

# Ch 8 Textbook Problems

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MATH 10  
Section 2838  
SID: 154 5566

8.1 2. a)  $P_n = n P_{n-1}$

Note  $P_0 = 1$  b/c one permutation of a set w/ no objects (the empty sequence)

b)  $P_n = n P_{n-1}$

$$= n(n-1) P_{n-2}$$

$$= \dots$$

$$= n(n-1) \dots 2 \cdot 1 \cdot P_0$$

$$= n!$$

n	$P_n$
0	1
1	1
2	$2 \cdot 1$
3	$3 \cdot 2 \cdot 1$
4	$4 \cdot 3 \cdot 2 \cdot 1$
5	$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

3. one-dollar coins, \$1 bills, \$5 bills

a) Let  $a_n$  be the # of ways to deposit  $n$  dollars in the vending machine. We must express  $a_n$  in terms of earlier terms in the sequence. If we want to deposit  $n$  dollars, we may start w/ a dollar coin and then deposit  $n-1$  dollars. This gives us  $a_{n-1}$  ways to deposit  $n$  dollars. We can also start w/ a dollar bill and then deposit  $n-1$  dollars. This gives us  $a_{n-1}$  more ways to deposit  $n$  dollars. Finally, we can deposit a \$5 bill and follow that w/  $n-5$  dollars; there are  $a_{n-5}$  ways to do this.  $\therefore$  the recurrence relation is  $a_n = 2a_{n-1} + a_{n-5}$ . Note this is valid for  $n \geq 5$ , since otherwise  $a_{n-5}$  makes no sense.

b) We need initial conditions for all subscripts from 0 to 4. It is clear that  $a_0 = 1$  (deposit nothing) and  $a_1 = 2$  (deposit either the dollar coin or the dollar bill). It is also not hard to see that  $a_2 = 2^2 = 4$ ,  $a_3 = 2^3 = 8$ , and  $a_4 = 2^4 = 16$ , since each sequence of  $n$  C's and B's corresponds to a way to deposit  $n$  dollars - a C meaning to deposit a coin and a B meaning to deposit a bill.

$$\begin{aligned} c) \quad a_{10} &= 2a_{10-1} + a_{10-5} \\ &= 2a_9 + a_5 \\ &= 2(2a_{9-1} + a_{9-5}) + 2a_{5-1} + a_{5-5} \\ &= 2(2a_8 + a_4) + 2a_4 + a_0 \\ &= 2(2(2a_{8-1} + a_{8-5}) + 16) + 2(16) + 1 \\ &= 2(2(2(2a_{7-1} + a_{7-5}) + 16) + 33) \\ &= 2(2(2(2a_6 + a_2) + 8) + 16) + 33 \\ &= 2(2(4a_6 + 2(4) + 8) + 16) + 33 \end{aligned}$$

$$\begin{aligned}
&= 2(8a_6 + 16 + 16 + 16) + 33 \\
&= 16a_6 + 96 + 33 \\
&= 16a_6 + 129 \\
&= 16(2a_{6-1} + a_{6-5}) + 129 \\
&= 32a_5 + 32 + 129 \\
&= 32(2a_{5-1} + a_{5-5}) + 161 \\
&= 32(32 + 1) + 161 \\
&= 1056 + 161 \\
&= \boxed{1217}
\end{aligned}$$

7. a) Let  $a_n$  be the # of bit strings of length  $n$  containing a pair of consecutive 0's.

**Case 1** starts with 1

1 — — — —  
 $\underbrace{\hspace{1.5cm}}_{n-1}$   
 contains 00

so  $a_{n-1}$

**Case 2** starts with 0

0 1 — — — —  
 $\underbrace{\hspace{1.5cm}}_{n-2}$   
 contains 00

so  $a_{n-2}$

00 — — — —  
 $\underbrace{\hspace{1.5cm}}_{n-2}$   
 any bit string

so  $2^{n-2}$

Thus, the recurrence relation is

$$a_n = a_{n-1} + a_{n-2} + 2^{n-2} \quad \text{for } n \geq 2$$

b)  $a_0 = 0, a_1 = 0$

b/c there are no bit strings of length 0 or 1 that contain 00

$$\begin{aligned}
c) \quad a_2 &= a_1 + a_0 + 2^0 = 0 + 0 + 1 = 1 \\
a_3 &= a_2 + a_1 + 2^1 = 1 + 0 + 2 = 3 \\
a_4 &= a_3 + a_2 + 2^2 = 3 + 1 + 4 = 8 \\
a_5 &= a_4 + a_3 + 2^3 = 8 + 3 + 8 = 19 \\
a_6 &= a_5 + a_4 + 2^4 = 19 + 8 + 16 = 43 \\
a_7 &= a_6 + a_5 + 2^5 = 43 + 19 + 32 = \boxed{94}
\end{aligned}$$

11. a) Let  $a_n$  be the # of ways to climb  $n$  stairs. In order to climb  $n$  stairs, a person must either start w/ a step of one stair and then climb  $n-1$  stairs (and this can be done in  $a_{n-1}$  ways) or else start w/ a step of two stairs and then climb  $n-2$  stairs (and this can be done in  $a_{n-2}$  ways). From this analysis we can immediately write down the recurrence relation, valid for all  $n \geq 2$ :  $a_n = a_{n-1} + a_{n-2}$ .

b)  $a_0 = 1$  one way to climb no stairs (do nothing)

$a_1 = 1$  one way to climb one stair

Note that the recurrence relation is the same as that for the Fibonacci sequence, and the initial conditions are that  $a_0 = f_1$  and  $a_1 = f_2$ , so it must be that  $a_n = f_{n+1}$  for all  $n$ .

$$\begin{aligned}
c) \quad a_2 &= a_1 + a_0 = 1 + 1 = 2 \\
a_3 &= a_2 + a_1 = 2 + 1 = 3 \\
a_4 &= a_3 + a_2 = 3 + 2 = 5 \\
a_5 &= a_4 + a_3 = 5 + 3 = 8 \\
a_6 &= a_5 + a_4 = 8 + 5 = 13 \\
a_7 &= a_6 + a_5 = 13 + 8 = 21 \\
a_8 &= a_7 + a_6 = 21 + 13 = \boxed{34}
\end{aligned}$$

12. a) Let  $a_n$  be the # of ways to climb  $n$  stairs if the person climbing the stairs can take one, two, or three stairs at a time. In order to climb  $n$  stairs, a person must either start w/ a step of one stair then climb  $n-1$  stairs (this can be done in  $a_{n-1}$  ways), start w/ a step of two stairs then climb  $n-2$  stairs (this can be done in  $a_{n-2}$  ways), or else start w/ a step of three stairs then climb  $n-3$  stairs (this can be done in  $a_{n-3}$  ways).

$$\therefore a_n = a_{n-1} + a_{n-2} + a_{n-3} \quad n \geq 3$$

b)  $a_0 = 1$  do nothing

$a_1 = 1$  climb one stair

$a_2 = 2$  two ways to climb two stairs

c)  $a_3 = a_2 + a_1 + a_0 = 2 + 1 + 1 = 4$

$a_4 = a_3 + a_2 + a_1 = 4 + 2 + 1 = 7$

$a_5 = a_4 + a_3 + a_2 = 7 + 4 + 2 = 13$

$a_6 = a_5 + a_4 + a_3 = 13 + 7 + 4 = 24$

$a_7 = a_6 + a_5 + a_4 = 24 + 13 + 7 = 44$

$a_8 = a_7 + a_6 + a_5 = 44 + 24 + 13 = \boxed{81}$

14. a) Let  $a_n$  be the # of ternary strings that contain two consecutive 0's.

**Case 1** starts with 1

1 -----  
 $\underbrace{\hspace{1.5cm}}_{n-1}$   
 contains 00

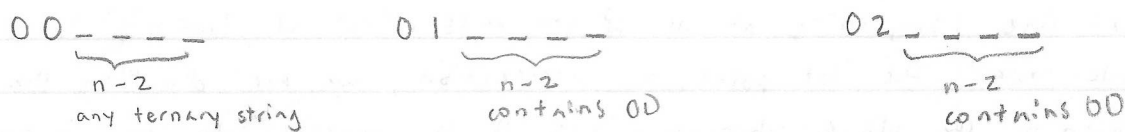
so  $a_{n-1}$

**Case 2**

2 -----  
 $\underbrace{\hspace{1.5cm}}_{n-1}$   
 contains 00

so  $a_{n-1}$

**Case 3** Starts with 0



so  $3^{n-2}$

so  $a_{n-2}$

so  $a_{n-2}$

$$\therefore a_n = 2a_{n-1} + 2a_{n-2} + 3^{n-2} \quad n \geq 2$$

b)  $a_0 = 0$

$a_1 = 0$

c)  $a_2 = 2a_1 + 2a_0 + 3^0 = 1$

$a_3 = 2a_2 + 2a_1 + 3^1 = 2 + 0 + 3 = 5$

$a_4 = 2a_3 + 2a_2 + 3^2 = 10 + 2 + 9 = 21$

$a_5 = 2a_4 + 2a_3 + 3^3 = 42 + 10 + 27 = 79$

$a_6 = 2a_5 + 2a_4 + 3^4 = 158 + 42 + 81 = \boxed{281}$

19. a) This is the same as #11, since a sequence of signals exactly corresponds to a sequence of steps in that exercise.

$$\therefore a_n = a_{n-1} + a_{n-2} \quad n \geq 2$$

b)  $a_0 = 1$  do nothing (empty msg)

$a_1 = 1$  one way to send a msg in one  $\mu s$

c)  $a_2 = a_1 + a_0 = 1 + 1 = 2$

$a_3 = a_2 + a_1 = 2 + 1 = 3$

$a_4 = a_3 + a_2 = 3 + 2 = 5$

$a_5 = a_4 + a_3 = 5 + 3 = 8$

$a_6 = a_5 + a_4 = 8 + 5 = 13$

$a_7 = a_6 + a_5 = 13 + 8 = 21$

$a_8 = a_7 + a_6 = 21 + 13 = 34$

$a_9 = a_8 + a_7 = 34 + 21 = 55$

$a_{10} = a_9 + a_8 = 55 + 34 = \boxed{89}$

$$\begin{array}{r} 158 \\ + 123 \\ \hline 281 \end{array}$$

21. a) Consider the plane already divided by  $n-1$  lines into  $R_{n-1}$  regions. The  $n$ th line is now added, intersecting each of the other  $n-1$  lines in exactly one point,  $n-1$  intersections in all. Think of drawing that line, beginning at one of its ends (out at "infinity"). As we move toward the 1st point of intersection, we are dividing the unbounded region of the plane through which it is passing into two regions; the division is complete once we reach the 1st point of intersection. Then as we draw from the 1st point of intersection to the 2nd, we cut off another region (in other words we divide another of the regions that were already there into two regions). This process continues as we encounter each point of intersection. By the time we have reached the last point of intersection, the number of regions have increased by  $n-1$  (one for each point of intersection). Finally, as we move off to infinity, we divide the unbounded region through which we pass into two regions, increasing the count by yet 1 more. Thus there are exactly  $n$  more regions than there were before the  $n$ th line was added. The analysis we have just completed shows that the recurrence relation we seek is  $R_n = R_{n-1} + n$ . The initial condition is  $R_0 = 1$  (since there is just one region - the whole plane - when there are no lines). Alternatively, we could specify  $R_1 = 2$  as the initial condition.

□ clarify

$$\begin{aligned}
 \text{b) } R_n &= R_{n-1} + n \\
 &= n + R_{n-1} \\
 &= n + [(n-1) + R_{n-2}] \\
 &= n + (n-1) + (n-2) + \dots + 3 + 2 + 1 + R_0 \\
 &= \frac{n(n+1)}{2} + R_0 \\
 &= \frac{n^2 + n}{2} + 1 \\
 &= \frac{n^2 + n + 2}{2}
 \end{aligned}$$

sum of first  $n$  natural numbers

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

(R) (E) (A)  
red, green, gray

- no two red tiles adjacent
- tiles of the same color considered indistinguishable

27. We assume that the walkway is one tile in width and  $n$  tiles long, from start to finish. Thus we are talking about ternary sequences of length  $n$  that do not contain two consecutive R's. Let  $a_n$  represent the desired quantity.

Case 1 starts with E

E — — — —  
n-1  
doesn't contain RR

so  $a_{n-1}$

Case 2 starts with A

A — — — —  
n-1  
doesn't contain RR

so  $a_{n-1}$

Case 3 starts with R

~~RR — — — —~~

discard

RE — — — —  
n-2  
doesn't contain RR

so  $a_{n-2}$

RA — — — —  
n-2  
doesn't contain RR

so  $a_{n-2}$

$$\therefore a_n = 2a_{n-1} + 2a_{n-2} \quad n \geq 2$$

b)  $a_0 = 1$  (empty sequence)

$a_1 = 3$

c)  $a_2 = 2a_1 + 2a_0 = 6 + 2 = 8$

$a_3 = 2a_2 + 2a_1 = 16 + 6 = 22$

$a_4 = 2a_3 + 2a_2 = 44 + 16 = 60$

$a_5 = 2a_4 + 2a_3 = 120 + 44 = 164$

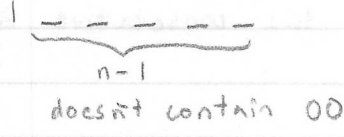
$a_6 = 2a_5 + 2a_4 = 328 + 120 = 448$

$a_7 = 2a_6 + 2a_5 = 896 + 328 = 1224$

Ex 3

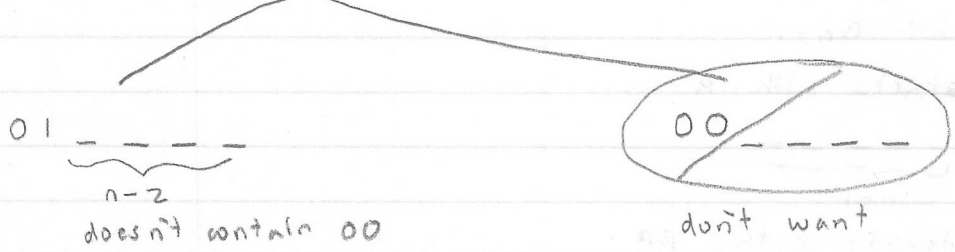
Let  $a_n$  be the # of bit strings of length  $n$  that do not contain 00

**Case 1** starts with 1



so  $a_{n-1}$

**Case 2** starts with 0



$\therefore a_n = a_{n-1} + a_{n-2} \quad n \geq 2$

□ note book says this is  $n \geq 3$  (doesn't count empty string?)

How many such bit strings of length 5?

$a_0 = 1$  empty string

$a_1 = 2$

$a_2 = 3$

$a_3 = a_2 + a_1 = 3 + 2 = 5$

$a_4 = a_3 + a_2 = 5 + 3 = 8$

$a_5 = a_4 + a_3 = 8 + 5 = 13$



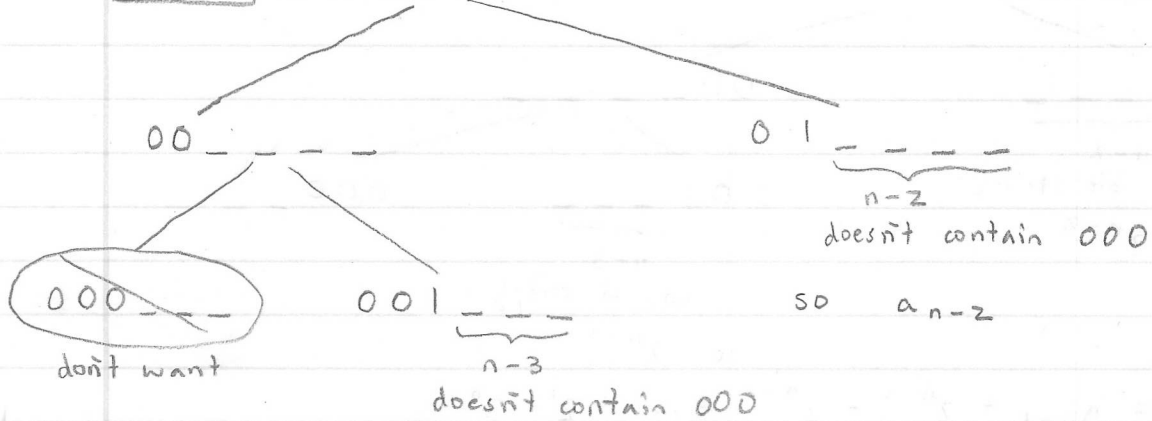
9. a) Let  $a_n$  be the # of bit strings of length  $n$  that do not contain 000

Case 1 starts with 1

$\underbrace{\quad\quad\quad}_{n-1}$   
 doesn't contain 000

so  $a_{n-1}$

Case 2 starts with 0



SD  $a_{n-3}$

$$\therefore a_n = a_{n-1} + a_{n-2} + a_{n-3} \quad n \geq 3$$

initial conditions {

b)  $a_0 = 1$  {""} empty string

$$a_1 = 2 \quad \{0, 1\}$$

$$a_2 = 4 \quad \{00, 01, 10, 11\}$$

$$a_3 = 7 \quad |T_{\text{total}}| - |\{000\}| = 8 - 1 = 7$$

$$a_y = 13 \quad | \text{Total} | - | \{ 1000, 0000, 0001 \} | = 16 - 3 = 13$$

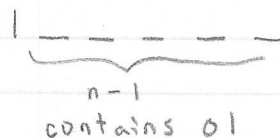
c)  $a_5 = a_4 + a_3 + a_2 = 13 + 7 + 4 = 24$

$$a_6 = a_5 + a_4 + a_3 = 24 + 13 + 7 = 44$$

$$a_7 = a_6 + a_5 + a_4 = 44 + 24 + 13 = 81$$

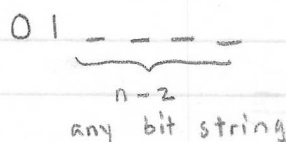
10. a) Let  $a_n$  be the # of bit strings of length  $n$  that contain the string 01

**Case 1** starts with 1

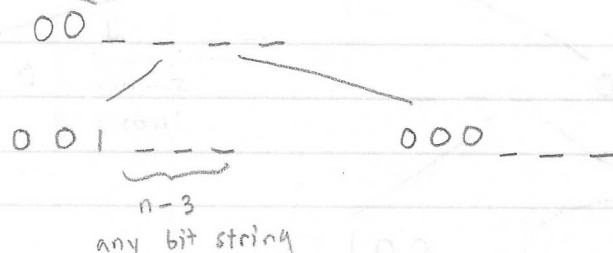


so  $a_{n-1}$

**Case 2** starts with 0



so  $2^{n-2}$



so  $2^{n-3}$

$$\therefore a_n = a_{n-1} + 2^{n-2} + 2^{n-3} + \dots + 2^1 + 2^0$$

$$a_n = a_{n-1} + 2^{n-1} - 1 \quad n \geq 2$$

initial conditions

b)  $a_0 = 0$

$a_1 = 0$

$a_2 = 1$

$$a_2 = a_1 + 2^{2-1} - 1$$

$$= 0 + 2 - 1 = 1$$

c)  $a_3 = a_2 + 2^2 - 1 = 1 + 4 - 1 = 4$

$a_4 = a_3 + 2^3 - 1 = 4 + 8 - 1 = 11$

$a_5 = a_4 + 2^4 - 1 = 11 + 16 - 1 = 26$

$a_6 = a_5 + 2^5 - 1 = 26 + 32 - 1 = 57$

$a_7 = a_6 + 2^6 - 1 = 57 + 64 - 1 = \boxed{120}$

$n = 4$

$$2^2 + 2^1 + 2^0 = 4 + 2 + 1 = 7$$

$$2^3 - 1 = 8 - 1 = 7$$

8.2 Ex 3.  $a_n = a_{n-1} + 2a_{n-2}$   $a_0 = 2, a_1 = 7$

$$c_1 = 1 \quad c_2 = 2$$

$$r^2 - (1)r - 2 = 0$$

$$r^2 - r - 2 = 0$$

$$(r - 2)(r + 1) = 0$$

$$r^2 + r - 2r - 2$$

$$r_1 = 2, r_2 = -1$$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n \quad \text{Thm 1}$$

$$a_n = \alpha_1 2^n + \alpha_2 (-1)^n, \quad \text{for some constants } \alpha_1, \alpha_2$$

$$a_0 = 2 = \alpha_1 2^0 + \alpha_2 (-1)^0$$

$$a_1 = 7 = \alpha_1 2^1 + \alpha_2 (-1)^1$$

$$2 = \alpha_1 + \alpha_2$$

$$7 = 2\alpha_1 - \alpha_2$$

$$\alpha_2 = 2 - \alpha_1$$

$$7 = 2\alpha_1 - (2 - \alpha_1)$$

$$\alpha_2 = 2 - 3 = -1$$

$$7 = 2\alpha_1 - 2 + \alpha_1$$

$$9 = 3\alpha_1$$

$$\alpha_1 = 3$$

$$a_n = 3 \cdot 2^n + (-1)(-1)^n$$

$$a_n = 3 \cdot 2^n - (-1)^n$$

1. a) yes, degree 3

b) no, 1st coeff. not constant ( $2n$ )

c) yes, degree 4

d) no, not homogeneous (2)

e) no, not linear ( $a_{n-1}^2$ )

f) yes, degree 2

g) no, no homogeneous

linear - terms  $a_i$  all appear  
to the 1st power

constant coefficients

homogeneous - no terms are  
not multiples of the  $a_j$ 's

$$3. a) a_n = 2a_{n-1} \text{ for } n \geq 1, a_0 = 3$$

$$c_1 = 2 \quad c_2 = 0$$

$$r^2 - (2)r - 0 = 0$$

$$r^2 - 2r = 0$$

$$r - 2 = 0$$

$$r_1 = 2$$

$$a_n = \alpha 2^n$$

Thm 3 for some constant  $\alpha$

$$a_0 = 3 = \alpha 2^0$$

$$\alpha = 3$$

$$a_n = 3 \cdot 2^n$$

$$b) a_n = a_{n-1} \text{ for } n \geq 1, a_0 = 2$$

$$c_1 = 1 \quad c_2 = 0$$

$$r^2 - (1)r - 0 = 0$$

$$r^2 - r = 0$$

$$r - 1 = 0$$

$$r_1 = 1$$

$$a_n = \alpha \cdot (1)^n$$

Thm 3 for some constant  $\alpha$

$$a_0 = 2 = \alpha \cdot 1^0$$

$$\alpha = 2$$

$$a_n = 2 \cdot (1)^n = 2 \text{ for all } n$$

$$c) a_n = 5a_{n-1} - 6a_{n-2} \text{ for } n \geq 2, a_0 = 1, a_1 = 0$$

$$c_1 = 5 \quad c_2 = -6$$

$$r^2 - (5)r - (-6) = 0$$

$$r^2 - 5r + 6 = 0$$

$$(r - 3)(r - 2) = 0$$

$$r_1 = 3 \quad r_2 = 2$$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$

Thm 1 for some constants  $\alpha_1, \alpha_2$

$$a_n = \alpha_1 \cdot 3^n + \alpha_2 \cdot 2^n$$

$$a_0 = 1 = \alpha_1 \cdot 3^0 + \alpha_2 \cdot 2^0$$

$$\alpha_1 = 1 - \alpha_2$$

$$\alpha_1 = 1 - 3 = -2$$

$$a_n = -2 \cdot 3^n + 3 \cdot 2^n$$

$$a_1 = 0 = \alpha_1 \cdot 3^1 + \alpha_2 \cdot 2^1$$

$$0 = 3\alpha_1 + 2\alpha_2$$

$$0 = 3(1 - \alpha_2) + 2\alpha_2$$

$$0 = 3 - 3\alpha_2 + 2\alpha_2$$

$$-3 = -\alpha_2$$

$$\alpha_2 = 3$$

$$d) a_n = 4a_{n-1} - 4a_{n-2} \text{ for } n \geq 2, a_0 = 6, a_1 = 8$$

$$c_1 = 4 \quad c_2 = -4$$

$$r^2 - (4)r - (-4) = 0$$

$$r^2 - 4r + 4 = 0$$

$$(r - 2)(r - 2) = 0$$

$$r_0 = 2$$

Thm 2

$$a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$$

for some constants  $\alpha_1, \alpha_2$

$$a_n = \alpha_1 \cdot 2^n + \alpha_2 n \cdot 2^n$$

$$a_0 = 6 = \alpha_1 \cdot 2^0 + 0$$

$$\alpha_1 = 6$$

$$a_1 = 8 = \alpha_1 \cdot 2^1 + \alpha_2 \cdot 1 \cdot 2^1$$

$$8 = 2\alpha_1 + 2\alpha_2$$

$$8 = 2(6) + 2\alpha_2$$

$$-4 = 2\alpha_2$$

$$\alpha_2 = -2$$

$$a_n = 6 \cdot 2^n - 2n \cdot 2^n$$

$$= 2^n(6 - 2n)$$

$$e) a_n = -4a_{n-1} - 4a_{n-2} \text{ for } n \geq 2, a_0 = 0, a_1 = 1$$

$$c_1 = -4 \quad c_2 = -4$$

$$r^2 - (-4)r - (-4) = 0$$

$$r^2 + 4r + 4 = 0$$

$$(r + 2)^2 = 0$$

$$r_0 = -2$$

$$a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$$

Thm 2

For some constants  $\alpha_1, \alpha_2$

$$a_n = \alpha_1 (-2)^n + \alpha_2 n (-2)^n$$

$$a_0 = 0 = \alpha_1 \cdot (-2)^0 + 0$$

$$\alpha_1 = 0$$

$$a_1 = 1 = \alpha_1 (-2)^1 + \alpha_2 (1)(-2)^1$$

$$1 = -2\alpha_1 + -2\alpha_2$$

$$1 = -2(0) + -2\alpha_2$$

$$1 = -2\alpha_2$$

$$\alpha_2 = -\frac{1}{2}$$

$$a_n = \left(-\frac{1}{2}\right)n(-2)^n$$

$$f) a_n = 4a_{n-2} \text{ for } n \geq 2, a_0 = 0, a_1 = 4$$

$$c_1 = 0 \quad c_2 = 4$$

$$r^2 - (0)r - 4 = 0$$

$$r^2 - 4 = 0$$

$$(r+2)(r-2) = 0$$

$$r_1 = -2, r_2 = 2$$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n \quad \text{Thm 1} \quad \text{For some constants } \alpha_1, \alpha_2$$

$$a_n = \alpha_1 (-2)^n + \alpha_2 (2)^n$$

$$a_0 = 0 = \alpha_1 (-2)^0 + \alpha_2 (2)^0$$

$$\alpha_1 = -\alpha_2$$

$$\alpha_1 = -(1) = -1$$

$$a_1 = 4 = \alpha_1 (-2)^1 + \alpha_2 (2)^1$$

$$4 = -2\alpha_1 + 2\alpha_2$$

$$4 = -2(-\alpha_2) + 2\alpha_2$$

$$4 = 4\alpha_2$$

$$\alpha_2 = 1$$

$$a_n = -1(-2)^n + 2^n$$

$$g) a_n = \frac{a_{n-2}}{4} \text{ for } n \geq 2, a_0 = 1, a_1 = 0$$

$$c_1 = 0 \quad c_2 = \frac{1}{4}$$

$$r^2 - (0)r - \frac{1}{4} = 0$$

$$r^2 - \frac{1}{4} = 0$$

$$(r + \frac{1}{2})(r - \frac{1}{2}) = 0$$

$$r_1 = -\frac{1}{2}, r_2 = \frac{1}{2}$$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n \quad \text{Thm 1} \quad \text{For some constants } \alpha_1, \alpha_2$$

$$a_n = \alpha_1 (-\frac{1}{2})^n + \alpha_2 (\frac{1}{2})^n$$

$$a_0 = 1 = \alpha_1 (-\frac{1}{2})^0 + \alpha_2 (\frac{1}{2})^0$$

$$\alpha_1 = 1 - \alpha_2$$

$$\alpha_1 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$a_1 = 0 = \alpha_1 (-\frac{1}{2})^1 + \alpha_2 (\frac{1}{2})^1$$

$$0 = -\frac{1}{2}\alpha_1 + \frac{1}{2}\alpha_2$$

$$0 = -\frac{1}{2}(1 - \alpha_2) + \frac{1}{2}\alpha_2$$

$$0 = -\frac{1}{2} + \frac{1}{2}\alpha_2 + \frac{1}{2}\alpha_2$$

$$\frac{1}{2} = \alpha_2$$

$$a_n = \frac{1}{2}(-\frac{1}{2})^n + \frac{1}{2}(\frac{1}{2})^n$$