OT 10

~~HW 9 Math 10 TRY2~~

c) No, you can't contradict a theorem. The reason parts (a) and (b) contradict each other is that part (a) is missing the premise of the basis step. Therefore, it was not an inductive proof and does not contradict the principle of mathematical induction. This shows why there are two steps required for an inductive proof.

Purpose to show why there are two parts of induction steps required.

Given the statement $P(n)$ "10^n is divisible by 7"

a) Prove that $P(n) \to P(n+1)$ is a tautology.

Comments. You are doing step 2 first! Case 1: $P(n)$ is false for all nonnegative integers n then the conditional will always be true. Case 2: $P(n)$ is true for some nonnegative integer n. YOU FILL IN THE REST and explain why P(n+1) must also be true.

b) Prove that $P(n)$ is not true for any nonnegative integer

c) Do the results in part a and part b contradict the principle of mathematical induction. Explain.

a) For the inductive hypothesis we assume that $P(k)$ is true. That is, we assume 10^k is divisible by 7 for an arbitrary positive integer k. To complete the inductive step, we must show that when we assume the inductive hypothesis, it follows that $P(k+1)$, the statement that 10^{k+1} is divisible by 7, is also true. That is, we must show that 10^{k+1} is divisible by 7.

Note that:

$$10^{k+1} = 10 \cdot 10^k$$

We can now use the inductive hypothesis and part (ii) of Theorem 1 from Section 4. By the inductive hypothesis, 10^k is divisible by 7. By part (ii) of the theorem, we conclude $10 \cdot 10^k = 10^{k+1}$ is also divisible by 7. This completes the inductive step.

Combining the two cases, we conclude $P(n) \to P(n+1)$ is a tautology.

b) To show $P(n)$ is not true for any nonnegative integer, we must show $P(n)$ is false for every x.

We can prove this by induction. To construct the proof, let $Q(n)$ be the statement "10^n is not divisible by 7".

BASIS STEP: The statement $Q(0)$ is true b/c $10^0 = 1$ is not divisible by 7. This completes the basis step.

INDUCTIVE STEP: For the inductive hypothesis we assume that $Q(k)$ is true. That is, we assume 10^k is not divisible by 7 for an arbitrary positive integer k. To complete the inductive step, we must show that when we assume the inductive hypothesis, it follows that $Q(k+1)$, the statement that 10^{k+1} is not divisible by 7, is also true. That is, we must show that 10^{k+1} is not divisible by 7.

Note that:

$$10^{k+1} = 10 \cdot 10^k$$

We can now use the inductive hypothesis and part (ii) of Theorem 1 from Section 4.1. By the inductive hypothesis, 10^k is not divisible by 7. By part (ii) of the theorem, we conclude $10 \cdot 10^k = 10^{k+1}$ is also not divisible by 7. This completes the inductive step.