Chapter 1

hoge

Chapter 2

Transverse Ising Chain

2.1 Symmetries and the Critical Point

2.1.1 Duality Symmetry of the Transverse Ising Model

The duality transformation is defined as following.

$$\tau_j^x = S_j^z S_{j+1}^z \tag{2.1}$$

$$\tau_j^z = \prod_{k\leq j} S_k^x \ . \tag{2.2}$$

The duality transformed spin operators at the same site anticommute with each other.

$$\begin{aligned}
\{\tau_i^x, \tau_i^z\} &= S_i^z S_{i+1}^z \left(\prod_{k\leq i} S_k^x\right) + \left(\prod_{k\leq i} S_k^x\right) S_i^z S_{i+1}^z \\
&= S_i^z S_i^x S_{i+1} \left(\prod_{k<i} S_k^x\right) + \left(\prod_{k<i} S_k^x\right) S_i^x S_i^z S_{i+1}^z \\
&= -S_i^x S_i^z S_{i+1} \left(\prod_{k<i} S_k^x\right) + \left(\prod_{k<i} S_k^x\right) S_i^x S_i^z S_{i+1}^z = 0 \ .
\end{aligned} \tag{2.3}$$

The third equality of the above calculation uses anti-commutation relation of S_i^x and S_i^z.

The τ_i^α operators that contain only spins on different sites clearly commute. Therefore, we consider the τ_i^α operators that contain a spin on the same site.

$$
\begin{aligned}
\left[\tau_i^x, \tau_j^z\right] &= \left[S_i^z S_{i+1}^z, \prod_{k<j} S_k^x\right] \\
&= S_1^x \cdots S_{i-1}^x \left[S_i^z S_{i+1}^z, S_i^x S_{i+1}^x\right] S_{i+2}^x \cdots S_{j-1}^x \ .
\end{aligned} \tag{2.4}
$$

この中で注意しなければならないのは、$k=i$ となる場合の連続する $k, k+1$ 番目についてである。

$$
\begin{aligned}
\left[S_i^z S_{i+1}^z, S_i^x S_{i+1}^x\right] &= S_i^z S_{i+1}^z S_i^x S_{i+1}^x - S_i^x S_{i+1}^x S_i^z S_{i+1}^z \\
&= S_i^z S_i^x S_{i+1}^z S_{i+1}^x - S_i^x S_i^z S_{i+1}^x S_{i+1}^z \\
&= S_i^z S_i^x S_{i+1}^z S_{i+1}^x - S_i^z S_i^x S_{i+1}^z S_{i+1}^x \\
&= 0 \ .
\end{aligned} \tag{2.5}
$$