1: Page 583 #50

Suppose that R and S are reflexive relations on a set A.

Prove or disprove each of these statements.

(a) $R \cup S$ is reflexive. T

(b) $R \cap S$ is reflexive. T

(c) $R \oplus S$ is irreflexive. T

(d) $R - S$ is irreflexive. T

(e) $S \circ R$ is reflexive. T

a) Let $x \in A$ and xRx since R is reflexive. Thus, $x(R \cup S)x$ b/c $R \subseteq R \cup S$. $\therefore$ $R \cup S$ is reflexive.

b) Let $x \in A$ and xRx since R is reflexive. Also, xSx since S is reflexive. Thus, $x(R \cap S)x$ b/c $(x,x) \in R$ and $(x,x) \in S$. $\therefore$ $R \cap S$ is reflexive.

c) Let $x \in A$ and xRx since R is reflexive. Also, xSx since S is reflexive. Let $a(R \oplus S)b$. Then $a \neq b$ because $(a,a) \in R$ and $(a,a) \in S$. $\therefore$ $R \oplus S$ is irreflexive.

d) Let $x \in A$ and xRx since R is reflexive. Also, xSx since S is reflexive. Let $a(R-S)b$. Then $a \neq b$ b/c $(a,a) \in S$ so $(a,a) \notin R-S$. $\therefore$ $R-S$ is irreflexive.

e) Let $x \in A$ and xRx since R is reflexive. Also, xSx since S is reflexive. Thus, $x(S \circ R)x$ b/c $(x,x) \in S$ and $(x,x) \in R$. $\therefore$ $S \circ R$ is reflexive.