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ECON 4420

7 December 2020

Evaluating the Optimal Monetary Policy Model During the Financial Crisis

In this paper, I seek evaluate the nominal interest rates suggested by the optimal policy equilibrium for the period 2006 to 2019. I use the IS curve and the Fisher equation to solve for the optimal nominal interest rate. For simplicity, the model will be in steady-state so that there are no exogenous shocks. The model is as follows:

$$y_t = \theta_t - \sigma(r_t - \bar{r}) \tag{1}$$

$$= \theta_t - \sigma(i_t - E_{t-1}\pi_t - \bar{r}) \tag{2}$$

Given quarterly data on total factor productivity from John Fernald, I calculate the natural level of output using equation (3). Then I calculate equilibrium output and inflation for each period. Using the modified IS curve in equation (2), I solve for the nominal interest rate for each period given the values for $y_t,\ E_{t-1}\pi_t,\ \theta,\ \sigma,$ and $\bar{r}$.

$$y_t^n = z_t - \xi\mu^* \tag{3}$$

To solve for equilibrium output I first solve for y_t^n using the total factor productivity data from John Fernald. I detrend the data on the change in total factor productivity by regressing it against time and using the residuals as the change in each period. This process also sets the mean of total factor productivity, z_t, to zero. Because $\xi\mu^*$ does not affect the fluctuations of y_t^n, it will not impact the variation of other variables in the model. Therefore, we can use z_t in place of y_t^n so that in the model, y_t^n has a mean of zero.

Once values for y_t^n are established, we can calculate y_t, π_t and $E_{t-1}\pi_t$. The equations and initial parameter values are as follows:

$$\pi_t = \left(\frac{\lambda}{\kappa}\right)\bar{y} - \left(\frac{\kappa}{1+\frac{\kappa^2}{\lambda}}\right)y_t^n \tag{4}$$

$$y_t = \left(\frac{1}{1+\frac{\lambda}{\kappa^2}}\right)y_t^n \tag{5}$$

$$E_{t-1}\pi_t = \left(\frac{\lambda}{\kappa}\right)\bar{y} \tag{6}$$

ξ	μ	σ	ω	λ	κ	$\bar{r}$	θ^{ss}, y^{nss}	$\bar{y}$
2	0.2	1	0.67	1	0.25	0.02	~0	0.005

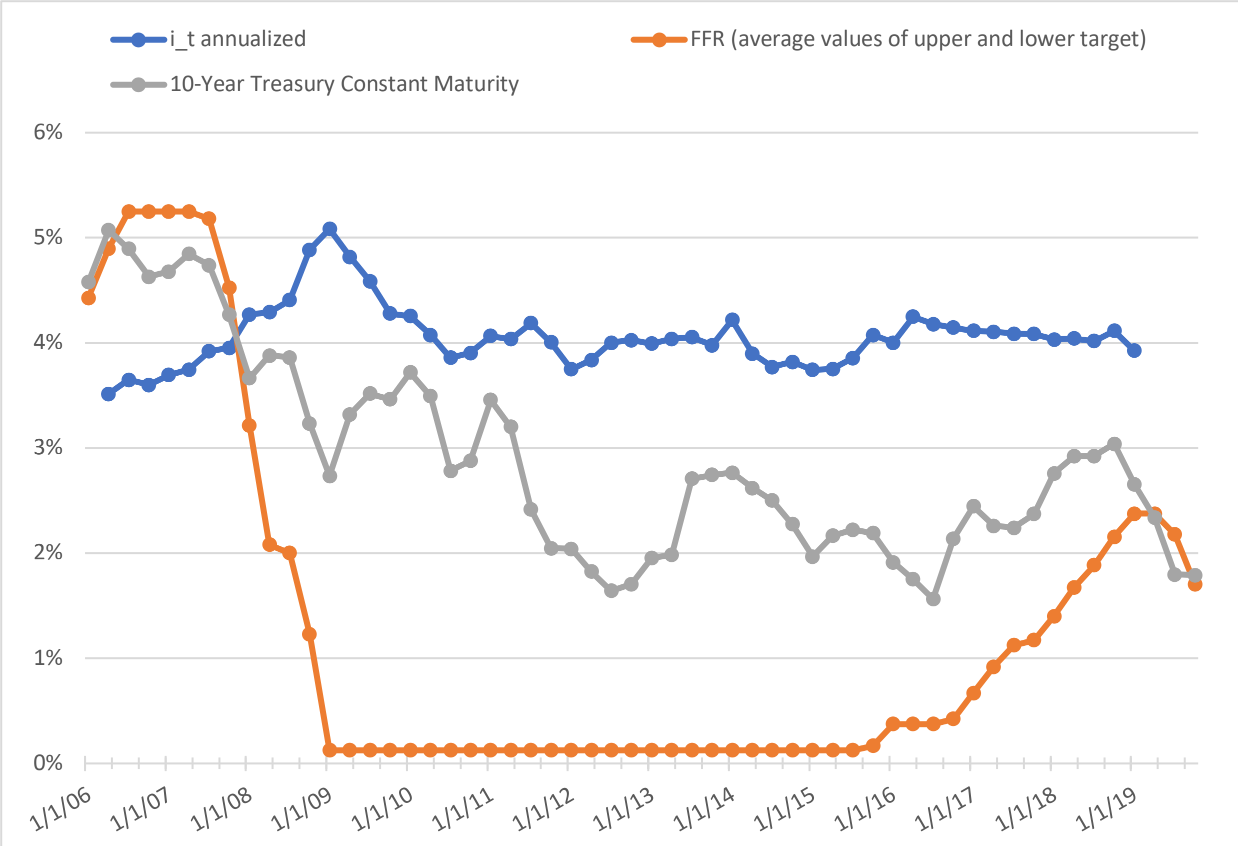
Parameters $\xi, \mu, \sigma, \omega, \lambda$ and $\bar{r}$ were all chosen as such because they were values we had worked with throughout the semester. $\kappa = (1-\omega)/\xi\omega$, so, the values chosen for ω and ξ will determine κ. $1-\omega$ represents the proportion of firms that reset prices each period and ξ is the elasticity of labor supply. μ is the markup charged by firms and $\mu^* = \ln(1+\mu)$. Also note that $\bar{r}$ and $\bar{y}$ are given as annual values here, but are converted to quarterly values in equation (7) for use in the model.

$$r^q = (1+r^a)^{0.25} - 1 \tag{7}$$

Once we have y_t for all periods, we can use the AS curve, along with the Fisher equation, to solve for i_t, the nominal interest rate. It's important to use the Fisher equation here because the central bank can actually control the nominal interest rate.

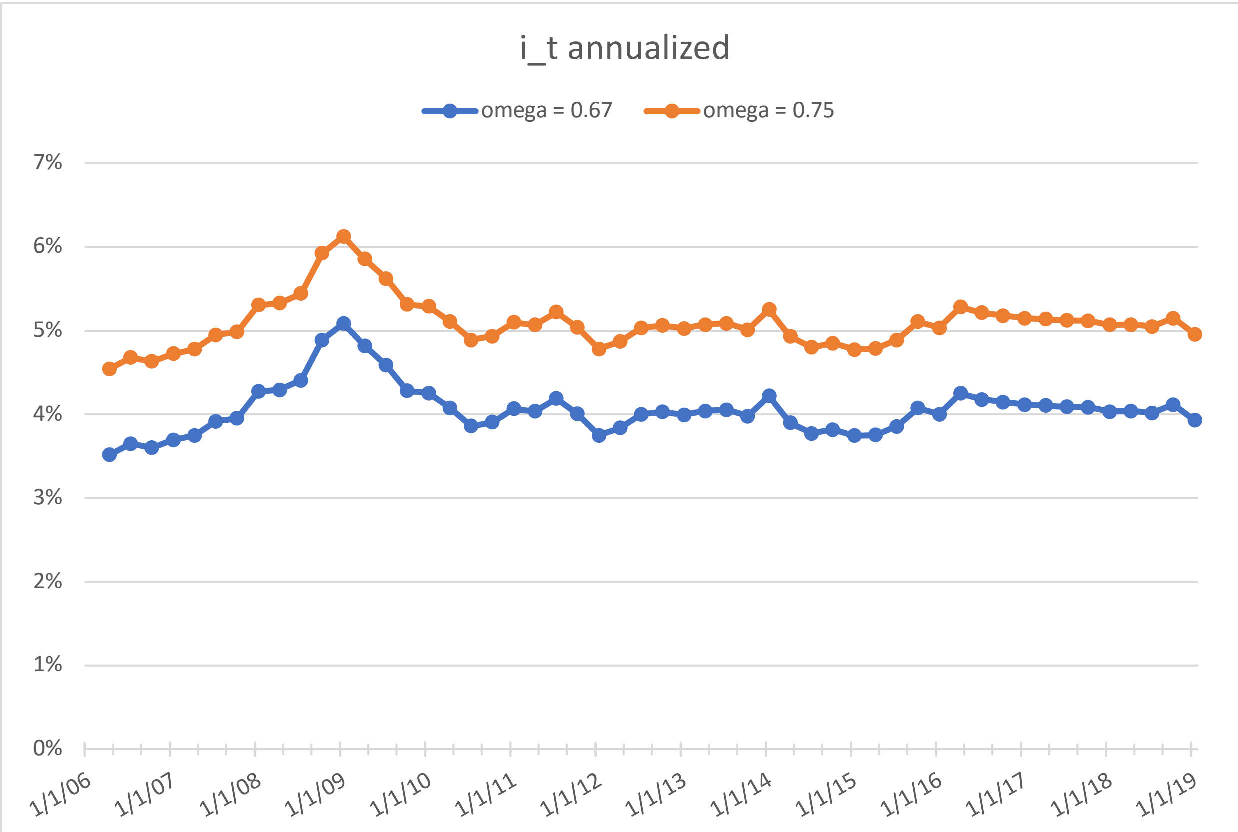
$$i_t = \frac{\theta_t - y_t}{\sigma} + E_{t-1}\pi_t + \bar{r} \tag{8}$$

Given all the parameter values above, the graph below illustrates all of the annualized values for i_t, along with the Federal Funds Rate and the 10-year treasury constant maturity rate for comparison.



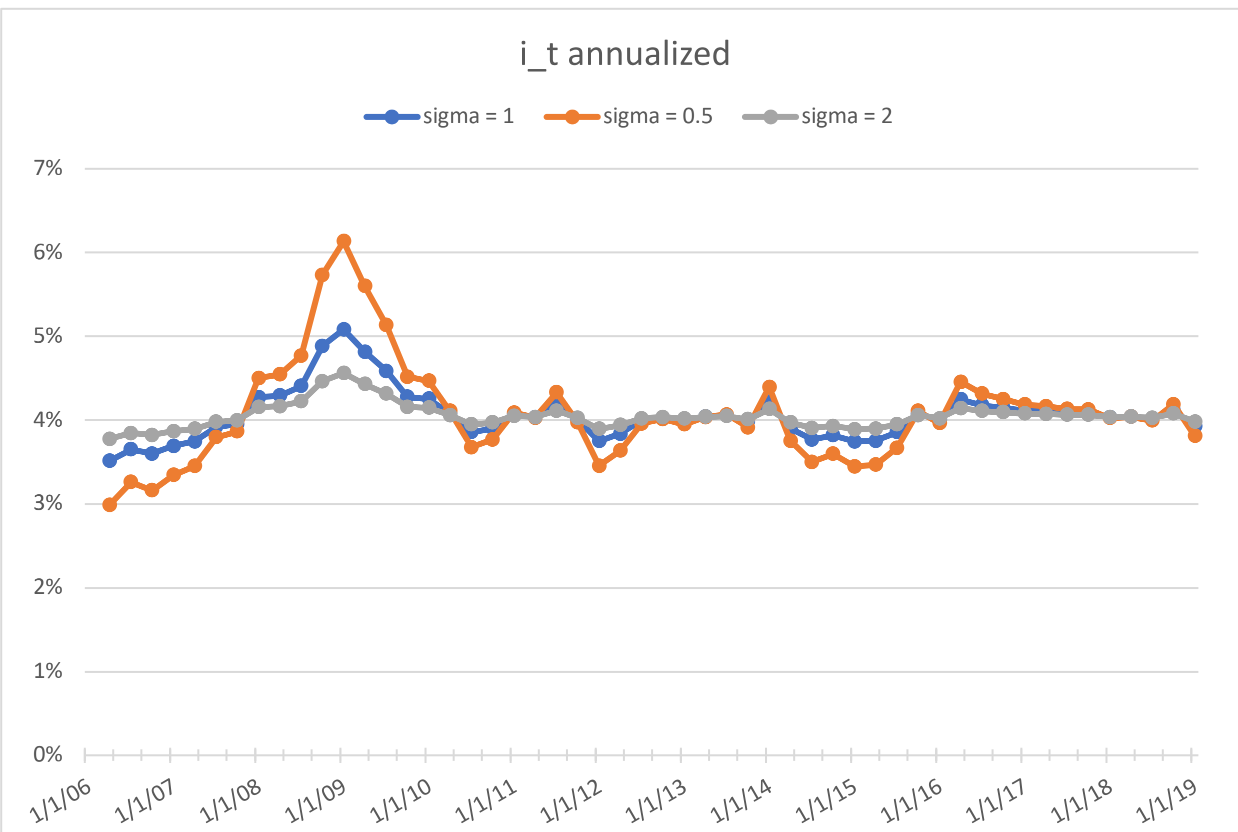
There is a clear dip in i_t in early 2009 as the economy is well into the recession following the financial crisis; however, it never goes below 3%. In comparison, the FFR goes to nearly zero by Q1 2009 and the 10-year treasury rate declines to around 2% by Q1 2012. In the model, i_t does not respond like the other rates because it is operating in steady-state. In steady-state there are no exogenous shocks; however, the real economy experienced massive exogenous shocks from 2007-09. Only once total factor productivity starts to decline does the decline of the real economy affect the model.

Next, I check the sensitivity of the model. I change the degree of price stickiness such that $\omega = 0.75$ and all other parameters remain the same. This means that more firms index their prices to past inflation; in other words, they cannot reset their prices each period based on actual inflation.



Here we see that when a larger percentage of firms cannot reset prices each period and must simply index to previous inflation, the optimal level for i_t increases. This makes sense mathematically, as a higher ω leads to a smaller κ. As κ decreases, expected inflation in the model increases, thus increasing i_t. From an economic standpoint, as ω increases fewer firms reset their prices every period; instead, they index to past inflation. Under discretionary policy, it is optimal for the central bank to exploit the AS curve by raising y_t with surprisingly high inflation. In this case, a higher proportion of firms are unable to react to unexpected inflation, so the central bank has more of an incentive to create positive unexpected inflation because y_t will increase more than normal. This also increases the nominal interest rate, which is seen in the graph above; the i_t curve shifts up when a higher percentage of firms cannot reset prices each period.

Another parameter to evaluate is σ. In the initial model, $\sigma = 1$, but here I set it to 0.5. σ describes the response of aggregate demand to the real interest rate. For a given r_t, σ determines how much aggregate demand will change based on the deviation of r_t from $\bar{r}$. When that relationship is flipped, the value of σ has the opposite affect on r_t (and thus also i_t) then it does on output. This relationship appears in equation (8) as σ is in the denominator. When σ increases, the i_t curve flattens, and when σ decreases, it expands. If sensitivity to changes in the interest rate is low, then a higher interest rate is required to match a given level of output then if responses to deviations in the interest rate were higher. These trends can be seen in the graph below.



Compared to $\sigma = 1$, $\sigma = 0.5$ produces a more volatile curve. In other words, the interest rate needs to increase or decrease by a greater magnitude to match the given level of output. On the other hand, $\sigma = 2$ produces a flatter curve where the interest rate doesn't need to change as much to match output. When $y_t^n \approx 0$, i_t is roughly the same for all values of σ. Any $\sigma < 1$ means that households and firms change their investment less than one-for-one to any deviations in r_t from $\bar{r}$, and vice versa for $\sigma > 1$. Compared to $\sigma = 1$, a given y_t requires a higher i_t if $\sigma < 1$ and $y_t^n < 0$ because households and firms will their investment less than one-for-one, so $i_t|\sigma = 1$ would not produce the equilibrium level of y_t. In other words, the central bank needs to set a higher i_t to discourage investment when $y_t^n < 0$ because households and firms will adjust their output at a lower magnitude than the gap in the real interest rate. The opposite is true if $y_t^n > 0$. If $\sigma > 1$, then households and firms react more than one-to-one to any deviation in r_t from $\bar{r}$. This means that when $y_t^n < 0$, the central bank does not need to raise i_t as much to match π_t, nor do they need to lower it as much when $y_t^n > 0$.

In all, the optimal policy model does not fare well during periods of unexpected events. The changes in the total factor productivity data do not fully describe the state of the economy. During the financial crisis, the massive macroeconomic shocks affected the economy more than anything. A future improvement of my model would be to not operate under the assumption that the economy is in steady-state and incorporate data on those shocks. However, during the periods of relative calm in the beginning and end of the period, the model made somewhat sensible recommendations. These conclusions reinforce the idea that unexpected circumstances can seriously diminish the usefulness of models, but they also have some utility during periods close to the steady-state.

References

Challe, Edouard, and Susan Emanuel. *Macroeconomic Fluctuations and Policies*. The MIT Press, 2019.

TFP Data. www.johnfernald.net/TFP.