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MATH 10
SID: 1545566
Section 2838

Section 4.5 Problems

19. a) 7 4 0 5 1 4 8 9 6 2 | 3

$3 = (7 + 4 + 0 + 5 + 1 + 4 + 8 + 9 + 6 + 2) \bmod 9$
$3 = (11 + 5 + 5 + 17 + 8) \bmod 9$
$3 = (16 + 22 + 8) \bmod 9$
$3 = (38 + 8) \bmod 9$
$3 = 46 \bmod 9$
$3 \neq 1$
$\therefore$ invalid

b) 8 8 3 8 2 0 1 3 4 4 | 5

$5 = (8 + 8 + 3 + 8 + 2 + 0 + 1 + 3 + 4 + 4) \bmod 9$
$5 = (16 + 11 + 2 + 4 + 8) \bmod 9$
$5 = (27 + 6 + 8) \bmod 9$
$5 = (33 + 8) \bmod 9$
$5 = 41 \bmod 9$
$5 = 5$
$\therefore$ valid

c) 5 6 1 5 2 2 4 0 7 8 | 4

$4 = (5 + 6 + 1 + 5 + 2 + 2 + 4 + 0 + 7 + 8) \bmod 9$
$4 = (11 + 6 + 4 + 4 + 15) \bmod 9$
$4 = (17 + 8 + 15) \bmod 9$
$4 = (25 + 15) \bmod 9$
$4 = 40 \bmod 9$
$4 = 4$
$\therefore$ valid

d) 6 6 6 0 6 6 3 11 7 | 8

$8 = (6 + 6 + 6 + 0 + 6 + 6 + 3 + 1 + 1 + 7) \bmod 9$

$8 = (12 + 6 + 12 + 4 + 8) \bmod 9$

$8 = (18 + 16 + 8) \bmod 9$

$8 = (34 + 8) \bmod 9$

$8 = 42 \bmod 9$

$8 \neq 6$

$\therefore$ invalid

21. a) 4 9 3 2 12 Q 0 6 8 | 8

Note that $0 \leq x_{11} \leq 8$

$8 = (4 + 9 + 3 + 2 + 1 + 2 + Q + 0 + 6 + 8) \bmod 9$

$8 = (13 + 5 + 3 + Q + 6 + 8) \bmod 9$

$8 = (18 + 3 + Q + 14) \bmod 9$

$8 = (21 + Q + 14) \bmod 9$

$8 = (35 + Q) \bmod 9$

$8 = (27 + 8 + Q) \bmod 9$

$8 = (Q + 8) \bmod 9$

$Q = 0$

$Q = 9$

$\therefore$ can't recover smudged digit

b) 8 5 0 Q 9 1 0 3 8 5 | 8

$8 = (8 + 5 + 0 + Q + 9 + 1 + 0 + 3 + 8 + 5) \bmod 9$

$8 = (13 + Q + 10 + 11 + 5) \bmod 9$

$8 = (Q + 23 + 16) \bmod 9$

$8 = (Q + 39) \bmod 9$

$8 = (Q + 36 + 3) \bmod 9$

$8 = (Q + 3) \bmod 9$

$Q = 5$

$\therefore$ yes can recover

c) 2 Q 9 4 1 0 0 7 7 3 | 4

$4 = (2 + Q + 9 + 4 + 1 + 0 + 0 + 7 + 7 + 3) \bmod 9$

$4 = (Q + 11 + 5 + 14 + 3) \bmod 9$

$4 = (Q + 16 + 17) \bmod 9$

$4 = (Q + 33) \bmod 9$

$4 = (Q + 27 + 6) \bmod 9$

$4 = (Q + 6) \bmod 9$

$Q = 7$

$\therefore$ can recover

d) 6 6 6 8 7 Q 0 3 2 0 | 1

$1 = (6 + 6 + 6 + 8 + 7 + Q + 0 + 3 + 2 + 0) \bmod 9$

$1 = (Q + 12 + 14 + 10 + 2) \bmod 9$

$1 = (Q + 26 + 12) \bmod 9$

$1 = (Q + 38) \bmod 9$

$1 = (Q + 36 + 2) \bmod 9$

$1 = (Q + 2) \bmod 9$

$Q = 8$

$\therefore$ can recover

23. Because the first ten digits are added, any transposition error involving them will go undetected — the sum of the first ten digits will be the same for the transposed number as it is for the correct number. Suppose the last digit is transposed w/ another digit; without loss of generality, we can assume it's the tenth digit and that $x_{10} \neq x_{11}$. Then the correct equation will be

$$x_{11} \equiv x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \pmod 9$$

but the equation resulting from the error will be

$$x_{10} \equiv x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{11} \pmod 9$$

Subtracting these two equations, we see that the erroneous equation will be true iff $x_{11} - x_{10} \equiv x_{10} - x_{11} \pmod 9$. This is equivalent to $2x_{11} \equiv 2x_{10} \pmod 9$, which, b/c 2 is relatively prime to 9, is equivalent to $x_{11} \equiv x_{10} \pmod 9$, which is false. This tells us that the check equation will fail. $\therefore$ we conclude that transposition errors involving the eleventh digits are detected.

33. a) 1059 - 102|7

$7 = (3(1) + 4(0) + 5(5) + 6(9) + 7(1) + 8(0) + 9(2)) \bmod 11$

$7 = (3 + 0 + 25 + 54 + 7 + 0 + 18) \bmod 11$

$7 = (28 + 61 + 18) \bmod 11$

$7 = (89 + 18) \bmod 11$

$7 = 107 \bmod 11$

$7 \neq 8$

$\therefore$ invalid

b) 0002 - 989|0

$0 = (3(0) + 4(0) + 5(0) + 6(2) + 7(9) + 8(8) + 9(9)) \bmod 11$

$0 = (0 + 0 + 0 + 12 + 63 + 64 + 81) \bmod 11$

$0 = (75 + 145) \bmod 11$

$0 = 220 \bmod 11$

$0 = 0$

$\therefore$ valid

c) 1530 - 866|9

$9 = (3(1) + 4(5) + 5(3) + 6(0) + 7(8) + 8(6) + 9(6)) \bmod 11$

$9 = (3 + 20 + 15 + 0 + 56 + 48 + 54) \bmod 11$

$9 = (23 + 71 + 102) \bmod 11$

$9 = (94 + 102) \bmod 11$

$9 = 196 \bmod 11$

$9 = 9$

$\therefore$ valid

$$\begin{array}{r} 11 \\ \times\, 17 \\ \hline 77 \\ 110 \\ \hline 187 \end{array}$$

d) 1007 - 120|X

$10 = (3(1) + 4(0) + 5(0) + 6(7) + 7(1) + 8(2) + 9(0)) \bmod 11$

$10 = (3 + 0 + 0 + 42 + 7 + 16 + 0) \bmod 11$

$10 = (45 + 23) \bmod 11$

$10 = 68 \bmod 11$

$10 \neq 2$

$\therefore$ invalid

□ Ask prof

34. For a single digit change we can say that if it has changed from x to y the change in the sum is equal to

$k(x-y)$ where $k \in \{1, 3, 4, 5, 6, 7, 8, 9\}$

If $k(x-y) \not\equiv 0 \mod 11$

Because if it is not so the sum modulo 11 is same and so not detectable.
11 and k are relatively prime and also 11 and $(x-y)$ are relatively prime. So 11 doesn't divide $k(x-y)$. Therefore $k(x-y) \not\equiv 0 \mod 11$.
So every single digit change is detectable including the check digit.

35. By subtracting d_8 from both sides and noting that $-1 \equiv 10 \pmod{11}$, we see that the checking congruence is equivalent to

$$3d_1 + 4d_2 + 5d_3 + 6d_4 + 7d_5 + 8d_6 + 9d_7 + 10d_8 \equiv 0 \pmod{11}.$$

It is now easy to see that transposing adjacent digits x and y (where x is on the left) causes the left-hand side to increase by x and decrease by y, for a net change of $x-y$. Because $x \not\equiv y \pmod{11}$, the congruence will no longer hold. $\therefore$ errors of this type are always detected.