

What is a random variable or vector? Define the distributions of interest over a random *vector*.

Problem: We toss a coin 3 times, and define X = number of heads on first toss (0 or 1) and Y = total number of heads in all 3 tosses. What is the marginal distribution of Y ? What is the conditional distribution of Y given that $X = 0$? It is helpful to draw a table.

How do you find PDFs of RVs that are functions of other RVs whose PDFs are already known?

Problem: Find the probability density of the RV Y given that you know that $Y = X^2$ and $X \sim U(-1, 1)$.

What are the Expectation (mean) and Variance of a random variable? What is the Covariance of two RVs? What is the expectation of a function of a random variable?

Problem: Derive the properties of the sample mean RV for an n -sized sample.

Problem: Define $Y = g(U) = U^2$ where $U \sim U(0, 1)$. Find $E(Y)$ directly from the definition, and then using the shortcut for mean of a function of an RV.

Problem: Find the expectation and variance of a binomially distributed RV using an indicator variable approach.

Define conditional expectation. Derive the Law of Total Expectation.

Problem: In a game show a person is brought into a room with 3 unmarked doors. Door1 leads to safety in 3 minutes, Door2 circles back into the room in 5 minutes, and Door3 circles back into the room in 7 minutes. Each time the person arrives in the room he is disoriented before choosing a door. What is the expected time it takes him to reach safety?

Problem: Find the mean of the geometric distribution.

What is the Law of Large Numbers? Prove Markov's Inequality.