ELL101: Introduction to Electrical Engineering

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IIT Delhi

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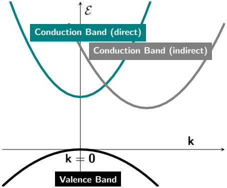
Lecture 13: Rudimentary semiconductor physics and pn junctions

Recap

- Step response. Definition of a unit step function. Step response for an RC circuit.
- The case of non-monotonic forcing stimuli. Square wave response. Minimum period of square wave for capacitor voltage to approach the forcing voltage levels.
- Motivation for the vital importance of semiconductor materials and devices. Courses at IIT Delhi that address this topic in greater detail. Audience and users for devices.
- Basics of semiconductor materials - periodic table, lattice types, forms, etc. Different types of conductivity. Deposition processes and device fabrication unit processes. Characterization. Packaging.
- Carriers in semiconductors in two different energy regimes. Bandstructure. Types of bandgaps.
- Doping. Intrinsic carrier concentration. Identities of donors and acceptors. Electrons and holes. Law of mass action.
- pn junction formation. Introduction to diffusion and drift. Effect of bias.

Carriers in semiconductors



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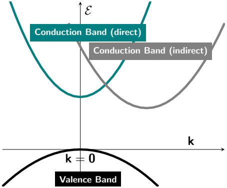

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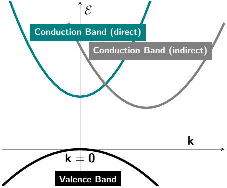

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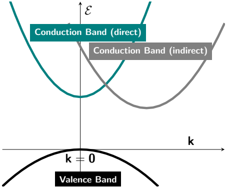

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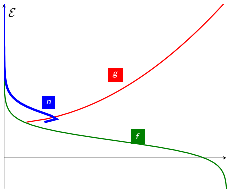

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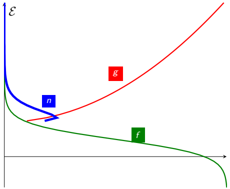

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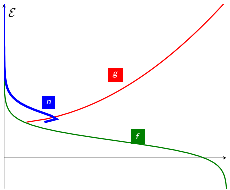

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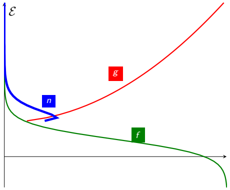

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Definition of effective mass.

Intrinsic semiconductor physics with the $3k_BT$ approximation

- Intrinsic carrier concentration. Approximation for cases where Fermi level is at least $3k_BT$ away from the band edges.

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- Expressions for carrier concentrations are not easy to use.
- Doping adds to individual carrier concentration. Law of mass action applies if we have equilibrium.

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Carrier mobility and sample resistivity

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- Resistivity can be measured using a lot of different methods - Hall effect, Haynes-Shockley, time of flight, CELIV, etc.

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$$\rho \equiv \frac{1}{\sigma} = \frac{1}{q\left(n\mu_n + p\mu_p\right)}$$

Semiconductor properties

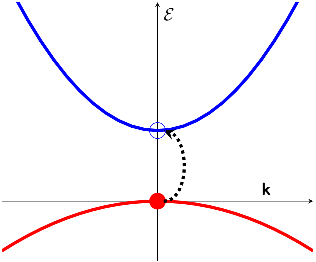
		E_g (eV)	μ_n (cm^2/V-s)	μ_p (cm^2/V-s)	m^*_n/m_o (m_l, m_t)	m^*_p/m_o (m_{lh}, m_{hh})	a (Å)	ϵ_r	Density (g/cm^3)	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(d/Z)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(d/Z)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(d/Z)	0.18	10^5	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(d/Z)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

All values at 300 K. *Vaporizes

Source: Streetman

Unlike doping, generation usually occurs in pairs.

- Electrons and holes are generated from heat. Thermal generation. Intrinsic.



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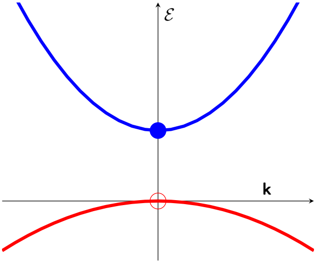

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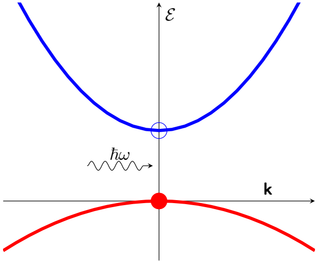

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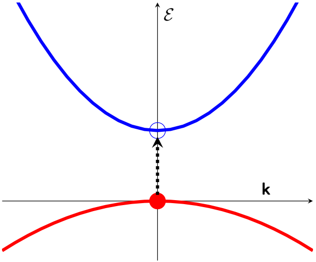

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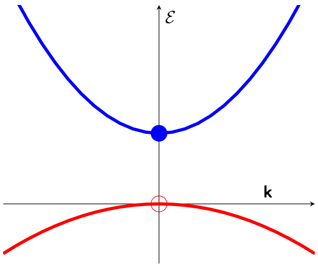

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Carrier recombination

- Equilibrium number of carriers are dictated by doping and the law of mass action.
- Thermal generation produces excess carriers as a function of temperature. This is the intrinsic carrier concentration.
- However, when light is incident, and then switched off, excess carriers generated will recombine.

$$\Delta n = (\Delta n)_0 \, e^{-t/\tau_n}$$

- It can be shown that the excess carrier population recombines as an exponential for low number of carriers. This process can produce photons.
- Electroluminescence: Excess carriers are injected electrically and emit light. This is a light emitting diode (LED). Only direct bandgap semiconductors can be used for this purpose.

Revisiting the pn junction

Materials used here are bound by Fair Use for educational purpose.

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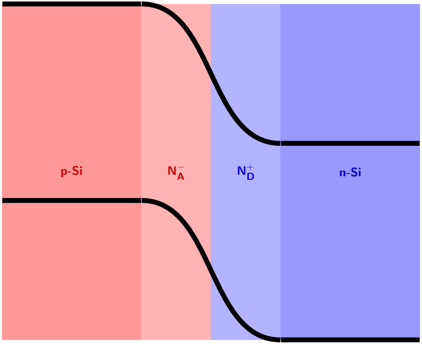

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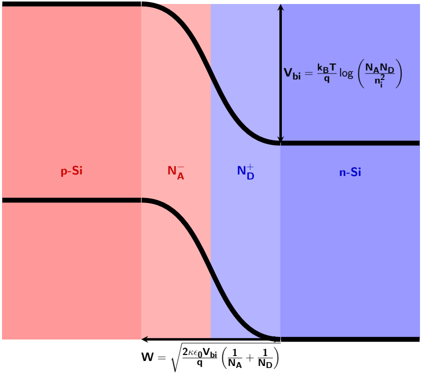

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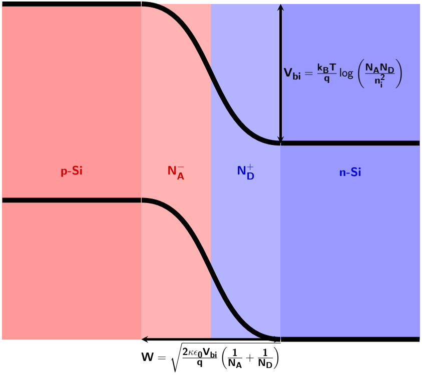

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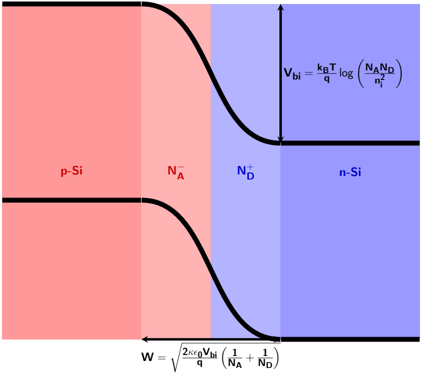
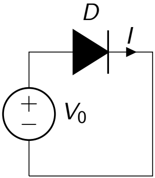
- Diode equation. Significant for several classes of devices.



$$J = \underbrace{q\left(\frac{D_p}{L_n}p_n + \frac{D_n}{L_n}n_p\right)}_{J_0}\left[e^{\frac{qV_{app}}{k_B T}} - 1\right]$$

pn junction under bias

- Forward bias reduces the barrier $\sim V_{bi} - V_{app}$ to majority carrier injection.



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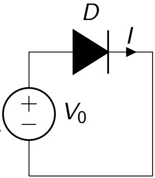

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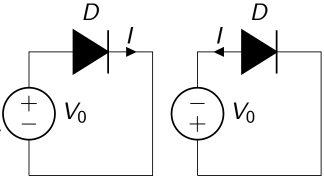

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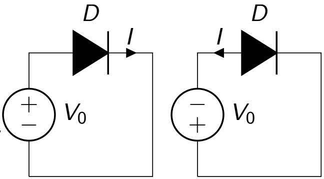

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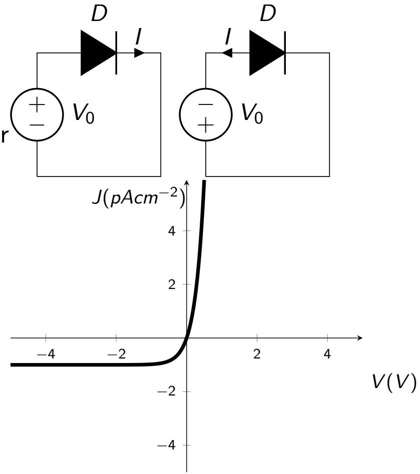
- Shape of diode response. Regimes of operation.



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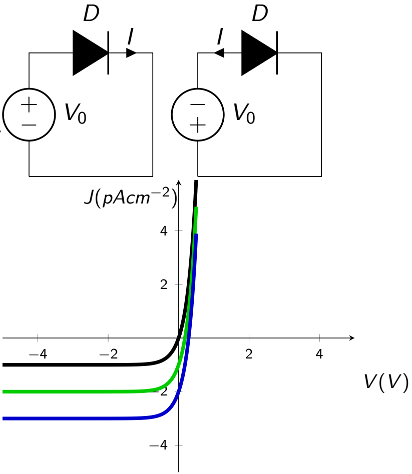
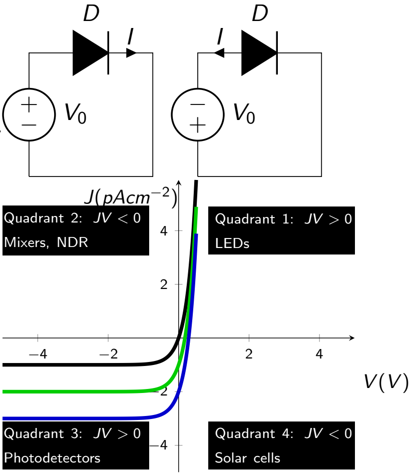
- Shape of diode response. Regimes of operation.
- Effect of light incidence.



pn junction under bias

- Forward bias reduces the barrier $\sim V_{bi} - V_{app}$ to majority carrier injection.
- Exponential increase in majority carrier current (diffusion dominated).
- Reverse bias increases the barrier $\sim V_{bi} + V_{app}$ to majority carrier injection.
- Current is dominated by minority carrier drift.
- Shape of diode response. Regimes of operation.
- Effect of light incidence.
- Different types of diode operation. Take ELL739/ELL743 for more details.



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  - **MRS (Materials Research Society)**: Become a student member today. There is an active student chapter on campus. Send email to the IIT Delhi MRS University Chapter to join. They hold very interesting webinars a few times every semester. Potential fiscal support for special projects. Annual lottery for partial travel support for the MRS Fall Meeting in Boston (as Chapter representative: restricted to presenters who are student members).