Math 10 Fall 2018 Exam 2 on selected sections from Ch 2, 5 and 9

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Directions: All work must be shown to receive full credit. No notes, study aids or cell phones may be used. Scientific calculators are allowed but not needed.

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Ch 2 1) Let $A=\{a,c,e,h,2,5,6\}$, $B=\{a,b,c,2,6,7\}$ and $C=\{\{a,c\},b,2,6\}$.

1 2 3 4 5 6 7

a) Find $A\cup B$

$A \cup B = \{a, b, c, e, h, 2, 5, 6, 7\}$

b) Find $A\cap B$

$A \cap B = \{a, c, 2, 6\}$

c) Find $B-A$

$B - A = \{b, 7\}$

d) Determine $|A|$

$|A| = 7$

e) Determine $|\mathcal{P}(A)|$

n = # elements

$|\mathcal{P}(A)| = 2^n = 2^7 = 128$

2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128

f) Which is the correct notation? $\{a,c\}\in C$ or $\{a,c\}\subseteq C$?

$\{a, c\} \in C$

~~5/5~~ 6/6

onto 1-1

Ch 2 2) Define $f:\mathbb{Z}\to\mathbb{Z}$ by $f(x)=x \bmod 10$. Determine if this is a surjection, injection, or bijection. Justify your answer.

This is not an injection since if $f(a)=f(b)$, a does not necessarily equal b. Counterexample: Let $a=16$ and $b=26$. So $f(a) = 16 \bmod 10 = 6$ and $f(b) = 26 \bmod 10 = 6 \Rightarrow f(a)=f(b)$ but $a \neq b$.

This is not a surjection b/c the range of f is $\{0,1,2,3,4,5,6,7,8,9\}$. So exists $y \in \mathbb{Z}$ such that $f(x) \neq y$. For example, let $y=10$ then $\forall x\,(f(x) \neq 10)$ b/c the remainder r is restricted to $0 \le r \le 9$.

Thus, f is not a bijection either.

	Countable	Uncountable
E	X	
N	X	
Z	X	
Q	X	
R		X
I		X
(0,1)		X

3/3

3) Let S be the set of bit strings $S = \{0, 1, 00, 11, ...\}$. Determine whether or not f is a function from S to

Ch 2 the set of integers if $f(x)$ is the position of a 1 in the bit string x. Justify your answer.

f is not a function from $S \to \mathbb{Z}$ b/c by definition a function must assign a unique value in the codomain to all values in the domain. f is ambiguous b/c f(x) has multiple values for a given value of x. For example, let a = 111. If positions indexed starting at 1, then f(a) could be 1, 2, or 3.

$^{13}/_{13}$

Ch 9 4) Let $A = \{1,2,3,12,15\}$ and R be a binary relation on A defined by $\forall m,n \in A \quad (m,n) \in R \text{ iff } m \mid n$.

a) Show R is a Poset.

b) Draw the corresponding Hasse diagram.

c) Give the minimal and maximal elements.

d) Give the least element and the greatest element, if they exist.

a) Let aRa, then $a \mid a$ is true b/c $a = ka$ for some integer k, namely $k=1$. Notice $0 \notin A$. So R is reflexive.

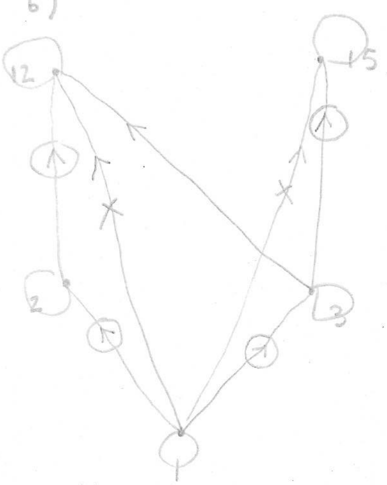
Let aRb and bRa. Then $b = ma$ for some integer m and $a = nb$ for some integer n. Goal to show $a=b$. $a = nb \Rightarrow b = \frac{a}{n}$, $b = ma \Rightarrow \frac{a}{n} = ma \Rightarrow mn = 1 \Rightarrow m = n = 1 \Rightarrow a = (1)b \Rightarrow a = b$

So R is antisymmetric.

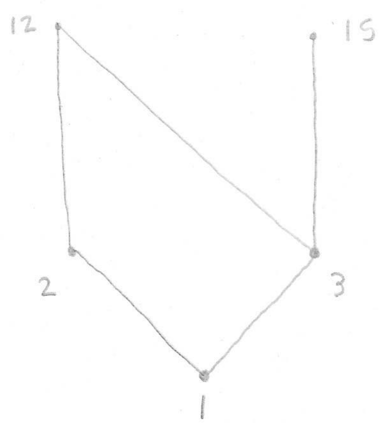
Let aRb and bRc. Then $b = ma$ for some integer m and $c = nb$ for some integer n. $\underbrace{\frac{c}{n} = \frac{nb}{n}}_{\text{divide both sides by } n} \Rightarrow b = \frac{c}{n}$ $\quad b = ma \Rightarrow \underbrace{\frac{c}{n} = ma}_{\text{substituting}} \Rightarrow c = (mn)a$

But $mn \in \mathbb{Z}$ so $a \mid c$ and aRc so R is transitive.

b)



digraph



hasse

c) minimal: 1

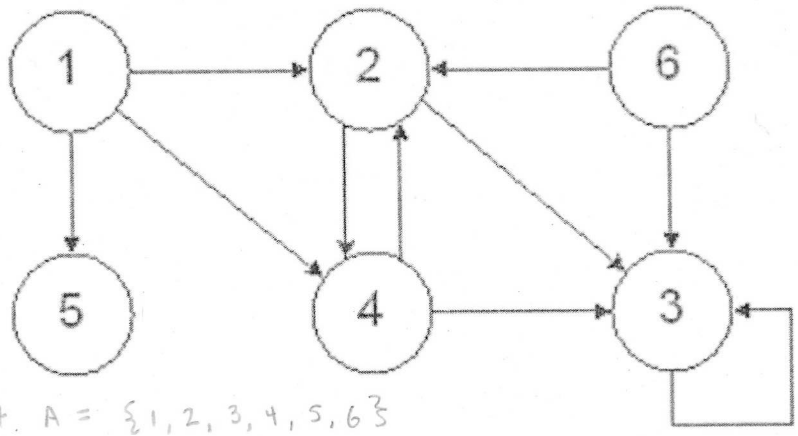
maximal: 12, 15

d) least: 1

greatest: none exists

ch 9

5) Let R be a relation on {1,2,3,4,5,6} represented by the directed graph below.



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Let $A = \{1, 2, 3, 4, 5, 6\}$

a) Is this relation reflexive? Why or why not?

No b/c it does not have loops for all $a \in A$.

e.g. $1 \not R 1$

b) Is this relation irreflexive? Why or why not?

No b/c it does have a loop $3 R 3$.

c) Is this relation symmetric? Why or why not?

No b/c not all edges go in both directions.

e.g. $1 R 5$ but $5 \not R 1$.

d) Is this relation transitive? Why or why not?

No. counterexample: $1 R 4$ and $4 R 3$ but $1 \not R 3$.

Ch 9 6) To determine if a relation is an equivalency relation explain what must be checked.

You must prove the relation is reflexive, symmetric, and transitive.

3/3

Ch 9 7) Let R be the set of all differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ consisting of all pairs (f, g) such that $f'(x) = g'(x)$ for all real numbers x. In homework you showed this was an equivalency relation. Which functions are in the same equivalency class as the function f where $f(x) = 2x + 3$?

2/3

$$[f] = \{ g \mid f'(x) = g'(x) \}$$
$$= \{ g \mid 2 = g'(x) \}$$
$$= \{ g \mid g(x) = 2x + C \text{ where } C \text{ some constant} \}$$

Ch 2

8) Show (0,1) has the same cardinality as (0,2). 1/6

The interval $(0,1)$ is uncountable

Make it $2x$, a bijection

Define $f: (0,1) \to (0,2)$ where $f(x) = x$. This is an injection so $|(0,1)| \leq |(0,2)|$ but $(0,1)$ uncountable so $(0,2)$ uncountable and same cardinality.

Does not follow.
Not all uncountable sets have the same cardinality.

Ch 2 $\mathbb{Q}^+$ $\frac{n}{d}$ $d < 3$

9) Let T be the set of all positive rational numbers that can be written with denominators less than 3. Determine if T is finite countable, infinite countable or uncountable. Justify your answer.

2/3

Define $f: \mathbb{Z} \times \mathbb{Z} \to T$ where $f(x, y) = \frac{x}{y}$ and $y < 3$

No, $f(0,0)$ is undefined

This is a bijection and $\mathbb{Z}$ infinite countable which means $\mathbb{Z} \times \mathbb{Z}$ also infinite countable $\Rightarrow$ $|\mathbb{Z} \times \mathbb{Z}| = |T|$ $\Rightarrow$ T infinite countable

w/ cardinality $\aleph_0$.

T is infinite subset (contains $\mathbb{Z}^+$) of countable set $\mathbb{Q}$, so is countably infinite.

Ch 5

10) Use the principal of mathematical induction to prove: $2n+3 \le 2^n$ for $n \ge 4$.

$10/10$

Let $P(n)$: $2n + 3 \le 2^n$ for $n \ge 4$

Basis Step: $n = 4$

$2n + 3 = 2(4) + 3 = 11$

$2^n = 2^4 = 16$

So $11 \le 16 \Rightarrow$ basis step true

Inductive Step: Inductive hypothesis: Assume $P(k)$ $2k + 3 \le 2^k$ for some integer k where $k \ge 4$

Goal to show $P(k+1)$

consider $\underbrace{2^{k+1} = \underbrace{2 \cdot 2^k \ge 2(2k+3)}_{\text{mult. both sides of ind. hyp. by 2}} = 4k + 6 \ge 2k + 5 = 2(k+1)+3}$

So $2(k+1) + 3 \le 2^{k+1}$ as desired

Thus $P(k) \Rightarrow P(k+1)$

The result follows by the principle of mathematical induction

$55/62 \approx 89/100$