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MATH 10
Section 2838
SID: 1545566

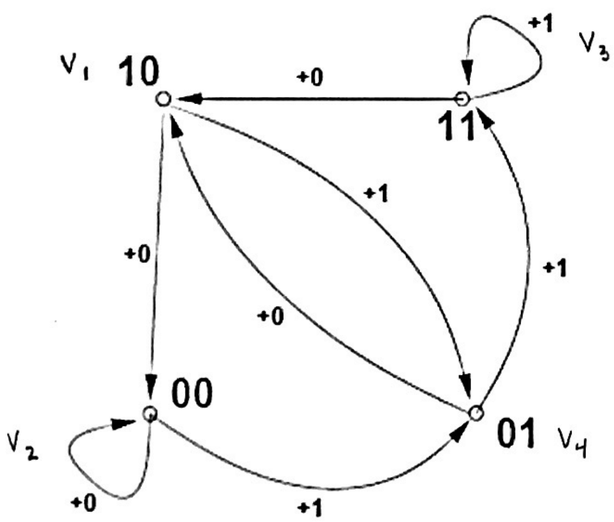
Math 10 OT 11

In class we were looking at permutations of the 26 letters in the English alphabet that did not include specific words. In particular we were looking at the words *harm* and *most*. They could be concatenated to the word Harmmost which is an 8-string but to avoid the use of the double m we instead made the word Harmost which is a 7-string.

Now I want you to look at a similar idea with bit strings.

Consider the set of 3-bit strings (000, 001, 101...}. These 3-bit strings could be concatenated to a 24 bit string 000 001 101........ but we could instead remove the first two 0's to get 0 001 101.... and get a 22 bit string that contains all 3-bit strings as 3 consecutive bits somewhere in it. A string is *3-good* if it contains every 3-bit string as 3 consecutive bits somewhere in it.

Consider the digraph



Meaning of weighting on the edges: Look at the node 10 and edge leaving it to the node 00 – this is interpreted as 10 drop the first bit (1) and add the +0 at the end and you get 00.

1. Explain why there is a one to one mapping between every walk in this digraph and a 3-good string.

   See page 2.

2. Explain why a walk in the graph that includes every edge exactly once provides a minimum length 3-good string.

   See page 2.

3. Find a walk of minimum length and the corresponding 3-good string.

   See page 3.

1. A 3-good string can be constructed by starting with the two bits at the vertex at the start of the walk and adding the bit that corresponds to each edge to the end of the string, for each edge in the walk in sequence. Let $v_1 = b_1 b_2$ and $v_2 = b_2 b_3$. Because the walk includes every edge, then any 3-bit string $b_1 b_2 b_3$ will appear as a substring when the edge (v_1, v_2) appears in the walk. Thus, every walk is a 3-good string. Because each walk results in a unique sequence of bits in the bit strings constructed in this way, there is a one-to-one mapping between every walk in this digraph and a 3-good string.

2. The string constructed by a walk that includes every edge exactly once will be of length 10 b/c there are 2 bits in the starting vertex of the walk and 8 edges. This is a minimal length 3-good string b/c you start with 2 bits and each additional bit can produce at most one new 3-bit string, so you need a minimum of $2 + 8 = 10$ bits to include all 8 3-bit strings.
In addition, we observe that using an edge more than once results in a duplicate of a 3-bit string in the spot that the edge is used a second time.
Let $v_1 = 10$, $v_2 = 00$, $v_3 = 11$, $v_4 = 01$
Consider the edge sequence with labeled repeated edges:

$$(v_2, v_2), \underbrace{(v_2, v_4)}, (v_4, v_3), (v_3, v_3), (v_3, v_1), (v_1, v_4), (v_4, v_1), (v_1, v_2), \underbrace{(v_2, v_4)}$$

repeated edge

Note the duplicates in the corresponding bit string occur where we repeated an edge.

$$0\underbrace{001}1101\underbrace{001}$$

duplicate 3-bit string

Thus, for a minimum length 3-good string we must use each edge exactly once.

3. Let $v_1 = 10$, $v_2 = 00$, $v_3 = 11$, $v_4 = 01$

One walk of minimum length is the following edge sequence:

$(v_2, v_2), (v_2, v_4), (v_4, v_3), (v_3, v_3), (v_3, v_1), (v_1, v_4), (v_4, v_1), (v_1, v_2)$

This walk corresponds to the 3-good string 0001110100 of length 10.