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Short-term price density forecasts in the lean hog futures market

Andres Trujillo-Barrera[†,*], **Philip Garcia**[‡] **and Mindy L. Mallory**[‡]

[†]*Wageningen University, The Netherlands;* [‡]*University of Illinois at Urbana-Champaign, USA*

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Abstract

We estimate and evaluate *ex-ante* density forecasts of lean hog futures prices using two approaches: forward-looking techniques using options market data and time series models. Our findings indicate that risk-neutral and risk-adjusted forward-looking market techniques are better calibrated and have superior predictive accuracy than time series GARCH models based on historical data. Improvements to goodness of fit and accuracy of the forecasts obtained by the calibration from risk-neutral to real-world densities imply that short-term risk premiums may be present in the lean hog futures markets, and they most likely appear in periods of market turmoil.

Keywords: density forecast, commodities, price analysis

JEL classification: Q11, Q13

1. Introduction

In recent years agricultural commodity markets have experienced heightened price volatility which can have significant implications on production, marketing, and risk management practices (Wang, Fausti and Qasmi, 2012). In this environment, Isengildina, Irwin and Good (2004) indicate many individuals rely on agricultural forecasts in their decision making and that the value of accurate information can be substantial. Traditionally, agricultural forecasting procedures have been based on a mean-variance framework, but this may not fully characterise the nature of risk in volatile markets. Agricultural prices and returns may exhibit non-Gaussian and non-linear properties, particularly at higher frequencies (daily, weekly). Further, decision makers' preferences in these markets are likely to differ from quadratic functions (Deaton and Laroque, 1992; Koekebakker and Lien, 2004), making standard forecasts less meaningful.

*Corresponding author: E-mail: andres.trujillobarrera@wur.nl

In this context, estimating density forecasts, the future conditional probability distribution of prices, offers a thorough description of future uncertainty and provides decision makers with more information than standard point forecasts (Tay and Wallis, 2000; Timmermann, 2000). Density forecasting procedures are not new, but it was not until the 1990s that significant interest began to emerge. Pioneering work in agriculture by Bessler and Kling (1989), Kling and Bessler (1989), and Fackler and King (1990) used calibration tests on the entire density based on ideas by Dawid (1984). Diebold, Gunther and Tay (1998) helped to popularise the development of density evaluation, which has seen widespread applications in econometrics, risk management, asset pricing and portfolio selection (Amisano and Giacomini, 2007).

For agricultural commodity prices, the importance of density forecasts was identified as early as Bottum (1966) and Timm (1966), who argued for the development of probabilistic outlook forecasts similar in form to those used in weather forecasting. Density forecasts can be generated using historical data (Taylor, 2005); alternatively forward-looking density forecasts can be generated using option price information (Shackleton, Taylor and Yu, 2010). In the agricultural economics literature, several papers have provided density estimation procedures (i.e. Fackler and King, 1990; Sherrick, Garcia and Tirupattur, 1996). Despite these advances, the use of density forecasts for agricultural commodity prices has been limited, in part perhaps because of the investment required to implement them.

Shackleton, Taylor and Yu (2010) and Ivanova and Puigvert Gutiérrez (2014), were among the first to provide *ex-ante* density forecasts. We follow their methods closely to estimate forecast densities for lean hog futures prices employing two general procedures: based on historical data using GARCH models, and based on forward-looking information content of options prices. The latter allows us to generate risk-neutral and risk-adjusted densities. We evaluate the out-of-sample forecast performance using calibration (i.e. whether the forecast distribution is correctly specified) and forecast accuracy. To assess calibration, we use the probability integral transform (*PIT*) (Diebold, Gunther and Tay, 1998) and the Berkowitz test (Berkowitz, 2001). To assess forecast accuracy, we measure the out-of-sample log likelihood (OLL) based on the Kullback–Leibler information criteria (Bao, Lee and Saltoğlu, 2007). The analysis is performed with a 2-week forecasting horizon using daily settlement futures and options prices for lean hogs from February 2002 to February 2017. The short-term nature of the forecast horizon reflects the observation that most market activity occurs in the nearby contract as hedgers and participants offset their market positions.

We focus on the hog market because considerable predictive research already exists. For instance, the reliability of hog futures prices to predict subsequent cash prices has been a traditional area of market research (Colino, Irwin and Garcia, 2011). More recently, researchers have begun to investigate the degree to which the implied volatilities from the hog options reflect subsequent realised volatility. While recent evidence is mixed, empirical findings using monthly and bimonthly horizons (e.g. 2 and 4 months) suggest that futures prices provide long-run unbiased forecasts, but short-run inefficiencies in forecasting may exist (McKenzie and Holt, 2002; Frank and Garcia, 2009). In terms of the options market, Szakmary *et al.* (2003) and Egelkraut and Garcia (2006) identify biases in implied forward volatility forecasts of subsequent realised volatility. Historical volatilities also add information to the market generated implied volatilities in predicting realised volatility, implying options prices do not contain all available information or may not account adequately for risk. Similarly, McKenzie, Thomsen and Phelan (2007) show that long hog straddle positions executed on Hogs and Pigs Report days are profitable if transaction costs are low enough.

In contrast, Urcola and Irwin (2010) analyse market efficiency of lean hog options contracts, looking at several trading strategies such as options straddles and strangles. They find that returns on options are often small, and even large returns are not statistically significant. They conclude that returns are not sufficiently large enough to allow for consistent speculative profits for off-floor traders. Hence while the bulk of the evidence suggests that short-term biases in market prices and their volatilities may exist, developing strategies to take advantage of them is challenging. Regardless, hog price density forecasts can play an important role in understanding spreads and can assist traders in managing daily risk. Packers and retailers interested in dynamic pricing and optimal inventories are able to take advantage of information to develop pricing strategies or protect themselves from added price volatility, skewness, and kurtosis. Accurate density estimates also can help commodity exchanges to determine appropriate margins and daily price limits and permit a clearer understanding of the existence and magnitude of volatility and tail risk premiums. Higher price volatility can reduce the effectiveness of traditional risk management tools, which may not be able to address the added variance and tail risk directly. To date no research has investigated the ability to generate short-term forecast densities in the hog market using either historical information or market generated forecasts.

Our findings indicate that risk-neutral and risk-adjusted forward-looking market techniques are better calibrated and have superior predictive accuracy than time series GARCH models based on historical data. These findings are consistent with Egelkraut and Garcia (2006), Egelkraut, Garcia and Sherrick (2007), and Brittain, Garcia and Irwin (2011) in agricultural commodity markets, and work by Szakmary *et al.* (2003) in a wide variety of markets, where implied volatility from options markets outperform historical-based forecasts. Forecast improvements obtained by adjusting risk-neutral to real-world densities suggest that short-term risk premiums may exist in the lean hog markets. These findings are consistent with Szakmary *et al.* (2003), Egelkraut and Garcia (2006) and McKenzie, Thomsen and Phelan (2007).

2. Density forecast measures

Density forecasts are derived using two approaches, historical and implied (Taylor, 2005; Liu *et al.*, 2007; Høg and Tsiaras, 2011). We obtain historical densities by estimating GARCH models and characterising their forecast errors by alternate functional forms. Implied densities rely on extracting the information in option prices, which should reflect aggregate risk-neutral market expectations of the underlying commodity when the option contract expires.

2.1. Historical density estimation

GARCH models of daily returns of lean hog futures prices are simulated to provide historical densities.[1] For the in-sample specification of the mean and variance dynamics, we use the GJR-GARCH specification (Glosten, Jagannathan and Runkle, 1993), which permits asymmetric volatility response to news and has been shown to function well with daily futures market data. The model is:

$$r_t = \mu_0 + \sum_{i=1}^{m} \delta_i r_{t-i} + \varepsilon_t \tag{1}$$

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 I(\varepsilon_{t-1} < 0) + \beta h_{t-1} \tag{2}$$

$$\varepsilon_t = \sqrt{h_t \eta_t}, \quad \eta_t \sim i.\, i.\, d\; D(0, 1). \tag{3}$$

In equation (1), $r_t = \log(P_t) - \log(P_{t-1})$ corresponds to a logarithmic return of price P_t, which is equal to an intercept, μ_0, plus the weighted sum of m lagged returns $(\delta_i r_{t-i})$ and the error term ε_t. In equation (2), the conditional variance of price returns h_t corresponds to the lagged squared error ε_{t-1}^2 plus the lagged conditional variance h_{t-1}. An asymmetric response emerges through an indicator function $(I(\varepsilon_{t-1} < 0))$ that takes a value of 1 if $(\varepsilon_{t-1} < 0)$ and 0 otherwise. Equation (3) describes the error term as the product of the conditional standard deviation $\sqrt{h_t}$ and a random error η_t, where $D(0, 1)$ is a zero mean unit variance probability distribution.

In addition to the standard normal distribution (N) identified in (3), we examine other error distributions, including: a standardised t distribution (STD), a generalised error distribution (GED), and a normal inverse Gaussian (NIG). These last distributions allow for skewness and kurtosis in the density forecasts. To assess the models, we use AIC and BIC criteria and perform conventional misspecification tests on the standardised residuals. Tests suggest that AR(5)-GJR-GARCH(1,1) is a robust specification during the period of analysis.[2] Although GARCH lag structures varied modestly in several estimations, we maintain the GJR-GARCH(1,1) for model consistency. In support, Bao, Lee and Saltoğlu (2007) find that density forecast accuracy

1 GARCH models are estimated in R using rugarch 1.3.6. (Ghalanos, 2015). This package and its vignettes offer detailed explanations of the univariate garch models, and the different underlying distributions used.

2 Misspecification and diagnostic tests are available from the authors.

depends more on the choice of the distribution than on the lags of the conditional variance.

2.2. Historical density simulation

The AR(5)-GJR-GARCH-based forecast densities are constructed using a procedure suggested by Taylor (2005). First, for a particular date t, we use the five most recent years of daily logarithmic returns to estimate the parameters of the model. By drawing a random number from distribution D and multiplying it by $\sqrt{h_t}$ a set of new residuals ε_t are generated (in equation (3)). These are used to update the conditional variance (in equation 2) and then to calculate simulated returns (in equation (1)). This is repeated from time t until the forecast horizon $t + n$. Simulated returns are compounded and multiplied by the price at time t to generate the forecast, $P_{t+n} = P_t \times \exp(r_{t+1} + r_{t+2} + \ldots + r_{t+n})$. To create the density forecast we repeat this process 100,000 times. To produce a smooth distribution we apply a Gaussian kernel density with bandwidth equal to $0.9N^{-1/5}\sigma$, where σ is the standard deviation of the forecast value and N the number of simulations.[3] Here, n corresponds to ten business days, resulting in a density prediction of the final price of the futures/options contract 2 weeks before expiration.

2.3. Risk-neutral densities

An option contract gives the holder the right to make a transaction on an underlying asset at a later date for a specific price (strike price). The owner of a call option has the right but not the obligation to buy the underlying asset, while the owner of a put option has the right but not the obligation to sell. The price of a European call option is equal to the present value of its final payoffs, which allows us to write:

$$\begin{aligned} c(X) &= \mathrm{e}^{-r_f T} E^Q [S_T - X] \\ &= \mathrm{e}^{-r_f T} \int_0^\infty \max(x - X,\ 0) f_Q(x) dx \end{aligned} \tag{4}$$

where X is the strike price, $c(X)$ is the price of the call option, S_T is the price of the underlying contract, r_f is the free risk rate, T is the time to maturity, f_Q is the risk-neutral probability distribution and E^Q is an expectation. This holds for a complete set of exercise prices $X \geq 0$, and $\int_0^\infty f_Q(x)dx = 1$. Breeden and Litzenberger (1978) show that the existence and uniqueness of a risk-neutral density f_Q can be inferred from European call prices $c(X)$ from contracts with continuous strike prices that lack of arbitrage opportunities. The risk-neutral density (RND) is given by

3 Results before and after smoothing the density forecast with a Gaussian kernel density are virtually identical.

$$f(x) = e^{r_f T} \frac{\partial^2 C}{\partial x^2}. \tag{5}$$

The estimation task is to find a risk-neutral density $f_Q(x)$ that provides a reasonable approximation to observed market prices.

Several methods are available to recover risk-neutral densities from option prices (see Jackwerth, 2000; Taylor, 2005). Examples in the agricultural economics literature include Fackler and King (1990), Sherrick, Garcia and Tirupattur (1996) and Egelkraut, Garcia and Sherrick (2007). We follow a similar approach but use the Generalised Beta distribution of the second kind (GB2) as the implied density (Liu *et al.*, 2007; Høg and Tsiaras, 2011).[4] In addition to its flexibility, Taylor (2005) recommends the GB2 because of its desirable characteristics, including: the tails are fat relative to lognormal distributions, estimates are not sensitive to the discreteness in options prices, closed-form expressions for the probability density and cumulative distribution functions exist, and solutions and calibrations are relatively easy to obtain.

The GB2 density has four parameters $\Theta = (a, b, p, q)$, allowing for the estimation of the mean, variance, skewness and kurtosis. Its probability distribution function is defined as

$$f_{GB2}(x|a, b, p, q) = \frac{a}{b^a B(p, q)} \frac{x^{ap-1}}{\left[1 + \left(\frac{x}{b}\right)^{p+q}\right]}, \quad x > 0 \tag{6}$$

with $B(p, q) = \Gamma(p)\Gamma(q)/\Gamma(p, q)$ where Γ is the gamma function. The density is risk-neutral when the underlying futures price F is

$$F = E^Q[S_T] = \frac{bB\left(p + \frac{1}{a}, q - \frac{1}{a}\right)}{B(p, q)}. \tag{7}$$

To obtain the risk-neutral density, we find the parameter vector Θ that minimises the sum of the squared differences between the observed market and theoretical option prices (Ji and Brorsen, 2009):

$$\min h(\Theta) = \sum_{i=1}^{n} (C_{\text{market}}(x_i) - C(x_i|\Theta))^2 + (P_{\text{market}}(x_i) - P(x_i|\Theta))^2 \tag{8}$$

where $C_{\text{market}}(x_i)$ and $P_{\text{market}}(x_i)$ are the market call and put prices at the strikes x_i, and $C(x_i|\Theta)$ and $P(x_i|\Theta)$ are the theoretical prices at the strikes x_i. Theoretical prices are generated by replacing f_q by $f_{GB2}(x|a, b, p, q)$ in

4 Sherrick, Garcia and Tirupattur (1996) use the Burr-3 distribution, a special case of the GB2 when $q = 1$.

equation (4) and by applying the constraint in equation (7). The European call option price then is given by

$$c = (X|\Theta) = e^{-r_f T}\int_x^{\infty}(x - X)f_{GB2}(x|\Theta)dx$$

$$Fe^{-r_f T}\left[1 - F_{GB2}\left(x\middle|a, b, p + \frac{1}{a}, q - \frac{1}{a}\right)\right] - Xe^{-r_f T}[1 - F_{GB2}(x|\Theta)] \tag{9}$$

where F_{GB2} is the cumulative distribution function of the GB2 density. The functional form in equation (9) is used in the minimisation problem, in equation (8).

2.4. Real-world densities

A fundamental idea in pricing theory is that the value of an asset is equal to its expected discounted cash flows. Risk-neutral densities assume that risk is irrelevant for pricing future cash flows. But if an investor is risk-averse, then risk-neutral implied densities from option contracts may provide inaccurate forecasts. A possible approach is to adjust forecast densities so that they incorporate risk, assuming a particular utility function and a degree of risk aversion (Liu *et al.*, 2007). However, such transformations are problematic since the estimated stochastic discount factors often do not match expected risk-averse behaviour (Høg and Tsiaras, 2011). For agricultural commodity futures the situation may be more complex because it has been difficult to establish whether a risk premium exists. For instance, Frank and Garcia (2009) find no evidence of time-varying risk premium on corn, soybean meal and lean hogs at 2 and 4 month horizons. However, Egelkraut and Garcia (2006) find some evidence that the lean hog markets may demand a premium for bearing risk when volatility becomes less predictable at distant horizons.

An alternative approach that avoids these difficulties is to obtain real-world densities via statistical calibration of the risk-neutral densities. The procedure searches for a function that adjusts or calibrates the risk-neutral density to the observed data without making assumptions about the utility functions that represent risk preferences (de Vincent-Humphreys and Noss, 2012). The process described by Fackler and King (1990) follows the idea of a probability integral transform (*PIT*) (Rosenblatt, 1952), and focuses on improving density estimates against the assumption that random variables defined by their cumulative distribution functions (cdf) are uniformly distributed.

Let $f(y)$ and $F(y)$ denote the probability and cumulative distribution function of a random variable y where X corresponds to the actual realisation of the random variable. The probability integral transform (*PIT*) is given by

$$PIT = \int_{-\infty}^{X} f(y)dy \equiv F(X). \tag{10}$$

Rosenblatt (1952) shows that if X is a continuous random variable, the PIT is a uniform random variable in the interval [0,1]. Therefore if the actual observed data X has been generated from function $f(y)$ its PIT is distributed $U(0, 1)$.

Operationally, we obtain risk-neutral densities, and link risk-neutral and real-world densities through a calibration function. Let $f_Q(v)$ and $F_Q(v)$ be the risk-neutral density and the risk-neutral cumulative distribution function of the underlying asset v at time T, v_t. Denote $G(u)$ as the real-world cumulative distribution of random variable $U = F_Q(V_t)$, and $g(u)$ its first derivative. Then the real-world cumulative distribution function $F_p(v)$ and probability density function $f_p(v)$ of v_t are:

$$F_p(v) = G(F_Q(v)) \tag{11}$$

$$f_p(v) = \frac{dF_p(v)}{dv} = \frac{dG(F_Q(v))}{dv} = \frac{dG}{dF_Q}\frac{dF_Q}{dv} = g(F_Q(v))f_q(v). \tag{12}$$

In effect, the real-world density is generated through point-wise multiplication of the calibration function and the option-implied risk-neutral density. Here, to estimate the real-world densities, we first use the risk-neutral densities obtained from the solution to equation (8), Θ_{GB2}, and then use two calibration functions, a parametric function based on the Beta distribution, and a non-parametric function based on a kernel density from the empirical distribution.

For the parametric calibration, we follow Fackler and King (1990) who used the Beta distribution. This distribution has a number of advantages (Taylor, 2005), it is parsimonious as it only depends on two parameters (α, β). It nests the uniform distribution and allows for the risk-neutral and real-world measures to be identical without imposing transformations (when $\alpha = 1, \beta = 1$), but still it has a flexible shape that allows with simple transformations, shifts in mean, variance, and skewness. Further, the parameters can be estimated by maximum likelihood. If $G(.)$ is the cumulative distribution function of the Beta distribution defined as:

$$G(u|\alpha, B) = \frac{1}{B(\alpha, \beta)} = \int_0^u s^{a-1}(1-s)^{\beta-1}ds \tag{13}$$

where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(B)}{\Gamma(\alpha+\beta)}$. Then calibration density $g(.)$ is its derivative:

$$g(u|\alpha, \beta) = \frac{u^{\alpha-1}(1-u)^{\beta-1}}{B(\alpha, \beta)}. \tag{14}$$

The parameters of the Beta density α and β are estimated by maximising the following log-likelihood function:

$$\log(L(v_1, v_2, \ldots, v_t)) = \sum_{t=1}^{n} \log\left(f_p(v_t|\Theta_{GB2}, \alpha, \beta)\right). \tag{15}$$

This selects the parameter values α and β that produce the beta distribution that are most likely to have produced the actual data. Therefore, estimating the real-world densities from the set of risk-neutral densities consist of estimating the parameters α and β of the beta distribution.

Fackler and King (1990) acknowledge that a parametric approach may not represent the calibration function well. Therefore, we also employ a non-parametric procedure. Following Shackleton, Taylor and Yu (2010) (for more detail, see their Appendix A), and Ivanova and Puigvert Gutiérrez (2014), we calibrate the *PITS* of the risk-neutral distribution with a Gaussian kernel smoothing function to obtain the real-world distribution. More specifically, we construct the real-world density using the past realisations of $u_t = F_{Q,t}(v_t)$, that denote the cumulative risk-neutral probability at the observed futures prices (*PITS*). Then let $\phi(.)$ and $\Phi(.)$ be the pdf and cdf of a standard normal distribution. The series of u_t is transformed into a new series $z_t = \Phi^{-1}(u_t)$. The series u_t is smoothed via a Gaussian kernel density h(z), obtained with empirical distribution $H(z)$, with bandwidth $0.9N^{-1/5}\sigma$. The empirical calibration of series u_t is $G(u) = H(\Phi^{-1}(u_t)) = H(z_t)$. From equation (11), we get that the calibrated RWD CDF:

$$F_p(v) = G(F_Q(v)) = G(u) = H(z) \tag{16a}$$

and the resulting real-world pdf of the forecast (analogous to equation (12)) is:

$$f_p(v) = \frac{d}{dv}H(z) = \frac{dz}{dv}\frac{dH(z)}{dz} = \frac{du}{dv}\frac{dz}{du}\,h(z) = f_q(v)\,\frac{h(z)}{\phi(z)}. \tag{16b}$$

3. Density forecast evaluation

Density forecasts are assessed using two criteria: goodness of fit which evaluates whether the density forecast correctly matches the actual realisation of the underlying random variable, and forecast accuracy.

3.1. Goodness of fit

To measure goodness of fit performance, Diebold, Gunther and Tay (1998) popularised the probability integral transform (*PIT*) developed by Rosenblatt (1952). Plugging into equation (10), the probability and cumulative density forecast function $f(y_t)$ and $F(y_t)$ of a random variable y_t at time t, and representing Y_{t+n} as the actual realisation of the random variable at the forecast horizon yields:

$$PIT_t = \int_{-\infty}^{t+n} f(y_t)\, dy \equiv F(Y_{t+n}). \tag{17}$$

In this case, the *PIT* is the value that the predictive cdf attains at the observation Y_{t+n}. Although the true random variable distribution is often unobservable, Diebold, Gunther and Tay (1998) exploit the fact that when the forecast density equals the true density, then the *PIT* follows a $U(0, 1)$ and is independent and identically distributed (iid). Hence, evaluation of whether the conditional forecast density matches the true conditional density can be performed by a test of the joint hypothesis of independence and uniformity of the sequence of *PIT*s.

Berkowitz (2001) suggests transforming the *PIT* distribution from Uniform to Normal. Several advantages are obtained by this transformation (Mitchell and Wallis, 2011). For instance, there are more tests available for normality than uniformity; it is easier to test autocorrelation, and the normal likelihood can be used to construct likelihood ratio tests. Let φ^{-1} denote the inverse of the standard normal distribution, Berkowitz (2001) shows that for any sequence of *PIT* that is iid $U(0,1)$, it follows that $z_t = \varphi^{-1}(PIT_t)$ is an iid $N(0,1)$. Under this transformation, independence and normality are tested jointly by using a likelihood ratio test on the following model:

$$z_t - \mu = \varrho(z_{t-1} - \mu) + \varepsilon_t,\ \ \varepsilon_t \sim i.\, i.\, d\ N(0, \sigma^2). \tag{18}$$

The null hypothesis is that z_t follows an uncorrelated Gaussian process with zero mean unit variance against an AR(1) with unspecified mean and variance. Therefore, the likelihood ratio can be set as $LR_3 = -2(L(0, 1, 0) - L(\mu, \hat{\sigma}^2, \hat{\varrho}))$, which follows a χ^2 distribution with three degrees of freedom.

3.2. Out-of-sample forecast comparisons

The preceding methods offer measures of the reliability of density forecasts relative to the data generating process. However, we are also interested in comparing the accuracy of competing forecasting methods. To evaluate predictive densities, we use scoring rules which are functions of predictive distributions and realised outcomes (Gneiting and Raftery, 2007). Here, as a scoring rule we use the out-of-sample log likelihood values (OLL) (Bao, Lee and Saltoğlu, 2007; Shackleton, Taylor and Yu, 2010; Mitchell and Wallis, 2011).

As explained in Bjørnland *et al.* (2011) logarithmic scores are linked to the Kullback–Leibler information criterion (*KLIC*), where the *KLIC* of the *i*th model is given by

$$KLIC_i = E\left(log\left(\frac{h(y_t)}{f_{i(y_t)}} \right) \right) \tag{19}$$

and the expectation is with respect to the true unknown density $h(y_t)$. The *KLIC* represents the expected divergence of the model density relative to the true unobservable density across the domain of the true density. The *KLIC* reaches a lower bound of zero when $h(y_t) = f_i(y_t)$. In practice, although the expected value of $h(y_t)$ is unknown, it is considered as a fixed constant, therefore *KLIC* is minimised by maximising

$$OLL_i = \sum_{t=0}^{n-1} \log(f_t(y_t)). \tag{20}$$

The out-of-sample log-likelihood statistic (OLL_i) for model i, can be used to rank predictive accuracy of alternative procedures. The best forecast method yields the highest value, generating the closest to the true but unknown density.

To assess whether the predictive accuracy of alternate procedures differ statistically, we test the out-of-sample log likelihood differences by regressing the differences in OLL between competing forecasts i and j on a constant and use HAC standard errors to determine significance (Mitchell and Hall, 2005).

$$OLL_i - OLL_j = c \tag{21}$$

where c is the constant. If the difference between forecasts is positive and significantly different than zero, then density forecast i is superior to density forecast j. The HAC covariance matrix is estimated using Newey and West (1994) non-parametric bandwidth selection procedure.

4. Data

Data consist of daily settlement prices of lean hog futures and options traded at the Chicago Mercantile Exchange (CME) obtained from the Commodity Research Bureau (CRB).[5] The futures data start 31 January 1996 and end 14 February 2017; the options data start 16 January 2002 and end 14 February

5 We use settlement prices as they determine the final value of the contract for a trading session. Note not all settlement prices reflect actual transactions. While settlement prices usually reflect market value in the last minutes of trading, disparities can occur in less liquid markets like the hog options market. CME has an elaborate set of procedures to determine settlement prices for both futures and options contracts. Specific details of the futures settlement procedures can be found at:
http://www.cmegroup.com/confluence/display/EPICSANDBOX/Livestock
Information on option settlement can be found at:
http://www.cmegroup.com/confluence/display/EPICSANDBOX/CME++CBOT++NYMEX++COMEX+Daily+Option+Settlement+Procedures

2017. To estimate the GARCH models, logarithmic returns calculated as $r_t = [\ln(P_t) - \ln(P_{t-1})]$, are obtained using the nearby contract, except when there are ten days or less to delivery. In this case, returns are calculated using the next nearest delivery contract. Returns are always calculated using the same delivery contract. We proxy the short-run interest rate (r_f) with the 3-month Treasury Bill rate that is obtained from the Federal Reserve Bank.

The options data are American-style written on futures contracts of lean hogs.[6] The underlying futures contract expires on the 10th business day of the expiration month, the same day as the option contract. There are eight contracts in a calendar year for lean hog options and futures, with expirations in February, April, May, June, July, August, October and December. The lean hog future contract uses cash settlement to the CME Lean Hog Index, that is a two-day weighted average of lean hog values collected by USDA from the Western Cornbelt, Eastern Cornbelt and MidSouth regions, this ensures convergence between futures and cash prices.

We collect option prices 10 business days before expiration, which usually corresponds to 15 calendar days. Although trading activity in many markets declines rapidly during the expiration month, it remains similar to the next nearby contract in the hog option market. This may be the result of a market with low liquidity. Regardless, at expiration and nearby contracts, sufficient trading activity exists for a range of strike prices to extract the implied distribution.

The final option price data consist of 116 sets. To construct real-world densities from risk-neutral ones, previous data are required to estimate the calibration function. We start using the first 2 years of data (16 observations) for the initial calibration, after which calibration is done recursively by adding observations to the calibration set. We filtered the options data eliminating strikes with no volume trade or open interest, and not complying with the put-call parity conditions. We also eliminated the cases of extreme outliers, since those were the result of lack of observations of the put and calls options. As a result, there are 100 real-world densities.[7]

5. Results

Table 1 presents summary statistics of daily prices and returns of lean hogs from January 1996 to February 2017. Lean hog prices move in a \$104.70/cwt range (\$27.95–\$132.65). However the interquartile (25th and 75th percentile) range is small, only \$20.42. Similarly for returns, the overall range is between −6.9 and 6.9 per cent, but the interquartile range is between −0.87 and 0.86 per cent. Mean and median returns are close to zero which is

6 Equation (9) indicates that European option prices are used in the theory underlying the risk-neutral distributions, but the lean hogs market trades American-style options. This is unlikely to affect our results appreciably because we focus on short horizons near expiration. Indeed, as a robustness check we use the Barone-Adesi and Whaley (1987) method to approximate European option prices and the results were indistinguishable. For simplicity of exposition, we do our main analysis with the unaltered American-style option prices.

7 Code used in the analysis is available from the authors. However, historical data for options are not public and must be obtained directly from the CME or a data vendor.

Table 1. Descriptive statistics of daily futures prices Jan 1996–Jan 2017

	Prices	Returns
Observations	5288.00	5288.00
Mean	68.07	−0.02
Median	66.05	0.00
SD	16.36	0.02
Minimum	27.95	−6.90
Maximum	132.65	6.90
First quartile	56.75	−0.87
Third quartile	77.17	0.86
Skewness	0.74	−0.12
Excess kurtosis	1.21	1.24
Coefficient of variation	0.24	−69.78

Note: Prices are the daily settlement futures price in dollars/cwt of the nearby CME lean hogs contract rolled on the first day of the expiration month (10 business days before expiration). The corresponding returns are multiplied by 100.

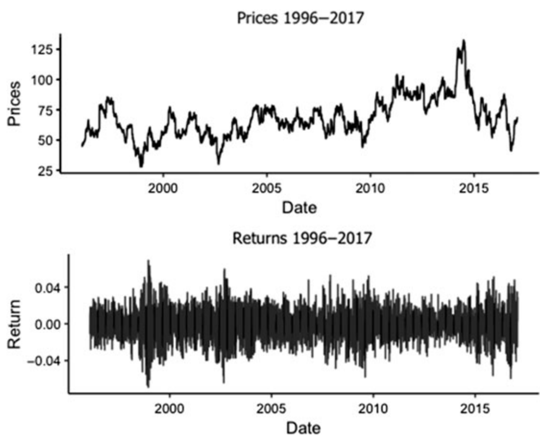

Fig. 1. Lean hog price and returns.

frequently observed in commodity prices. The price distribution for the whole period is slightly negatively skewed and shows excess kurtosis.

Figure 1 plots the temporal price and return movements during the period. Overall, prices exhibit a slight positive trend, with strong swings and considerable volatility. Observed prices and their volatility (i.e. returns squared) are often influenced by large changes in cyclical hog supplies. Similar to other commodity prices, beginning in 2006 hog prices increased, and then declined during the financial crisis in 2008–2009 as demand for commodities declined. Beginning in 2010, prices increase somewhat steadily, but volatility is stable. In spring 2014, prices spiked sharply due to the Porcine Epidemic Diarrhea Virus crisis that threatened US hog supplies. Rather quickly, prices returned to more normal patterns and levels once it became clear that the threat to hog supplies was smaller than expected. In our analysis, we examine two sets of results, the first are 65 observations generated for the contracts expiring from March 2004 until March 2012, and 100 observations generated for the contracts expiring from March 2004 until February 2017. This last portion of the sample was more volatile and comparing results across the periods will allow us to assess the robustness of our results under different price behaviour.

5.1. Density descriptions

We start the analysis generating graphs for each density forecast.[8] The figures were first visually examined by investigating patterns among the historical GARCH, and then among the forward-looking risk-neutral and real-world densities. Densities were also examined on selected dates including periods that contained changing volatility. Analysis revealed several patterns. While the GARCH density forecasts vary modestly with time, the patterns they display are quite similar. In a few occasions all the distributions generate nearly the same shape, however the GARCH estimations that allow higher moments often exhibit more leptokurtic distributions which are also slightly skewed to the right.

Although the risk-neutral density and the GARCH densities often produce similar looking distributions, the risk-neutral densities are usually less leptokurtic and display less mass concentrated in the right tail. The parametrically calibrated real-world densities show patterns that do not seem to deviate from the risk-neutral densities, but the non-parametric calibrated densities exhibit less leptokurtosis than the risk-neutral densities. During periods of relative stability, many of the distributions exhibit similar patterns, but during periods of sharp changes in prices and their volatility, the risk-neutral and real-world densities are less leptokurtic and seem to centre more closely on the value of price at expiration.

5.2. *PIT* histograms and Berkowitz tests

Histograms of *PIT* values are used as a preliminary assessment of uniformity. If the *PIT* values are spread evenly in the [0,1] interval, then the bins in the histogram will be uniform, and the predicted densities will match closely the observed densities. The histograms are divided into 10 bins, corresponding to deciles. In Figure 2 we present the histograms of the density forecast models that correspond to 65 observations from March 2004 to March 2012. While in Figure 3 we present the histograms for 100 observations that correspond to observations from March 2004 to February 2017.

Figures 2 and 3 have very similar patterns, but rather large differences in the ability of the predictive densities to match with observed densities appear. Following Carney and Cunningham's (2006) interpretation, all the GARCH

8 Fourteen hundred density forecast figures were generated and are available from the authors.

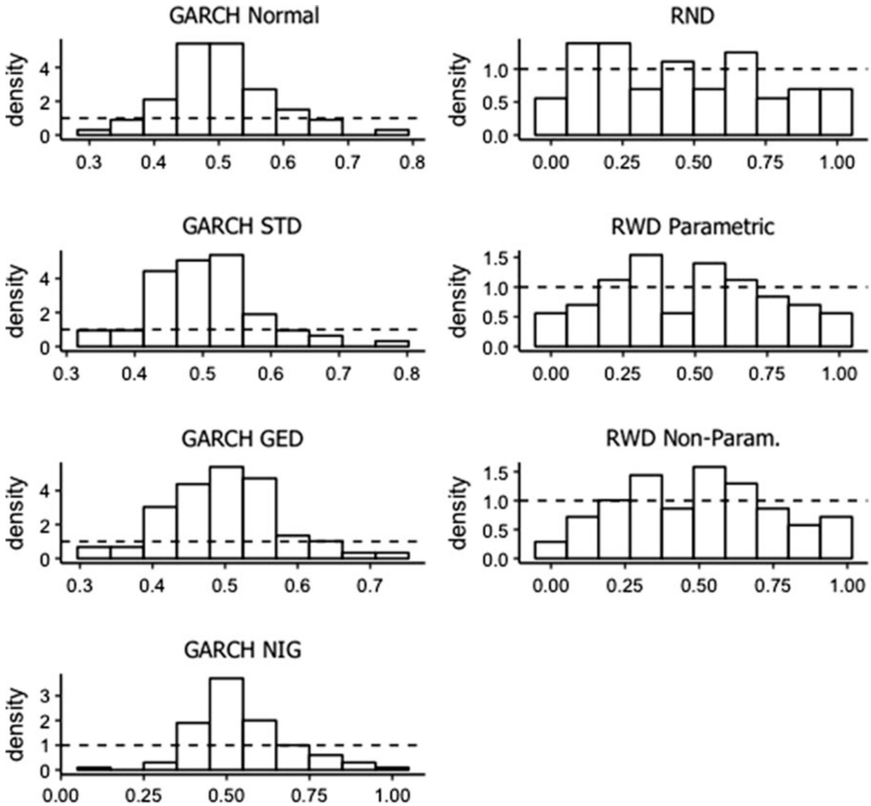

Fig. 2. Probability integral transforms (*PIT*) histograms, March 2004–March 2012 (65 observations).

Note: A uniform density with a value of 1.0 across deciles represents a perfect match between the conditional forecast and true density.

models strongly exhibit an under-confident prediction, meaning that the prediction variance is wider than the observed values which results in less *PIT* observations in the tails. The GARCH models also appear to have long tails to the right, suggesting that the skewness and kurtosis of the target densities are underestimated. In contrast, the risk-neutural distribution (RND) and the real-world distributions (RWD) appear relatively uniform. An exception is their first bin, which points to a degree of kurtosis in the observed densities that is not identified by the predicted distribution.

To evaluate the uniformity and independence of the *PITs* formally, we use the Berkowitz test. The results (Table 2) indicate that the forward-looking forecasts—the real-world parametric, real-world non-parametric and risk-neutral density—provide densities that match observed densities since the test fails to reject the null hypothesis at the 10 per cent level. Real-world densities outperform the risk-neutral, and forward-looking densities exhibit a much better goodness of fit than the historical models in both periods.

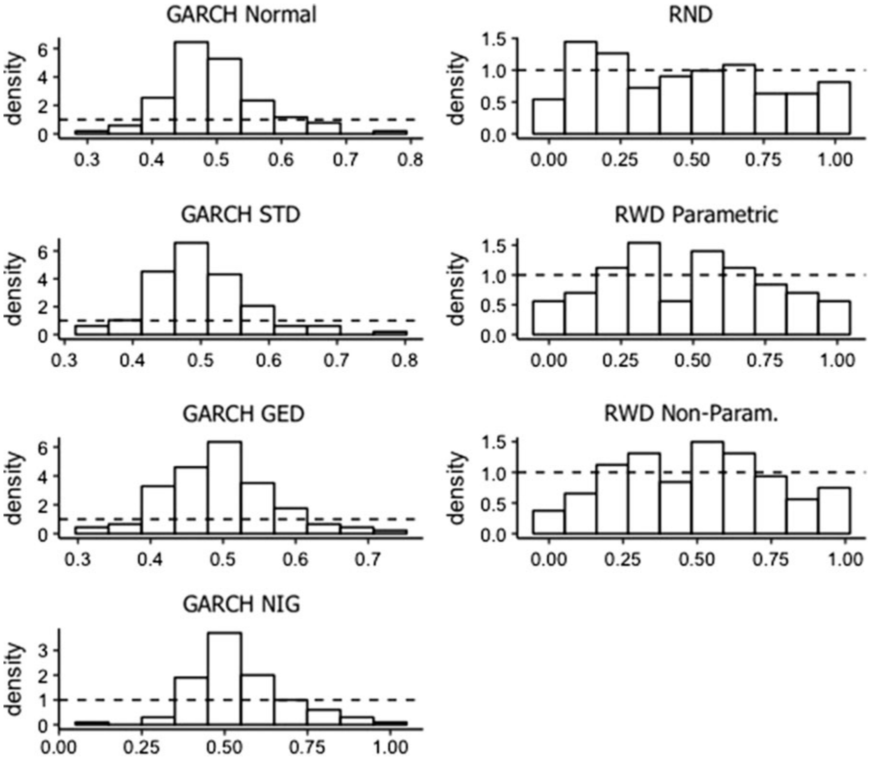

Fig. 3. Probability integral transforms (*PIT*) histograms March 2004–February 2017 (100 observations).

Note: A uniform density with a value of 1.0 across deciles represents a perfect match between the conditional forecast and true density

Overall analysis of the goodness of fit indicate that the GARCH models overestimate the variance of the observed distributions, while the forward-looking methods succeed in matching the distributions.

5.3. Out-of-sample log-likelihood

Table 3 presents the results of the out-of-sample log-likelihood (OLL). Based on the Kullback–Leibler information criterion, densities that are more accurate forecasts of the true density have the highest (less negative) values. In the first period (March 2004–March 2012), the real-world distributions exhibit the highest OLL, with the RWD non-parametric distribution providing the best performance. The GARCH models perform poorly, with the GARCH-NIG the best among these historical models. A similar pattern in the second period (March 2004–February 2017) emerges. To account for differences in observations in the two periods, we generate average OLL measures. On average, the values are similar across the periods, with GARCH models

Table 2. Berkowitz tests

Models	65 obs		100 obs	
	LR_3	*p*-Value	LR_3	*p*-Value
GARCH-GED	143.323	<0.000***	236.535	<0.000***
GARCH-NIG	47.680	<0.000***	87.384	<0.000***
GARCH-Normal	142.841	<0.000***	238.381	<0.000***
GARCH-STD	142.356	<0.000***	238.999	<0.000***
Real World Density-Non-Parametric (RWD-NP)	2.263	0.520	3.504	0.320
Real World Density-Parametric (RWD-P)	0.336	0.953	0.276	0.965
Risk-Neutral Density (RND)	3.908	0.272	5.395	0.145

Note: LR_3 is the Berkowitz likelihood ratio test (equation (18)), that follows a χ^2 distribution with 3 degrees of freedom, and assesses whether the predicted and observed densities match. GARCH models are GJR-GARCH(1,1). Their error distributions differ: GED is the generalised error distribution, NIG is the normal inverse gaussian distribution, Normal is the normal distribution, and STD is the standardised *t* distribution. ***Significant at 1%. 65 observations are from March 2004 to March 2012, 100 observations are from March 2004 to February 2017.

Table 3. Out-of-sample log-likelihoods of predictive accuracy

Models	65 obs	Average 65 obs	100 obs	Average 100 obs
GARCH-Normal	−255.84	−3.94	−397.90	−3.98
GARCH-STD	−255.27	−3.93	−397.46	−3.97
GARCH-GED	−253.13	−3.89	−393.35	−3.93
GARCH-NIG	−215.97	−3.32	−339.06	−3.39
Risk-Neutral Density (RND)	−170.53	−2.62	−260.51	−2.61
Real World Density-Parametric (RWD-P)	−170.77	−2.63	−260.41	−2.60
Real World Density-Non-Parametric (RWD-NP)	−165.51	−2.55	−253.52	−2.54

Note: The out-of-sample log-likelihood statistic (equation (20)) is used to rank predictive accuracy of alternate models. The best forecast method yields the highest value, generating the closest to the true but unknown density. GARCH models are GJR-GARCH (1,1). Their error distributions differ: GED is the generalised error distribution, NIG is the normal inverse gaussian distribution, Normal is the normal distribution and STD is the standardised *t* distribution. Sixty-five observations are from March 2004 to March 2012; 100 observations are March 2004 to January 2017. Averages are obtained by dividing each model by the number of observations

declining in forecast ability modestly and the forward-looking models improving modestly in the second period.

In Table 4 we test predictive accuracy between competing models. We compare the forecast accuracy of GARCH-NIG, the historical model with the highest OLL, against alternate models, and then compare forward-looking models. The results indicate the NIG model provides forecasts that are significantly better than the other GARCH models in both periods. However, the risk-neutral (RND) and the non-parametric real-world (RWD-NP)

Table 4. Pairwise predictive accuracy tests for selected models

	65 obs			100 obs		
Models	Coef	SE	p-Value	Coef	SE	p-Value
NIG vs. GED	0.5717	0.1228	<0.000***	0.5429	0.0839	<0.000***
NIG vs. Normal	0.6135	0.1134	<0.000***	0.5884	0.0770	<0.000***
NIG vs. STD	0.6046	0.1226	<0.000***	0.5841	0.0824	<0.000***
RND vs. NIG	0.6991	0.1544	<0.000***	0.7855	0.1149	<0.000***
RWD-NP vs. NIG	0.7762	0.1343	<0.000***	0.8554	0.1030	<0.000***
RWD-NP vs. RND	0.0771	0.0584	0.1917	0.0699	0.0395	0.0799
RWD-NP vs. RWD-P	0.0808	0.0423	0.0605	0.0689	0.0304	0.0257
RWD-P vs. RND	−0.0037	0.0266	0.8894	0.0010	0.0190	0.9584

Note: The table presents the statistical results for pairwise differences in the out-of-sample log likelihoods between competing models (equation (21)). A positive (negative) coefficient and significant p-value indicate the first (second) model is statistically more accurate. GARCH models are GJR-GARCH(1,1). Their error distributions differ: GED is the generalised error distribution, NIG is the normal inverse gaussian distribution, Normal is the normal distribution, and STD is the standardised t distribution. ***Significant at 1%. 65 observations are from March 2004 to March 2012, 100 observations are from March 2004 to February 2017.

forecasts strongly outperform the NIG. For the forward-looking forecasts, no significant differences in predictive accuracy appear between the real-world parametric and the risk-neutral forecasts. However, the real-world non-parametric (RWD-NP) forecasts are significantly more accurate than the real-world parametric (RWD-P) forecasts in both periods, particularly in the later period. Similarly, the RWD-NP forecasts appear to be generally more accurate than the RND forecasts, especially in the latter, more volatile period where the difference is significant at the 10 per cent level. The findings point to usefulness of the flexibility of the non-parametric framework in adjusting options-generated densities and its increased desireability in periods of large price shifts (2012–2017).

6. Conclusions and implications

We estimate and evaluate *ex-ante* density forecasts of lean hog futures prices using two approaches. The first method generates forecasts based on historical data, using a GARCH-type model and alternative error distributions which allow for non-normal distributions. The second method is a forward-looking approach that obtains an implied risk-neutral density from options prices. Since RNDs can fail to adequately account for risk, RNDs are adjusted parametrically and non-parametrically to form real-world densities.

Overall, the findings suggest that the risk-neutral and real-world density functions, particularly those generated using non-parametric procedures, provide the most accurate representations of the price distributions in terms of goodness of fit and predictive accuracy. Forecast densities based on historical data (GARCH) markedly underperformed showing larger variance and lower predictive accuracy. Forward-looking methods appear to adjust more rapidly, providing a closer reflection of changing market conditions. These results are consistent with findings in related literature (Szakmary *et al.*, 2003; Egelkraut and Garcia, 2006; Egelkraut, Garcia and Sherrick, 2007) that show implied volatility estimated from options outperform historical-based methods. Adjusting forward-looking risk-neutral densities further improves forecasts. This is consistent with results found in other markets at short-term horizons (Shackleton, Taylor and Yu, 2010; Høg and Tsiaras, 2011). Here, real-world densities are superior to historical and risk-neutral density forecasts for a 2-week horizon. Both parametric and non-parametric adjustments or calibrations offered an improvement on the goodness of fit relative to risk-neutral density forecasts. However, only the non-parametric calibration offered improvement in predictive accuracy, and that only occurred over the longer out-of-sample-period (2004–2017) which included episodes of heightened and changing volatility. These findings point to the importance of correcting for the changing risk during these highly volatile periods to obtain an accurate forecast.

Improvements to goodness of fit and accuracy of the forecasts obtained by the calibration from risk-neutral to real-world densities imply that short-term risk premiums may be present in the lean hog futures markets. While our findings suggest that risk premiums are time varying and may appear most noticeably in periods of market turmoil, their exact source and form, whether they exist at more distant horizons, and economic magnitude are not clear. Nevertheless, our findings offer an informative avenue for further research. In this context, it is important to recall that the analysis here was performed in a market (which is rather illiquid particularly in options), focused on short-term forecast performance, and used rather standard historical-based methods. Further, settlement difficulties and illquidity can create potential biases since prices may not reflect actual transaction prices and may be subject to market distortions near expiration. Clearly, much research is needed to identify the effectiveness of price density forecasting procedures at longer horizons, and in other agricultural markets some of which may be more actively traded. Additionally, forecast densities may be improved by allowing for non-linearity in the conditional means of the historical methods, and by combining individual density forecasts (Mitchell and Wallis, 2011; Aastveit *et al.*, 2014). Such research will provide more comprehensive insights into the usefulness of density forecasting procedures, and help to identify how their forecasts may be more effectively used in hedging at different horizons and other decision making.

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References

Aastveit, K. A., Gerdrup, K. R., Jore, A. S. and Thorsrud, L. A. (2014). Nowcasting GDP in real time: a density combination approach. *Journal of Business & Economic Statistics* 32(1): 48–68.

Amisano, G. and Giacomini, R. (2007). Comparing density forecasts via weighted likelihood ratio tests. *Journal of Business & Economic Statistics* 25(2): 177–190.

Bao, Y., Lee, T.-H. and Saltoğlu, B. (2007). Comparing density forecast models. *Journal of Forecasting* 26(3): 203–225.

Barone-Adesi, G. and Whaley, R. E. R. E. (1987). Efficient analytic approximation of American option values. *Journal of Finance* 42(2): 301–320.

Berkowitz, J. (2001). Testing density forecasts, with applications to risk management. *Journal of Business & Economic Statistics* 19(4): 465–474.

Bessler, D. A. and Kling, J. L. (1989). The forecast and policy analysis. *American Journal of Agricultural Economics* 71(2): 503–506.

Bjørnland, H. C., Gerdrup, K., Jore, A. S., Smith, C. and Thorsrud, L. A. (2011). Weights and pools for a Norwegian density combination. *North American Journal of Economics and Finance* 22(1): 61–76.

Bottum, J. C. (1966). Changing functions of outlook in the U.S. *Journal of Farm Economics* 48(5): 1154–1159.

Breeden, D. T. and Litzenberger, R. H. (1978). Prices of state-contingent claims implicit in option prices. *Journal of Business* 51(4): 621–651.

Brittain, L., Garcia, P. and Irwin, S. H. (2011). Live and feeder cattle options markets: returns, risk, and volatility forecasting. *Journal of Agricultural and Resource Economics* 36(1): 28–47.

Carney, M. and Cunningham, P. (2006). *Evaluating Density Forecasting Models*, Vol. 21, Dublin, Ireland: Trinity College Dublin, Department of Computer Science.

Colino, E. V., Irwin, S. H. and Garcia, P. (2011). Improving the accuracy of outlook price forecasts. *Agricultural Economics* 42(3): 357–371.

Dawid, A. P. (1984). Present position and potential developments: some personal views: statistical theory: the prequential approach. *Journal of the Royal Statistical Society. Series A* 147(2): 278–292.

Deaton, A. and Laroque, G. (1992). On the behaviour of commodity prices. *The Review of Economic Studies* 59(1): 1–23.

de Vincent-Humphreys, R. and Noss, J. (2012). Estimating probability distributions of future asset prices: empirical transformations from option-implied risk-neutral to real-world density functions. Bank of England Working Paper Series 455. pp. 1–37.

Diebold, F. X., Gunther, T. A. and Tay, A. S. (1998). Evaluating density forecasts with applications to financial risk management. *International Economic Review* 39(4): 863–883.

Egelkraut, T. and Garcia, P. (2006). Intermediate volatility forecasts using implied forward volatility: the performance of selected agricultural commodity options. *Journal of Agricultural and Resource Economics* 31(3): 508–528.

Egelkraut, T. M., Garcia, P. and Sherrick, B. J. (2007). The term structure of implied forward volatility: recovery and informational content in the corn options market. *American Journal of Agricultural Economics* 89(1): 1–11.

Fackler, P. L. and King, R. P. (1990). Calibration of option-based probability assessments in agricultural commodity markets. *American Journal of Agricultural Economics* 72(1): 73–83.

Frank, J. and Garcia, P. (2009). Time-varying risk premium: further evidence in agricultural futures markets. *Applied Economics* 41(6): 715–725.

Ghalanos, A. (2015). *rugarch: Univariate GARCH models*. (R Package, Version 1.3.6).

Glosten, L. R., Jagannathan, R. and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance* 48(5): 1779–1801.

Gneiting, T. and Raftery, A. E. (2007). Strictly proper scoring rules, prediction, and estimation. *Journal of the American Statistical Association* 102(477): 359–378.

Høg, E. and Tsiaras, L. (2011). Density forecasts of crude-oil prices using option-implied and ARCH-type models. *Journal of Futures Markets* 31(8): 727–754.

Isengildina, O., Irwin, S. H. and Good, D. L. (2004). Evaluation of USDA interval forecasts of corn and soybean prices. *American Journal of Agricultural Economics* 86(4): 990–1004.

Ivanova, V. and Puigvert Gutiérrez, J. M. (2014). Interest rate forecasts, state price sensities and risk premium from Euribor options. *Journal of Banking & Finance* 48: 210–223.

Jackwerth, J. C. (2000). Recovering risk aversion from option prices and realized returns. *Review of Financial Studies* 13(2): 433–451.

Ji, D. and Brorsen, W. B. (2009). A relaxed lattice option pricing model: implied skewness and kurtosis. *Agricultural Finance Review* 69(3): 268–283.

Kling, J. L. and Bessler, D. A. (1989). Calibration-based predictive distributions: an application of prequential analysis to interest rates, money, prices, and output. *Journal of Business* 62(4): 477–499.

Koekebakker, S. and Lien, G. (2004). Volatility and price jumps in agricultural futures prices—evidence from wheat options. *American Journal of Agricultural Economics* 86(4): 1018–1031.

Liu, X., Shackleton, M. B., Taylor, S. J. and Xu, X. (2007). Closed-form transformations from risk-neutral to real-world distributions. *Journal of Banking & Finance* 31(5): 1501–1520.

McKenzie, A. M. and Holt, M. T. (2002). Market efficiency in agricultural futures markets. *Applied Economics* 34(12): 1519–1532.

McKenzie, A., Thomsen, M. and Phelan, J. (2007). How do you straddle hogs and pigs? Ask the Greeks! *Applied Financial Economics* 17(7): 511–520.

Mitchell, J. and Hall, S. G. (2005). Evaluating, comparing and combining density forecasts using the KLIC with an application to the Bank of England and NIESR 'Fan' charts of inflation. *Oxford Bulletin of Economics and Statistics* 67(s1): 995–1033.

Mitchell, J. and Wallis, K. F. (2011). Evaluating density forecasts: forecast combinations, model mixtures, calibration and sharpness. *Journal of Applied Econometrics* 26(6): 1023–1040.

Newey, W. K. and West, K. D. (1994). Automatic lag selection in covariance matrix estimation. *Review of Economic Studies* 61(4): 631–653.

Rosenblatt, M. (1952). Remarks on a multivariate transformation. *Annals of Mathematical Statistics* 23(3): 470–472.

Shackleton, M. B., Taylor, S. J. and Yu, P. (2010). A multi-horizon comparison of density forecasts for the S&P 500 using index returns and option prices. *Journal of Banking & Finance* 34(11): 2678–2693.

Sherrick, B. J., Garcia, P. and Tirupattur, V. (1996). Recovering probabilistic information from option markets: tests of distributional assumptions. *Journal of Futures Markets* 16(5): 545–560.

Szakmary, A., Ors, E., Kyoung Kim, J. and Davidson, W. N. (2003). The predictive power of implied volatility: evidence from 35 futures markets. *Journal of Banking & Finance* 27(11): 2151–2175.

Tay, A. S. and Wallis, K. F. (2000). Density forecasting: a survey. *Journal of Forecasting* 19(4): 235–254.

Taylor, S. J. (2005). *Asset Price Dynamics, Volatility, and Prediction*. Princeton, NJ: Princeton University Press.

Timm, T. R. (1966). Proposals for improvement of the agricultural outlook program of the United States. *Journal of Farm Economics* 48(5): 1179.

Timmermann, A. (2000). Density forecasting in economics and finance. *Journal of Forecasting* 19(4): 231–234.

Urcola, H. and Irwin, S. H. (2010). Hog options: contract redesign and market efficiency. *Journal of Agricultural & Applied Economics* 42(4): 773.

Wang, Z., Fausti, S. W. and Qasmi, B. A. (2012). Variance risk premiums and predictive power of alternative forward variances in the corn market. *Journal of Futures Markets* 32(6): 587–608.