1: Prove if $f : A \to A$, where A is finite. Then f is one to one if and only if f is onto.

Since this problem is of the form $P \Leftrightarrow Q$, we must show $P \Rightarrow Q$ and $Q \Rightarrow P$.

Answer:

Let $P =$ "$f : A \to A$", $Q =$ "A is finite", $R =$ "f is one-to-one", $S =$ "f is onto".

(Step 1) Show $(P \wedge Q \wedge R) \Rightarrow S$

We can use a proof by contradiction to prove this conditional statement. We assume $(P \wedge Q \wedge R)$ is true and S is false, namely, that f is one-to-one and f is not onto. Because A is finite, let its cardinality be n. Let $a \in A$ be such that $f(b) \neq a$ for any $b \in A$. Therefore, we have a mapping from n elements to $n - 1$ elements. The definition of a function says each $a \in A$ must be assigned to a unique element of A, so we must have two elements in the domain mapping to the same element in the codomain. This contradicts the premise that f is one-to-one. $\Rightarrow\!\Leftarrow$

(Step 2) Show $(P \wedge Q \wedge S) \Rightarrow R$

We can use a proof by contradiction to prove this conditional statement. We assume $(P \wedge Q \wedge S)$ is true and R is false, namely, that f is onto and f is not one-to-one. Because A is finite, let its cardinality be n. Let $a, b \in A$ be such that $f(a) = f(b)$ but $a \neq b$. Therefore, we have a mapping from n elements to $n - 1$ elements. However, this means there must be one element of the codomain that is not mapped to by any element of the domain. This contradicts the premise that f is onto. $\Rightarrow\!\Leftarrow$