Math 10 Fall 2018 Exam 1 on selected sections from Ch 1, 4 and 12

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Directions: All work must be shown to receive full credit. No notes, study aids or cell phones may be used. Scientific calculators are allowed but needed.

Ch 1

1) State the negation of the statement: "Some box contains at least 12 items"

No box contains at least 12 items. ✓

Ch 1

2) Given the propositions
U: A person enters Utopia. D: A person shows a Driver's license. P: A person shows a passport.
and the statement
"To enter Utopia, you must show a driver's license or a passport."

a) Represent the statement symbolically

$U \rightarrow (D \vee P)$

b) Determine its negation.

$\neg(U \rightarrow (D \vee P)) \equiv \neg[\neg U \vee (D \vee P)]$

$\equiv \neg[\neg U \vee D \vee P]$

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Ch 1 3) Proof strategies

a) A student is asked to prove the statement "If a function f is continuous on [a,b] then it is integrable on [a,b]" using either proof by contradiction or proof by contraposition. Where do they start? In other words what is the first sentence or two on the students paper.

Let P = "a function f is continuous on [a,b]"

Q = "a function f is integrable on [a,b]"

Using a proof by contraposition, we will prove $\neg Q \to \neg P$ to show $P \to Q$ is also true. First, we assume $\neg Q$ is true, namely, that a function f is not integrable on [a,b]. ✓

b) What is a counterexample to the statement " If n is a positive integer and n^2 is divisible by 4 then n is divisible by 4."

One counterexample is $n = 2$ b/c $n^2 = (2)^2 = 4$ and $4 \mid 4$ but $4 \nmid 2$.

✓

Ch 1 4) Show $(P \wedge Q) \to P$ is a tautology.

P	Q	$P \wedge Q$	$(P \wedge Q) \to P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

✓

$(P \wedge Q) \to P$ always true ∴ tautology

Ch 1

5) Consider the argument

"Either I wear a red tie or I wear blue socks. I am wearing blue socks. Therefore, I am not wearing a red tie."

a) Translate this argument into propositional form. Make sure to clearly state the meaning of each of your propositions, and use logical connectors and mathematical operators

Let R = "I wear a red tie"

B = "I wear blue socks"

Note: made the assumption that this was an exclusive-or b/c inclusivity was not explicit per the problem (per the convention used in textbook)

① $R \oplus B$

② B

$\therefore \neg R$

b) Determine if this argument is valid. Justify your choice.

R	B	① $R \oplus B$	Both premises true	$\neg R$
T	T	F	n/a	F
T	F	T	n/a	F
F	T	T	yes	T
F	F	F	n/a	T

The conclusion is true in all possible rows that all premises are true, ∴ argument valid.

Ch 1 6) Prove that $2m^2 + 3n^2 = 40$ has no solution for integers m and n.

We will use an exhaustive proof by cases. Note $\cancel{0}\,-3 \le n \le 3$ b/c $3n^2 \le 40$ and $\cancel{0}\,-4 \le m \le 4$ b/c $2m^2 \le 40$.

$\boxed{\text{Case 1}}$: $m \ge 0$ and $n \ge 0$

m	n	$2m^2 + 3n^2$	$= 40$?
0	0	$2(0)^2 + 3(0)^2 = 0$	F
0	1	$2(0)^2 + 3(1)^2 = 3$	F
0	2	$2(0)^2 + 3(2)^2 = 12$	F
0	3	$2(0)^2 + 3(3)^2 = 27$	F
1	0	$2(1)^2 + 3(0)^2 = 2$	F
1	1	$2(1)^2 + 3(1)^2 = 5$	F
1	2	$2(1)^2 + 3(2)^2 = 2 + 12 = 14$	F
1	3	$2(1)^2 + 3(3)^2 = 2 + 27 = 29$	F
2	0	$2(2)^2 + 3(0)^2 = 8$	F
2	1	$2(2)^2 + 3(1)^2 = 8 + 3 = 11$	F
2	2	$2(2)^2 + 3(2)^2 = 8 + 12 = 20$	F
2	3	$2(2)^2 + 3(3)^2 = 8 + 27 = 35$	F
3	0	$2(3)^2 + 3(0)^2 = 18$	F
3	1	$2(3)^2 + 3(1)^2 = 18 + 3 = 21$	F
3	2	$2(3)^2 + 3(2)^2 = 18 + 12 = 30$	F
3	3	$2(3)^2 + 3(3)^2 = 18 + 27 = 45$	F
4	0	$2(4)^2 + 3(0)^2 = 32$	F
4	1	$2(4)^2 + 3(1)^2 = 32 + 3 = 35$	F
4	2	$2(4)^2 + 3(2)^2 = 32 + 12 = 44$	F
4	3	$2(4)^2 + 3(3)^2 = 32 + 27 = 59$	F

Because $(-k)^2 = k^2 = (k)^2$ for some integer k, WLOG it follows that no solutions in cases 2–4 below:

$\boxed{\text{Case 2}}$ $m \ge 0$ and $n < 0$

$\boxed{\text{Case 3}}$ $m < 0$ and $n \ge 0$

$\boxed{\text{Case 4}}$ $m < 0$ and $n < 0$

QED

7) Linear Congruential Method for creating Pseudorandom numbers (numbers generated from a weak random source that are "random enough"): Use four integers satisfying particular inequalities, m, a, c, and x_0, to define an infinite sequence $\{x_n\}_{n=1}^{\infty}$ of these Pseudorandom numbers by $x_{n+1} \equiv (ax_n + c) \bmod m$. Given $m = 17$, $a = 5$, $c = 2$, and $x_0 = 3$ to compute the first five numbers generated by this method.

ch 4

$n = 0$

$X_{0+1} \equiv ((5)(3) + 2) \bmod 17$

$X_1 \equiv 17 \bmod 17$

$\boxed{X_1 \equiv 0}$

$n = 1$

$X_{1+1} \equiv ((5)(0) + 2) \bmod 17$

$X_2 \equiv 2 \bmod 17$

$\boxed{X_2 \equiv 2}$

$n = 2$

$X_{2+1} \equiv ((5)(2) + 2) \bmod 17$

$X_3 \equiv 12 \bmod 17$

$\boxed{X_3 \equiv 12}$

$n = 3$

$X_{3+1} \equiv ((5)(12) + 2) \bmod 17$

$X_4 \equiv 62 \bmod 17$

$\boxed{X_4 \equiv 11}$

$n = 4$

$X_{4+1} \equiv ((5)(11) + 2) \bmod 17$

$X_5 \equiv 57 \bmod 17$

$\boxed{X_5 \equiv 6}$

$$\begin{array}{r} {}^{2}17 \\ \times\ 3 \\ \hline 51 \end{array} \qquad \begin{array}{r} {}^{2}17 \\ \times\ 4 \\ \hline 68 \end{array}$$

Ch 4 8) If the digits of an ISBN are denoted $a_1, a_2, \cdots a_{10}$ with the first nine in the range 0-9. The check digit is either a digit 0-9 or the letter X used to represent the case when $a_{10} = 10$ and it is chosen using the formula $(a_1 + 2a_2 + 3a_3 + \cdots + 10a_{10}) \equiv 0 \pmod{11}$.

a) Is 0-43-209105-5 is a valid ISBN number?

b) A books ISBN looks like 2-72-9188Q6-2 where Q denotes a smudged digit. Can you recover Q? If yes, what is the correct ISBN?

a) $(0 + 2(4) + 3(3) + 4(2) + 5(0) + 6(9) + 7(1) + 8(0) + 9(5) + 10(5)) \equiv 0 \pmod{11}$

$(8 + 9 + 8 + 54 + 7 + 45 + 50) \equiv 0 \pmod{11}$

$(17 + 62 + 52 + 50) \equiv 0 \pmod{11}$

$(79 + 102) \equiv 0 \pmod{11}$

$181 \equiv 0 \pmod{11}$

$176 + 5 \equiv 0 \pmod{11}$

$11 \cdot 16 + 5 \equiv 0 \pmod{11}$

$5 \not\equiv 0 \pmod{11}$

$\boxed{\therefore \text{ invalid ISBN}}$

b) $(2 + 2(7) + 3(2) + 4(9) + 5(1) + 6(8) + 7(8) + 8Q + 9(6) + 10(2)) \equiv 0 \pmod{11}$

$(2 + 14 + 6 + 36 + 5 + 48 + 56 + 8Q + 54 + 20) \equiv 0 \pmod{11}$

$(16 + 42 + 53 + 8Q + 110 + 20) \equiv 0 \pmod{11}$

$(58 + 163 + 8Q + 20) \equiv 0 \pmod{11}$

$(58 + 183 + 8Q) \equiv 0 \pmod{11}$

$(241 + 8Q) \equiv 0 \pmod{11}$

$(231 + 10 + 8Q) \equiv 0 \pmod{11}$

$(11 \cdot 21 + 10 + 8Q) \equiv 0 \pmod{11}$

$(10 + 8Q) \equiv 0 \pmod{11}$

There is only one solution, namely, $Q = 7$, $\therefore$ we can recover Q.

Shown where

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$$\begin{array}{r} 11 \\ 20 \\ \hline 00 \\ 220 \\ \hline 220 \end{array} \qquad \begin{array}{r} 11 \\ 183 \\ +\ 58 \\ \hline 241 \end{array} \qquad \begin{array}{r} 11 \\ \times\ 16 \\ \hline 66 \\ 110 \\ \hline 176 \end{array}$$

$10 \times 11 = 110$

9) Let $F(x, y, z) = \left(x + y\bar{z} \right)\overline{(yz)}$

Ch 12

a) Use a table to express the values of the Boolean function.
b) Find the disjunctive normal form of F.
c) Use a K-map to find a minimal expansion as a Boolean sum of Boolean products.
d) Construct the circuit of the minimal expansion from inverters, AND gates, and OR gates.

a)

	x	y	z	$\bar{z}$	$y\bar{z}$	$x + y\bar{z}$	yz	$\overline{(yz)}$	$(x + y\bar{z})(\overline{yz})$	
m_7	1	1	1	0	0	1	1	0	0	F(1,1,1)
m_6	1	1	0	1	1	1	0	1	1	
m_5	1	0	1	0	0	1	0	1	1	⋮
m_4	1	0	0	1	0	1	0	1	1	
m_3	0	1	1	0	0	0	1	0	0	
m_2	0	1	0	1	1	1	0	1	1	
m_1	0	0	1	0	0	0	0	1	0	
m_0	0	0	0	1	0	0	0	1	0	F(0,0,0)

b) $F(x,y,z) = m_2 + m_4 + m_5 + m_6$

$= \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z}$

c)

000	001	011	010
0	1	3	2
0	0	0	1
100	101	111	110
4	5	7	6
1	1	0	1

$$\begin{array}{ccc} 1 & 0 & 0 \\ 1 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 0 \\ 1 & 1 & 0 \end{array}$$

$x\bar{y}$ $\qquad$ $y\bar{z}$

$$\boxed{F(x,y,z) = x\bar{y} + y\bar{z}}$$

d)