ASSIGNMENT 3: 1D FLOW IN CHANNELS.

Computational Methods in Heat and Mass Transfer

Submitted by-

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Problem Specification:

In this analysis, we have to examine a fluid flow inside a tube with known internal (shown in figure 1), external diameter and the tube length. The flow inlet conditions, including inlet velocity, inlet pressure and inlet temperatures are known.

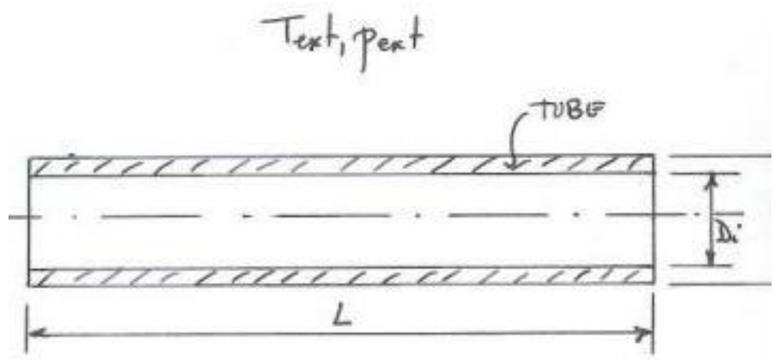

Figure 1: Representation of the proposed problem.

The physical parameters chosen for the analysis are given below:

Physical Data	
Fluid	Water
Tube material	Carbon steel C 1.5%
λ (W/mK)	36
L (m)	20
Di (mm)	20
Do (mm)	24
ε (smooth tube)	<0.0001
v_{in} (m/s)	1
P_{in} (bar)	2
T_{in} (^oC)	95
T_{ext} (^oC)	20
P_{ext} (bar)	1

Table 1: Physical parameters.

Methodology:

For this analysis, we need to discretize the tube materials and the fluid separately and interconnect both together to solve this conjugated problem as a whole. For the fluid part, we need to solve it using Step-by-step method and the solid tube has to be solved using conventional heat conduction numerical analysis.

We need to use proper indexing for the convenience of the code writing during the numerical solution. For that purpose, a small discretization has been demonstrated to explain how the discretization takes place. Here, the node which is being analyzed is called the P node and the index for this node is [i]. The left node of the analysis node is called the W node with index of [i-1], whole the right node of the analysis node is called the E node with index of [i+1]. Similarly, the left face of the of analysis node P is called ‘w’ face and the right face is called the ‘e’ face.

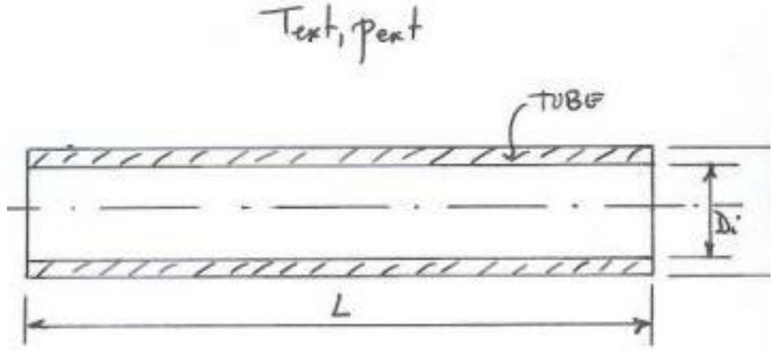

Figure 2: Tube characteristics.

1) Fluid solution:
For calculating the temperature inside the fluid flow, the following four equations were considered,
a) Mass conservation:

$$\dot{m}[i+1] = \dot{m}[i] = \dot{m}_{in}\ (1a)$$
$$\dot{m}[i+1] = \dot{m}_{in} = \rho[i+1]v[i+1]\ \ (1b)$$

b) Momentum equation:

$$\dot{m}_{in}\left[v[i+1] - v[i-1]\right] = p[i]S - p[i+1]S - \tau_{wi}P\Delta x\ (2)$$

c) Energy equation:

$$\dot{m}_{in}\, c_p\left[T[i+1] - T[i]\right] + \dot{m}_{in}\, (\frac{v^2[i+1]}{2} - \frac{v^2[i]}{2}) = q_w P\Delta x\ (3)$$
$$where,\ q_w = \alpha_i(T_t[i] - T_i)$$

d) State equation:

$$\rho_{out} = f(T_{out}, p_{out}) = f(T[i+1], p[i+1])$$

2) Tube solution:
For calculating the tube section, we need to consider conduction equation, where each side has convection heat transfer with the tube wall. However, one side has a forced convection fluid flow, and the exterior side of the wall incorporates the natural convection due to density change. Thus, the equation for solving the temperatures of the wall can be written as,

$$\dot{Q} = 0$$
$$Q_w - Q_e + Q_i - Q_{ext,i} = 0$$
$$\frac{-\lambda_t\cdot(T_t{}^p - T_t{}^w)}{d_{Pw}} S_t + \frac{\lambda_t\cdot\left(T_t{}^E - T_t{}^p\right)}{d_{PE}} S_t + \alpha_i(T_i - T_t{}^p)\, P_i\Delta x - \alpha_{ext,i}(T_t{}^p - T_{ext})\, P_o\Delta x = 0$$

$$a_P T_{tP} = a_E T_{tE} + a_W T_{tW} + b_P$$

$$a_E = \frac{\lambda_t S_t}{d_{PE}}$$

$$a_w = \frac{\lambda_t S_t}{d_{PW}}$$

$$a_p = a_E + a_w + \alpha_i\, P_i\Delta x + \alpha_{ext,i}\, P_o\Delta x$$

$$b_p = \alpha_i\ T_i P_i\Delta x + \alpha_{ext,i}\, T_{ext,i}\, P_o\Delta x$$

Code structure:

Steps to solve the set of equations:

1. **Input data**: We had two sets of input data: Physical data and Numerical data. Physical data included L, λ, D_0, D_i, tube material, T_{in}, T_{ext}, ε_r, v_{in}, P_{in} etc. and numerical data included N_{cv}, δ, $T_{initial}$ etc.
2. **Previous calculation and vector definition**: We discretized the domain by calculating x_{cv}, x_p, V_p, S, S_t, and then we defined matrices for the discretization coefficients.
3. **Initial temperature (t=0)**: In this stage, we defined the initial guess temperature T_t* for the solid tube.
4. **Evaluate internal fluid temperature (using step by step method).**
5. **External heat transfer coefficient:** In this step, we calculate the following values using the empirical data inputs.

$$Gr_i = \frac{\delta\beta_{ext}{\rho_{ext}}^2|T_t[i] - T_{ext}|D^3}{\mu^2}$$

$$Pr_i = \frac{\mu_{ext,i}c_{pext,i}}{\lambda_{ext,i}}$$

$$Ra_i = Gr_iPr_i$$

$$Nu_i = C{Ra_i}^nK$$

$$\alpha_{ext,i} = \frac{\lambda_{ext,i}Nu_i}{D_o}$$

6. **Solve the tube temperature:** In this step, using the data generated from the step-by-step method code, we calculated the temperature of the tubes again to current the guess temperature.
7. **Error check.**
8. **Final calculations and print results.**
9. **End.**

Results:

Using the methodology and the algorithm mentioned in the previous sections, the following results were obtained:

v_{in}=0.9974 m/s

P_{in}= 189440 Pa

T_{in}= 91.1°C

Re_i= 61686

Pr_i=1.94

f_i=0.0055

α_i=7732 W/m²K

α_o= 8.905 W/m²K

These values were verified using previous data. Note that these results were calculated considering constant thermophysical properties and other constant properties.