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MATH 10
Section 2838
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Ch 8 Textbook Problems

n	P_n
0	1
1	1
2	$2 \cdot 1$
3	$3 \cdot 2 \cdot 1$
4	$4 \cdot 3 \cdot 2 \cdot 1$
5	$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

8.1

2. a) $P_n = n P_{n-1}$

Note $P_0 = 1$ b/c one permutation of a set w/ no objects (the empty sequence)

b)
$$\begin{aligned} P_n &= n P_{n-1} \\ &= n(n-1) P_{n-2} \\ &= \cdots \\ &= n(n-1) \cdots 2 \cdot 1 \cdot P_0 \\ &= n! \end{aligned}$$

3. one-dollar coins, \$1 bills, \$5 bills

a) Let a_n be the # of ways to deposit n dollars in the vending machine. We must express a_n in terms of earlier terms in the sequence. If we want to deposit n dollars, we may start w/ a dollar coin and then deposit n-1 dollars. This gives us a_{n-1} ways to deposit n dollars. We can also start w/ a dollar bill and then deposit n-1 dollars. This gives us a_{n-1} more ways to deposit n dollars. Finally, we can deposit a \$5 bill and follow that w/ n - 5 dollars; there are a_{n-5} ways to do this. ∴ the recurrence relation is $a_n = 2a_{n-1} + a_{n-5}$. Note this is valid for $n \geq 5$, since otherwise a_{n-5} makes no sense.

b) We need initial conditions for all subscripts from 0 to 4. It is clear that $a_0 = 1$ (deposit nothing) and $a_1 = 2$ (deposit either the dollar coin or the dollar bill). It is also not hard to see that $a_2 = 2^2 = 4$, $a_3 = 2^3 = 8$, and $a_4 = 2^4 = 16$, since each sequence of n C's and B's corresponds to a way to deposit n dollars — a C meaning to deposit a coin and a B meaning to deposit a bill.

c)
$$\begin{aligned} a_{10} &= 2a_{10-1} + a_{10-5} \\ &= 2a_9 + a_5 \\ &= 2(2a_{9-1} + a_{9-5}) + 2a_{5-1} + a_{5-5} \\ &= 2(2a_8 + a_4) + 2a_4 + a_0 \\ &= 2(2(2a_{8-1} + a_{8-5}) + 16) + 2(16) + 1 \\ &= 2(2(2(2a_{7-1} + a_{7-5}) + a_3) + 16) + 33 \\ &= 2(2(2(2a_6 + a_2) + 8) + 16) + 33 \\ &= 2(2(4a_6 + 2(4) + 8) + 16) + 33 \end{aligned}$$

$= 2(8a_6 + 16 + 16 + 16) + 33$

$= 16a_6 + 96 + 33$

$= 16a_6 + 129$

$= 16(2a_{6-1} + a_{6-5}) + 129$

$= 32a_5 + 32 + 129$

$= 32(2a_{5-1} + a_{5-5}) + 161$

$= 32(32 + 1) + 161$

$= 1056 + 161$

$= \boxed{1217}$

7. a) Let a_n be the # of bit strings of length n containing a pair of consecutive 0's.

$\boxed{\text{Case 1}}$ starts with 1

$1\underbrace{\_\,\_\,\_\,\_\,\_}_{n-1}$

contains 00

so a_{n-1}

$\boxed{\text{Case 2}}$ starts with 0

$01\underbrace{\_\,\_\,\_\,\_}_{n-2}$

contains 00

so a_{n-2}

$00\underbrace{\_\,\_\,\_\,\_}_{n-2}$

any bit string

so 2^{n-2}

Thus, the recurrence relation is

$$a_n = a_{n-1} + a_{n-2} + 2^{n-2} \quad \text{for } n \geq 2$$

b) $a_0 = 0, \quad a_1 = 0$

b/c there are no bit strings of length 0 or 1 that contain 00

c) $a_2 = a_1 + a_0 + 2^0 = 0 + 0 + 1 = 1$

$a_3 = a_2 + a_1 + 2^1 = 1 + 0 + 2 = 3$

$a_4 = a_3 + a_2 + 2^2 = 3 + 1 + 4 = 8$

$a_5 = a_4 + a_3 + 2^3 = 8 + 3 + 8 = 19$

$a_6 = a_5 + a_4 + 2^4 = 19 + 8 + 16 = 43$

$a_7 = a_6 + a_5 + 2^5 = 43 + 19 + 32 = \boxed{94}$

11. a) Let a_n be the # of ways to climb n stairs. In order to climb n stairs, a person must either start w/ a step of one stair and then climb $n-1$ stairs (and this can be done in a_{n-1} ways) or else start w/ a step of two stairs and then climb $n-2$ stairs (and this can be done in a_{n-2} ways). From this analysis we can immediately write down the recurrence relation, valid for all $n \geq 2$: $a_n = a_{n-1} + a_{n-2}$.

b) $a_0 = 1$ one way to climb no stairs (do nothing)

$a_1 = 1$ one way to climb one stair

Note that the recurrence relation is the same as that for the Fibonacci sequence, and the initial conditions are that $a_0 = f_1$ and $a_1 = f_2$, so it must be that $a_n = f_{n+1}$ for all n.

c) $a_2 = a_1 + a_0 = 1 + 1 = 2$

$a_3 = a_2 + a_1 = 2 + 1 = 3$.

$a_4 = a_3 + a_2 = 3 + 2 = 5$

$a_5 = a_4 + a_3 = 5 + 3 = 8$

$a_6 = a_5 + a_4 = 8 + 5 = 13$

$a_7 = a_6 + a_5 = 13 + 8 = 21$

$a_8 = a_7 + a_6 = 21 + 13 = \boxed{34}$

12. a) Let a_n be the # of ways to climb n stairs if the person climbing the stairs can take one, two, or three stairs at a time. In order to climb n stairs, a person must either start w/ a step of one stair then climb $n-1$ stairs (this can be done in a_{n-1} ways), start w/ a step of two stairs then climb $n-2$ stairs (this can be done in a_{n-2} ways), or else start w/ a step of three stairs then climb $n-3$ stairs (this can be done in a_{n-3} ways).

$\therefore \quad a_n = a_{n-1} + a_{n-2} + a_{n-3} \qquad n \geq 3$

b) $a_0 = 1$ do nothing

$a_1 = 1$ climb one stair

$a_2 = 2$ two ways to climb two stairs

c) $a_3 = a_2 + a_1 + a_0 = 2 + 1 + 1 = 4$

$a_4 = a_3 + a_2 + a_1 = 4 + 2 + 1 = 7$

$a_5 = a_4 + a_3 + a_2 = 7 + 4 + 2 = 13$

$a_6 = a_5 + a_4 + a_3 = 13 + 7 + 4 = 24$

$a_7 = a_6 + a_5 + a_4 = 24 + 13 + 7 = 44$

$a_8 = a_7 + a_6 + a_5 = 44 + 24 + 13 = \boxed{81}$

14. a) Let a_n be the # of ternary strings that contain two consecutive 0's.

$\boxed{\text{Case 1}}$ starts with 1

$1\underbrace{\_\ \_\ \_\ \_\ \_}_{n-1}$

contains 00

so a_{n-1}

$\boxed{\text{Case 2}}$ $2\underbrace{\_\ \_\ \_\ \_\ \_}_{n-1}$

contains 00

so a_{n-1}

$\boxed{\text{Case 3}}$ Starts with 0

$00\underbrace{\_\ \_\ \_\ \_}_{n-2}$ any ternary string, so 3^{n-2}

$01\underbrace{\_\ \_\ \_\ \_}_{n-2}$ contains 00, so a_{n-2}

$02\underbrace{\_\ \_\ \_\ \_}_{n-2}$ contains 00, so a_{n-2}

$\therefore \quad a_n = 2a_{n-1} + 2a_{n-2} + 3^{n-2} \qquad n \geq 2$

b) $a_0 = 0$

$a_1 = 0$

c) $a_2 = 2a_1 + 2a_0 + 3^0 = 1$

$a_3 = 2a_2 + 2a_1 + 3^1 = 2 + 0 + 3 = 5$

$a_4 = 2a_3 + 2a_2 + 3^2 = 10 + 2 + 9 = 21$

$a_5 = 2a_4 + 2a_3 + 3^3 = 42 + 10 + 27 = 79$

$a_6 = 2a_5 + 2a_4 + 3^4 = 158 + 42 + 81 = \boxed{281}$

$$\begin{array}{r} \scriptstyle 1 \\ 158 \\ +\ 123 \\ \hline 281 \end{array}$$

19. a) This is the same as #11, since a sequence of signals exactly corresponds to a sequence of steps in that exercise.

$\therefore \quad a_n = a_{n-1} + a_{n-2} \qquad n \geq 2$

b) $a_0 = 1$ do nothing (empty msg)

$a_1 = 1$ one way to send a msg in one μs

c) $a_2 = a_1 + a_0 = 1 + 1 = 2$

$a_3 = a_2 + a_1 = 2 + 1 = 3$

$a_4 = a_3 + a_2 = 3 + 2 = 5$

$a_5 = a_4 + a_3 = 5 + 3 = 8$

$a_6 = a_5 + a_4 = 8 + 5 = 13$

$a_7 = a_6 + a_5 = 13 + 8 = 21$

$a_8 = a_7 + a_6 = 21 + 13 = 34$

$a_9 = a_8 + a_7 = 34 + 21 = 55$

$a_{10} = a_9 + a_8 = 55 + 34 = \boxed{89}$

21. a) Consider the plane already divided by $n-1$ lines into R_{n-1} regions. The nth line is now added, intersecting each of the other $n-1$ lines in exactly one point, $n-1$ intersections in all. Think of drawing that line, beginning at one of its ends (out at "infinity"). As we move toward the 1st point of intersection, we are dividing the unbounded region of the plane through which it is passing into two regions; the division is complete once we reach the 1st point of intersection. Then as we draw from the 1st point of intersection to the 2nd, we cut off another region (in other words we divide another of the regions that were already there into two regions). This process continues as we encounter each point of intersection. By the time we have reached the last point of intersection, the number of regions have increased by $n-1$ (one for each point of intersection). Finally, as we move off to infinity, we divide the unbounded region through which we pass into two regions, increasing the count by yet 1 more. Thus there are exactly n more regions than there were before the nth line was added. The analysis we have just completed shows that the recurrence relation we seek is $R_n = R_{n-1} + n$. The initial condition is $R_0 = 1$ (since there is just one region - the whole plane - when there are no lines). Alternatively, we could specify $R_1 = 2$ as the initial condition.

b) $R_n = R_{n-1} + n$

sum of first n natural numbers

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\begin{aligned}
R_n &= R_{n-1} + n \\
&= n + R_{n-1} \\
&= n + [(n-1) + R_{n-2})] \\
&= n + (n-1) + (n-2) + \ldots + 3 + 2 \cdot 1 + R_0 \\
&= \frac{n(n+1)}{2} + R_0 \\
&= \frac{n^2+n}{2} + 1 \\
&= \frac{n^2+n+2}{2}
\end{aligned}$$

clarify

clarify

(R) (E) (A)
red, green, gray

- no two red tiles adjacent
- tiles of the same color considered indistinguishable

27. We assume that the walkway is one tile in width and n tiles long, from start to finish. Thus we are talking about ternary sequences of length n that do not contain two consecutive R's. Let a_n represent the desired quantity.

$\boxed{\text{Case 1}}$ starts with E

$E\underbrace{\_\ \_\ \_\ \_\ \_}_{n-1}$

doesn't contain RR

so a_{n-1}

$\boxed{\text{Case 2}}$ starts with A

$A\underbrace{\_\ \_\ \_\ \_\ \_}_{n-1}$

doesn't contain RR

so a_{n-1}

$\boxed{\text{Case 3}}$ Starts with R

~~RR _ _ _ _~~

discard

$RE\underbrace{\_\ \_\ \_\ \_}_{n-2}$

doesn't contain RR

so a_{n-2}

$RA\underbrace{\_\ \_\ \_\ \_}_{n-2}$

doesn't contain RR

so a_{n-2}

$\therefore\ a_n = 2a_{n-1} + 2a_{n-2}$ $n \geq 2$

b) $a_0 = 1$ (empty sequence)

$a_1 = 3$

c) $a_2 = 2a_1 + 2a_0 = 6 + 2 = 8$

$a_3 = 2a_2 + 2a_1 = 16 + 6 = 22$

$a_4 = 2a_3 + 2a_2 = 44 + 16 = 60$

$a_5 = 2a_4 + 2a_3 = 120 + 44 = 164$

$a_6 = 2a_5 + 2a_4 = 328 + 120 = 448$

$a_7 = 2a_6 + 2a_5 = 896 + 328 = \boxed{1224}$

Ex 3 Let a_n be the # of bit strings of length n that do not contain 00

$\boxed{\text{Case 1}}$ starts with 1

$1\underbrace{\_\ \_\ \_\ \_\ \_}_{n-1}$

doesn't contain 00

so a_{n-1}

$\boxed{\text{Case 2}}$ starts with 0

$01\underbrace{\_\ \_\ \_\ \_}_{n-2}$ $\qquad$ $\cancel{00\_\ \_\ \_\ \_}$

doesn't contain 00 $\qquad$ don't want

$\therefore \quad a_n = a_{n-1} + a_{n-2} \qquad n \geq 2$

□ note book says this is $n \geq 3$ (doesn't count empty string?)

How many such bit strings of length 5?

$a_0 = 1$ empty string

$a_1 = 2$

$a_2 = 3$

$a_3 = a_2 + a_1 = 3 + 2 = 5$

$a_4 = a_3 + a_2 = 5 + 3 = 8$

$a_5 = a_4 + a_3 = 8 + 5 = \boxed{13}$

9. a) Let a_n be the # of bit strings of length n that do not contain 000

$\boxed{\text{Case 1}}$ starts with 1

$1\underbrace{\_\ \_\ \_\ \_\ \_}_{n-1}$

doesn't contain 000

so a_{n-1}

$\boxed{\text{Case 2}}$ starts with 0

$00\_\ \_\ \_\ \_$

$01\underbrace{\_\ \_\ \_\ \_}_{n-2}$

doesn't contain 000

so a_{n-2}

$\cancel{000\_\ \_\ \_}$

don't want

$001\underbrace{\_\ \_\ \_}_{n-3}$

doesn't contain 000

so a_{n-3}

$\therefore\ a_n = a_{n-1} + a_{n-2} + a_{n-3}$ $\quad n \geq 3$

initial conditions

b) $a_0 = 1$ $\{""\}$ empty string

$a_1 = 2$ $\{0, 1\}$

$a_2 = 4$ $\{00, 01, 10, 11\}$

$a_3 = 7$ $|\text{Total}| - |\{000\}| = 8 - 1 = 7$

$a_4 = 13$ $|\text{Total}| - |\{1000, 0000, 0001\}| = 16 - 3 = 13$

c) $a_5 = a_4 + a_3 + a_2 = 13 + 7 + 4 = 24$

$a_6 = a_5 + a_4 + a_3 = 24 + 13 + 7 = 44$

$a_7 = a_6 + a_5 + a_4 = 44 + 24 + 13 = \boxed{81}$

10. a) Let a_n be the # of bit strings of length n that contain the string 01

$\boxed{\text{Case 1}}$ starts with 1

$$1\underbrace{\_\ \_\ \_\ \_\ \_}_{\substack{n-1 \\ \text{contains } 01}}$$

so a_{n-1}

$\boxed{\text{Case 2}}$ starts with 0

$$01\underbrace{\_\ \_\ \_\ \_}_{\substack{n-2 \\ \text{any bit string}}}$$

so 2^{n-2}

$$00\_\ \_\ \_\ \_$$

$$001\underbrace{\_\ \_\ \_}_{\substack{n-3 \\ \text{any bit string}}}$$

so 2^{n-3}

$$000\_\ \_\ \_$$

$\therefore\ a_n = a_{n-1} + 2^{n-2} + 2^{n-3} + \ldots + 2^1 + 2^0$

$a_n = a_{n-1} + 2^{n-1} - 1 \qquad n \geq 2$

initial conditions

b) $a_0 = 0$

$a_1 = 0$

$a_2 = 1$

$a_2 = a_1 + 2^{2-1} - 1$
$= 0 + 2 - 1 = 1$

c) $a_3 = a_2 + 2^2 - 1 = 1 + 4 - 1 = 4$

$a_4 = a_3 + 2^3 - 1 = 4 + 8 - 1 = 11$

$a_5 = a_4 + 2^4 - 1 = 11 + 16 - 1 = 26$

$a_6 = a_5 + 2^5 - 1 = 26 + 32 - 1 = 57$

$a_7 = a_6 + 2^6 - 1 = 57 + 64 - 1 = \boxed{120}$

$n = 4$

$2^2 + 2^1 + 2^0 = 4 + 2 + 1 = 7$

$2^3 - 1 = 8 - 1 = 7$

8.2 Ex 3. $a_n = a_{n-1} + 2a_{n-2}$ $a_0 = 2,\ a_1 = 7$

$c_1 = 1 \quad c_2 = 2$

$r^2 - (1)r - 2 = 0$

$r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$r^2 + r - 2r - 2$

$r_1 = 2,\ r_2 = -1$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ Thm 1

$a_n = \alpha_1 2^n + \alpha_2 (-1)^n$, for some constants α_1, α_2

$a_0 = 2 = \alpha_1 \cancel{2^0} + \alpha_2 \cancel{(-1)^0}$

$2 = \alpha_1 + \alpha_2$

$\alpha_2 = 2 - \alpha_1$

$\alpha_2 = 2 - 3 = -1$

$a_1 = 7 = \alpha_1 2^1 + \alpha_2 (-1)^1$

$7 = 2\alpha_1 - \alpha_2$

$7 = 2\alpha_1 - (2 - \alpha_1)$

$7 = 2\alpha_1 - 2 + \alpha_1$

$9 = 3\alpha_1$

$\alpha_1 = 3$

$a_n = 3 \cdot 2^n + (-1)(-1)^n$

$a_n = 3 \cdot 2^n - (-1)^n$

1. a) yes, degree 3
b) no, 1st coeff. not constant ($2n$)
c) yes, degree 4
d) no, not homogeneous (2)
e) no, not linear (a_{n-1}^2)
f) yes, degree 2
g) no, no homogeneous

linear - terms a_i all appear to the 1st power

constant coefficients

homogeneous - no terms are not multiples of the a_j's

3. a) $a_n = 2a_{n-1}$ for $n \geq 1$, $a_0 = 3$

$c_1 = 2 \quad c_2 = 0$

$r^2 - (2)r - 0 = 0$

$r^2 - 2r = 0$

$r - 2 = 0$

$r_1 = 2$

$a_n = \alpha\, 2^n$ Thm 3 for some constant α

$a_0 = 3 = \alpha\, \cancel{2^0}^{1}$

$\alpha = 3$

$a_n = 3 \cdot 2^n$

b) $a_n = a_{n-1}$ for $n \geq 1$, $a_0 = 2$

$c_1 = 1 \quad c_2 = 0$

$r^2 - (1)r - 0 = 0$

$r^2 - r = 0$

$r - 1 = 0$

$r_1 = 1$

$a_n = \alpha \cdot (1)^n$ Thm 3 for some constant α

$a_0 = 2 = \alpha \cdot \cancel{(1)^0}^{1}$

$\alpha = 2$

$a_n = 2 \cdot (1)^n = 2$ for all n

c) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

$c_1 = 5 \quad c_2 = -6$

$r^2 - (5)r - (-6) = 0$

$r^2 - 5r + 6 = 0$

$(r-3)(r-2) = 0$

$r_1 = 3 \quad r_2 = 2$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ Thm 1 for some constants α_1, α_2

$a_n = \alpha_1 \cdot 3^n + \alpha_2 \cdot 2^n$

$a_0 = 1 = \alpha_1 \cdot \cancel{3^0}^{1} + \alpha_2 \cdot \cancel{2^0}^{1}$

$\alpha_1 = 1 - \alpha_2$

$\alpha_1 = 1 - 3 = -2$

$a_n = -2 \cdot 3^n + 3 \cdot 2^n$

$a_1 = 0 = \alpha_1 \cdot 3^1 + \alpha_2 \cdot 2^1$

$0 = 3\alpha_1 + 2\alpha_2$

$0 = 3(1 - \alpha_2) + 2\alpha_2$

$0 = 3 - 3\alpha_2 + 2\alpha_2$

$-3 = -\alpha_2$

$\alpha_2 = 3$

d) $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 6$, $a_1 = 8$

$c_1 = 4 \quad c_2 = -4$

$r^2 - (4)r - (-4) = 0$

$r^2 - 4r + 4 = 0$

$(r-2)(r-2) = 0$

$r_0 = 2$ Thm 2

$a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$ for some constants α_1, α_2

$a_n = \alpha_1 \cdot 2^n + \alpha_2 n \cdot 2^n$

$a_0 = 6 = \alpha_1 \cdot \cancel{2^0}^{1} + 0$

$\alpha_1 = 6$

$a_1 = 8 = \alpha_1 \cdot 2^1 + \alpha_2 \cdot 1 \cdot 2^1$

$8 = 2\alpha_1 + 2\alpha_2$

$8 = 2(6) + 2\alpha_2$

$-4 = 2\alpha_2$

$\alpha_2 = -2$

$a_n = 6 \cdot 2^n - 2n \cdot 2^n$

$= 2^n(6 - 2n)$

e) $a_n = -4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 1$

$c_1 = -4 \quad c_2 = -4$

$r^2 - (-4)r - (-4) = 0$

$r^2 + 4r + 4 = 0$

$(r+2)^2 = 0$

$r_0 = -2$

$a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$ Thm 2 For some constants α_1, α_2

$a_n = \alpha_1 (-2)^n + \alpha_2 n (-2)^n$

$a_0 = 0 = \alpha_1 \cancel{(-2)^0}^{1} + 0$

$\alpha_1 = 0$

$a_1 = 1 = \alpha_1 (-2)^1 + \alpha_2 (1)(-2)^1$

$1 = -2\alpha_1 + -2\alpha_2$

$1 = -2(0) + -2\alpha_2$

$1 = -2\alpha_2$

$\alpha_2 = \frac{-1}{2}$

$a_n = \left(\frac{-1}{2}\right) n (-2)^n$

f) $a_n = 4a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 4$

$c_1 = 0 \quad c_2 = 4$

$r^2 - (0)r - 4 = 0$

$r^2 - 4 = 0$

$(r+2)(r-2) = 0$

$r_1 = {}^-2, \quad r_2 = 2$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ Thm 1 For some constants α_1, α_2

$a_n = \alpha_1 (^-2)^n + \alpha_2 (2)^n$

$a_0 = 0 = \alpha_1 \cancel{(^-2)^0}^{1} + \alpha_2 \cancel{(2)^0}^{1}$

$\alpha_1 = -\alpha_2$

$\alpha_1 = -(1) = {}^-1$

$a_1 = 4 = \alpha_1(-2)^1 + \alpha_2 2^1$

$4 = {}^-2\alpha_1 + 2\alpha_2$

$4 = {}^-2(-\alpha_2) + 2\alpha_2$

$4 = 4\alpha_2$

$\alpha_2 = 1$

$a_n = {}^-1(^-2)^n + 2^n$

g) $a_n = \dfrac{a_{n-2}}{4}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

$c_1 = 0 \quad c_2 = \frac{1}{4}$

$r^2 - (0)r - \frac{1}{4} = 0$

$r^2 - \frac{1}{4} = 0$

$(r + \frac{1}{2})(r - \frac{1}{2}) = 0$

$r_1 = \frac{-1}{2}, \quad r_2 = \frac{1}{2}$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ Thm 1 For some constants α_1, α_2

$a_n = \alpha_1 (\frac{-1}{2})^n + \alpha_2 (\frac{1}{2})^n$

$a_0 = 1 = \alpha_1 \cancel{(\frac{-1}{2})^0}^{1} + \alpha_2 \cancel{(\frac{1}{2})^0}^{1}$

$\alpha_1 = 1 - \alpha_2$

$\alpha_1 = 1 - \frac{1}{2} = \frac{1}{2}$

$a_1 = 0 = \alpha_1 (\frac{-1}{2})^1 + \alpha_2 (\frac{1}{2})^1$

$0 = \frac{-1}{2}\alpha_1 + \frac{1}{2}\alpha_2$

$0 = \frac{-1}{2}(1 - \alpha_2) + \frac{1}{2}\alpha_2$

$0 = \frac{-1}{2} + \frac{1}{2}\alpha_2 + \frac{1}{2}\alpha_2$

$\frac{1}{2} = \alpha_2$

$a_n = \frac{1}{2}(\frac{-1}{2})^n + \frac{1}{2}(\frac{1}{2})^n$