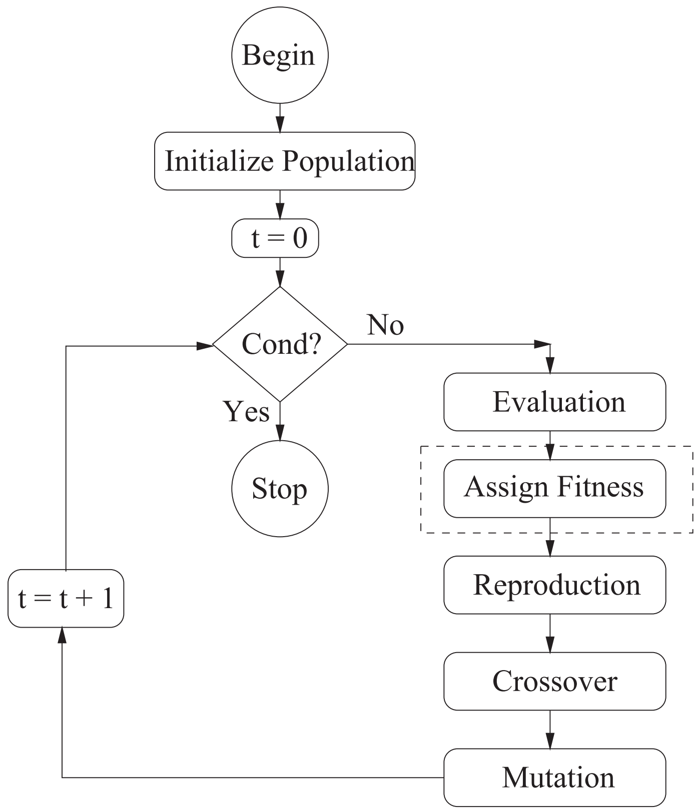
History of Multi-Objective Evolutionary Algorithms (MOEAs)

- Early penalty-based approaches
- VEGA (1984)
- Goldberg's suggestion (1989)
- MOGA, NSGA, NPGA (1993-95)
- Elitist MOEAs (SPEA, NSGA-II, PAES, MOMGA etc.) (1998 – Present)



What to Change in a Simple GA?

- Modify the fitness computation



Identifying the Non-dominated Set

Step 1 Set $i = 1$ and create an empty set P'.

Step 2 For a solution $j \in P$ (but $j \neq i$), check if solution j dominates solution i. If yes, go to Step 4.

Step 3 If more solutions are left in P, increment j by one and go to Step 2; otherwise, set $P' = P' \cup \{i\}$.

Step 4 Increment i by one. If $i \leq N$, go to Step 2; otherwise stop and declare P' as the non-dominated set.

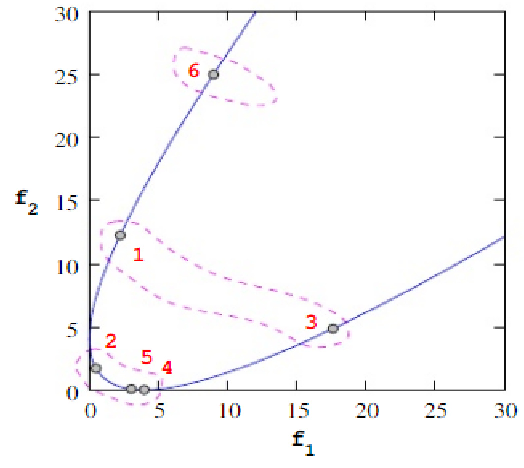
$O(MN^2)$ computational complexity

A Simple Non-dominated Sorting Algorithm

- Identify the best non-dominated set
- Discard them from population
- Identify the next-best non-dominated set
- Continue till all solutions are classified
- We discuss a $O(MN^2)$ algorithm later

Non-Dominated Sorting GA (NSGA)

x	f_1	f_2	Front	Fitness before	Fitness after
-1.50	2.25	12.25	2	3.00	3.00
0.70	0.49	1.69	1	6.00	6.00
4.20	17.64	4.84	2	3.00	3.00
2.00	4.00	0.00	1	6.00	3.43
1.75	3.06	0.06	1	6.00	3.43
-3.00	9.00	25.00	3	2.00	2.00



- Niching in *parameter* space
- Non-dominated solutions are emphasized
- Diversity among them is maintained

Applications of MOEAs

- Space-craft trajectory optimization
- Engineering component design
- Microwave absorber design
- Ground-water monitoring
- Extruder screw design
- Airline scheduling
- VLSI circuit design
- Other applications (refer Deb, 2001 and EMO-01 proceedings)