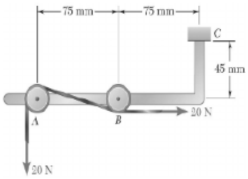
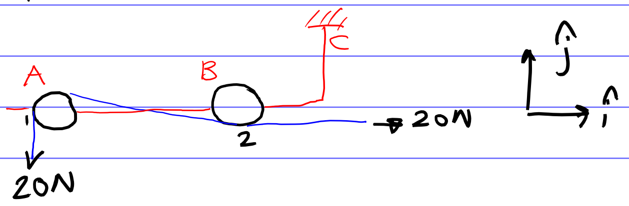
APL 100 : Tutorial 6A solution

1. B&J 4.46 A tension of 20N is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 10 mm, determine the reaction at C.



Solution;

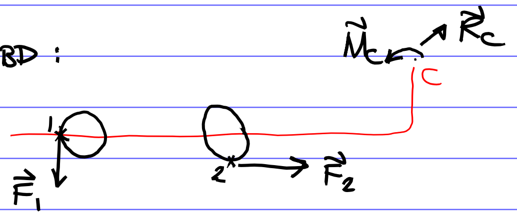


External forces due to belt are (C at origin)

$\vec{F}_1 = -20\hat{j}$ N at $\vec{r}_1 = (-45\hat{j} - 160\hat{i})$ mm

$\vec{F}_2 = 20\hat{i}$ N at $\vec{r}_2 = (-55\hat{j} - 75\hat{i})$ mm

FBD:



Reactions at C:
$\vec{R}_C$ (force)
$\vec{M}_C$ (moment)

For equilibrium: $\vec{R}_C + \vec{F}_1 + \vec{F}_2 = 0$ — (1)

$\vec{M}_C + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = 0$ — (2)

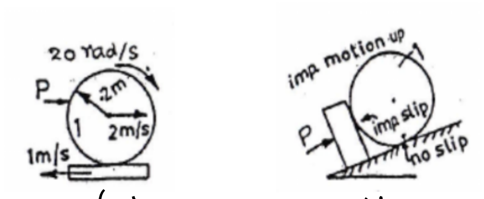
From (1) ∴

$\vec{R}_C = \boxed{(-20\hat{i} + 20\hat{j})\ N}$

$\vec{M}_C = -20 \times 0.16\,\hat{k} - 20 \times 0.055\,\hat{k}$

$= \boxed{-4.3\,\hat{k}\ N \cdot m}$

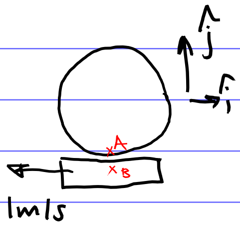
2. PCD 2.21 (p 132) Draw the FBD of body 1 of mass m for both the cases below. The coefficients of friction are μ_s, μ_k respectively.



(a) (b)

Solution :

(a)



$\vec{v}_A = 2\hat{i} + (-20\hat{k}) \times (-0.2\hat{j})$

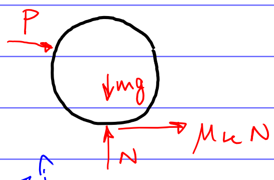
$= 2\hat{i} - 4\hat{i} = -2\hat{i}$ m/s

$\vec{v}_B = -\hat{i}$ m/s

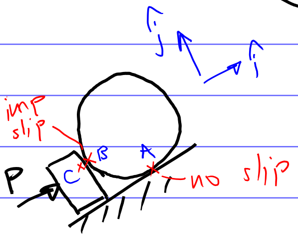
$\Rightarrow \vec{v}_{AB} = -\hat{i}$ m/s (slip)

Dynamic friction present, opposite to $\vec{v}_{AB}$

FBD :



(b)

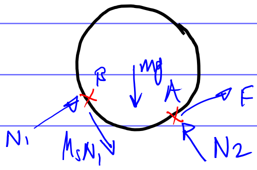


Clearly, $\vec{a}_{PC} \cdot \hat{j} > 0$

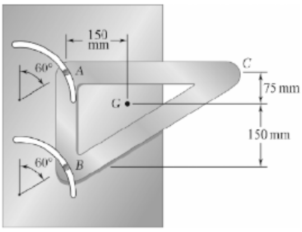
$\therefore$ Friction force $\mu_s N_1$ acts at B along $-\hat{j}$

Static friction force F acts at A

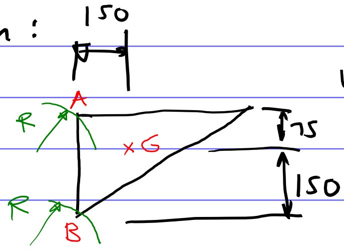
$|F| < \mu_s N_2$



3. B&J 16.19 The triangular weldment ABC is guided by two pins that slide freely in parallel curved slots of radius 150 mm cut in a vertical plate. The weldment weighs 8 kg and its mass center is located at point G. Knowing that at the instant shown the velocity of each pin is 750 mm/s downward along the slots, determine the reactions at A and B.



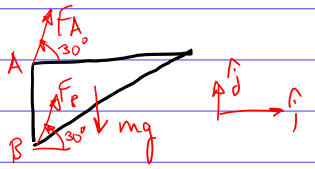
Solution :



$v_t = \dot{s}_A = \dot{s}_B = 0.75$ m/s

$R = 0.15$ m

$m = 8$ kg

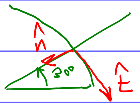


let $\ddot{s}_A = \ddot{s}_B = a_t$

FBD

clearly, acceleration of A, B is

$$\vec{a} = a_t \hat{t} + a_n \hat{n}$$
$$= a_t \hat{t} + \frac{v_t^2}{R} \hat{n}$$



where $\hat{t} = \sin 30^\circ \hat{i} - \cos 30^\circ \hat{j}$

$\hat{n} = -\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}$

Also note : $\vec{\omega} = 0$ for plate, since $\vec{v}_A = \vec{v}_B$

Hence weldment is translating $\Rightarrow \vec{a}_G = \vec{a}$

Euler's First axiom

$$\vec{F}_A + \vec{F}_B - mg\hat{j} = m\vec{a} \quad -(1)$$

Since weldment is translating, therefore

$$\vec{H}_{A/I} = \int_m \vec{r}_{PA} \times \vec{v}_{PA/I} \, dm = 0$$

since $\vec{v}_{P/I} = \vec{v}_{A/I} + \cancel{\vec{\omega}} \times \vec{AP}$

for any P in body

Euler's 2nd axiom then implies:

$$\dot{\vec{H}}_{A/I} = 0 = \vec{M}_A - \vec{r}_{GA} \times m\vec{a}$$

or $\vec{M}_A = \vec{r}_{GA} \times m\vec{a} \quad -(2)$

Eqns (1) and (2) need to be solved for 3 unknowns, F_A, F_B and a_t

Eqn (1) can be written as

$$-(F_A + F_B)\hat{n} - mg\hat{j} = m(a_n \hat{n} + a_t \hat{t})$$

$\Rightarrow a_t = -g\,\hat{j}\cdot\hat{t} = g\cos 30^\circ = 8.5 \text{ m/s}^2$

(3)

Also, $F_A + F_B = -ma_n - mg\,\hat{j}\cdot\hat{n} = m(-a_n + g\sin 30^\circ)$

$= m\left(-\frac{v_t^2}{R} + g\sin 30^\circ\right) = 8\cdot\left(-3.75 + \frac{9.8}{2}\right)$

$\Rightarrow F_A + F_B = 9.2 \text{ N} \quad -(4)$

Applying Eqn (2)

$$\vec{M}_A = -mg\cdot(0.15)\,\hat{k} + (F_B \cos 30^\circ \times 0.225)\,\hat{k}$$
$$= (-11.76 \quad + \quad 0.195\, F_B)\,\hat{k}$$

$$m\vec{a} = m\,(a_n\,\hat{n} + a_t\,\hat{t})$$
$$= (8\,\hat{i} - 73.89\,\hat{j})\ N$$
$$\vec{r}_{GA} = (0.15\,\hat{i} - 0.075\,\hat{j})\ m$$

$$\Rightarrow \quad \vec{r}_{GA} \times m\vec{a} = (-73.89 \times 0.15 + 0.075 \times 8)\,\hat{k}$$
$$= -10.48\ \hat{k}$$

$$\vec{M}_A = \vec{r}_{GA} \times m\vec{a} \Rightarrow 0.195\, F_B = 1.28$$
$$\text{or} \quad F_B = 6.56\ N \qquad -(5)$$

From eqns (4) and (5)
$$F_A = 2.64\ N$$

So,
$$\boxed{\begin{aligned} \vec{F}_A &= 2.64\,(\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j}) \\ \vec{F}_B &= 6.56\,(\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j}) \end{aligned}}$$