50/50

Summer 2018

Math 54

Test 3

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First Name Stewart

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1. In the Sullivan Statistics survey, individual households were asked to disclose the number of television in their household. The random variable X represent the number of television in hose holds

Number of Television, x	0	1	2	3	4	5	6	7	8	9
P(x)	0	0.161	0.261	0.176	0.186	0.116	0.055	0.025	0.010	0.010

a. [4 points] Verify this is discrete probability distribution.

4

(i) $\Sigma P(x) = 1$

$0 + 0.161 + 0.261 + 0.176 + 0.186 + 0.116 + 0.055 + 0.025 + 0.010 + 0.010 = 1$

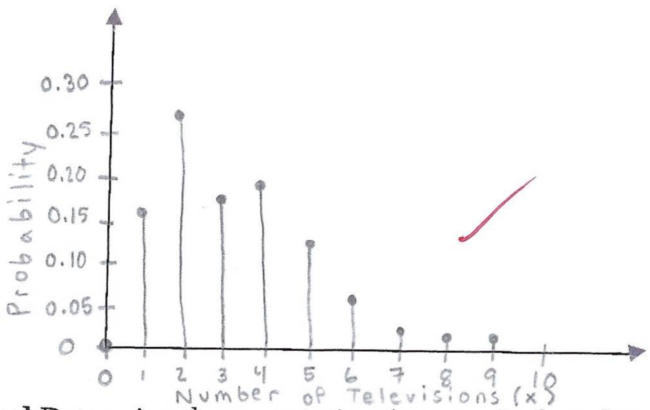
$1 = 1$ ✓

(ii) $0 \le P(x) \le 1$ ✓

∴ This is a discrete probability distribution

b. [2 points] Draw the probability histogram.

2



c. [2 point] Determine the expectation (or mean) of random variable, X

2

$E(X) = \mu_X = \Sigma x \cdot P(x) = 0(0) + 1(0.161) + 2(0.261) + 3(0.176) + 4(0.186) + 5(0.116) + 6(0.055) + 7(0.025) + 8(0.010) + 9(0.010)$

$= \boxed{3.21 \text{ televisions}}$

d. [2 point] What is the probability that a randomly selected house hold has four or more television?

2

$P(X \ge 4) = P(4 \text{ or } 5 \text{ or } 6 \text{ or } 7 \text{ or } 8 \text{ or } 9)$

$= P(4) + P(5) + P(6) + P(7) + P(8) + P(9)$

$= 0.186 + 0.116 + 0.055 + 0.025 + 0.010 + 0.010$

$= \boxed{0.402}$

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2.The National Health Interview Survey reports that 25% of telephone users no longer use landlines, and have switched completely to cell phone use. Suppose we take a random sample of 10 telephone users.

3

a.**[3 points]** verify that the binomial distribution properties are fulfilled.

(1) The number of trials is fixed: $n = 10$.

(2) The trials are independent: yes, $n \le 0.05N$ and one individual's cell phone use is independent of another's

(3) Two mutually exclusive outcomes: only use cell phone, do not only use cell phone

(4) Probability of success is the same for each trial: $p = 0.25$

2

b. **[2 points]** Find the probability that the sample contains **exactly 2** users who have abandoned their landlines.

$n = 10,\ x = 2,\ p = 0.25$

$P(2) = {}_{10}C_2 (0.25)^2 (1-0.25)^{10-2} = \boxed{0.2816}$

c. **[2 points]** Find the probability that the sample contains **at most 2** users who have abandoned their landlines

2

$n = 10,\ p = 0.25$

$P(x \le 2) = P(0 \text{ or } 1 \text{ or } 2)$

$= P(0) + P(1) + P(2)$

$= {}_{10}C_0 (0.25)^0 (1-0.25)^{10-0} + {}_{10}C_1 (0.25)^1 (1-0.25)^{10-1} + {}_{10}C_2 (0.25)^2 (1-0.25)^{10-2}$

$= 0.0563 + 0.1877 + 0.2816$

3

$= \boxed{0.5256}$

d.**[3 points]** Find the probability that the sample contains **at least 2** users who have abandoned their landlines

$P(x \ge 2) = 1 - P(x \le 2) + P(2)$

$= 1 - 0.5256 + 0.2816$

$= \boxed{0.7560}$

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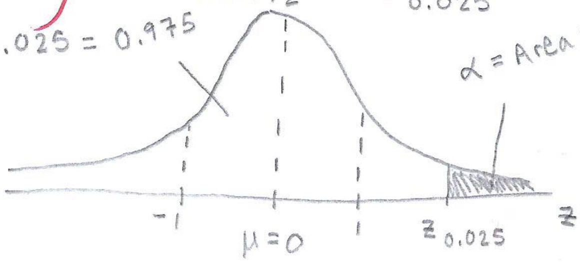
3. a.[3 points]. Find $Z_{\alpha/2}$ where $\alpha = 0.05$. Sketch the graph.

3

$z_{\alpha/2} = z_{0.05/2} = z_{0.025}$

Area = 1 − 0.025 = 0.975

α = Area = 0.025



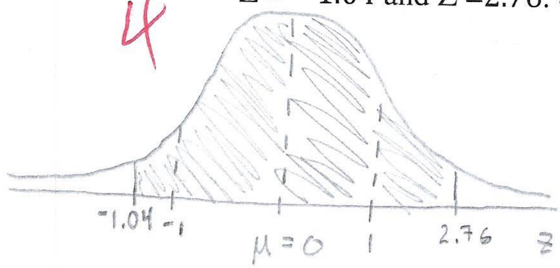
$P(z \geq z_{0.025}) = 0.025$

$z_{0.025} = \boxed{1.96}$

From Table V where area to the left = 1 − 0.025 = 0.975

b. **[4 points]** Determine the area under the standard normal curve between Z = −1.04 and Z =2.76. Sketch the graph

4



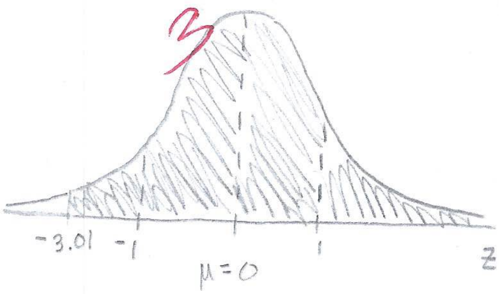
$P(-1.04 \leq z \leq 2.76) = P(z \leq 2.76) - P(z \leq -1.04)$

$= 0.9971 - 0.1492$

$= \boxed{0.8479}$

c. **[3 points]** Find the area under the normal curve to **the right** of $Z = -3.01$.

3



$P(z \geq -3.01) = 1 - P(z \leq -3.01)$

$= 1 - 0.0013$

$= \boxed{0.9987}$

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4. Assume that the random variable X is normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 7$

a. Describe the sample distribution:

i.[1 **points**] Find the inflection points

$\mu + \sigma = 50 + 7 = \boxed{57}$

$\mu - \sigma = 50 - 7 = \boxed{43}$

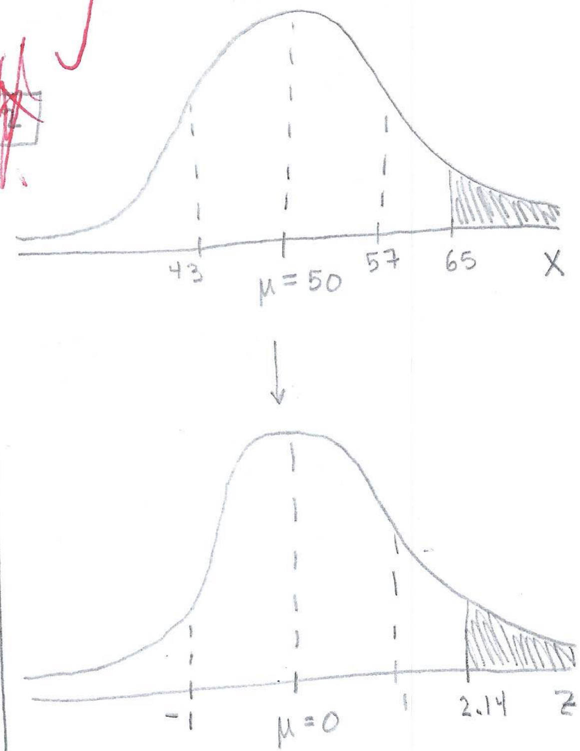
ii.[1 **points**] Find P(x ≥ 65)

$P(x \geq 65) = P\left(\frac{x-\mu}{\sigma} \geq \frac{65-\mu}{\sigma}\right) = P\left(\frac{x-50}{7} \geq \frac{65-50}{7}\right)$

$= P(z \geq 2.14)$

$= 1 - P(z \leq 2.14)$

$= 1 - 0.9838 = \boxed{0.0162}$



b.[3 **points**] Find $P(56 \leq X \leq 66)$.

$P(56 \leq X \leq 66) = P\left(\frac{56-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{66-\mu}{\sigma}\right)$

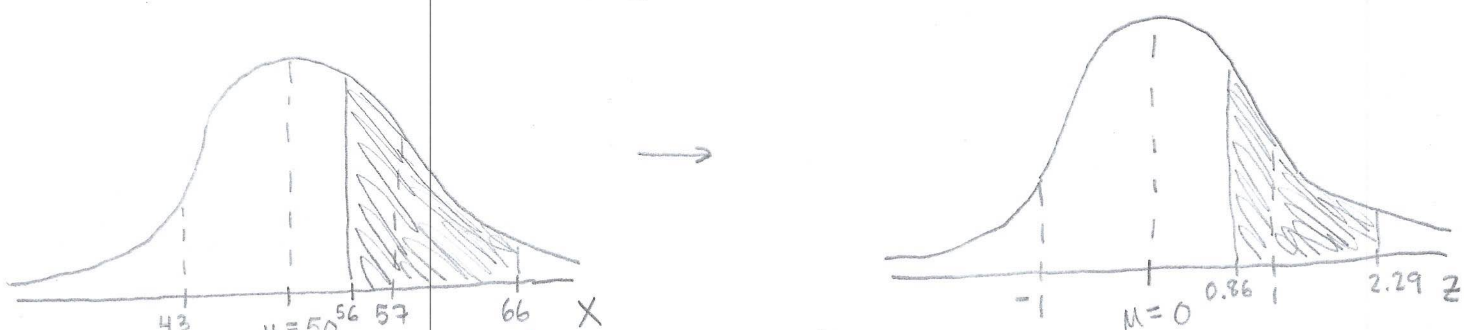
$= P\left(\frac{56-50}{7} \leq \frac{X-50}{7} \leq \frac{66-50}{7}\right)$

$= P(0.86 \leq z \leq 2.29)$

$= P(z \leq 2.29) - P(z \leq 0.86)$

$= 0.9890 - 0.8051$

$= \boxed{0.1839}$



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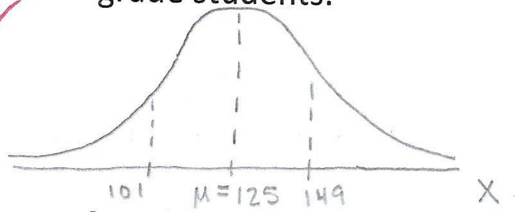
5. The reading speed of sixth-grade students is approximately normal, with a mean speed of 125 words per minute and a standard deviation of 24 words per minute.

a. **[2 points]** Draw a normal model that describes the reading speed of sixth-grade students.

2



Inflection points:

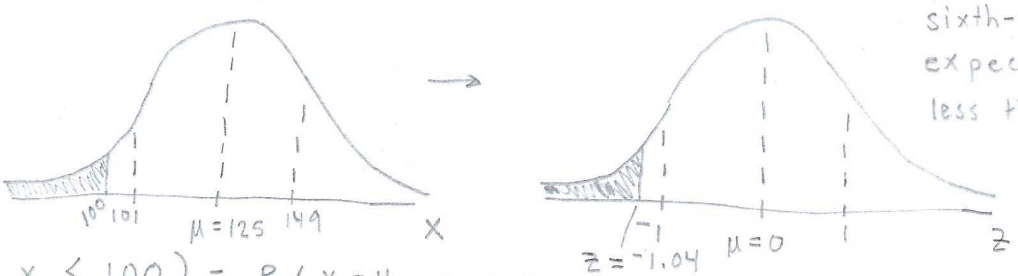
$\mu + \sigma = 125 + 24 = 149$ ✓

$\mu - \sigma = 125 - 24 = 101$ ✓

b. **[4 points]** Find and interpret the probability that randomly selected sixth-grade students reads less than 100 words per minute.

4

$P(x < 100)$

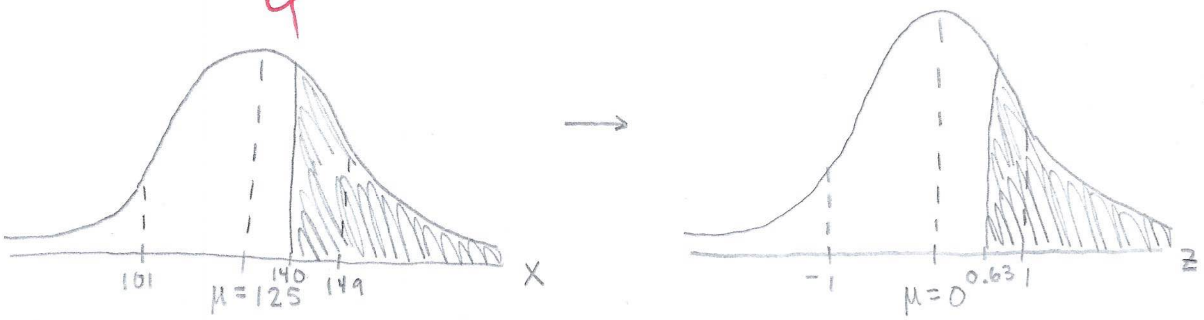


∴ If we randomly select 100 sixth-grade students, we would expect about 15 of them to read less than 100 wpm.

$$P(x < 100) = P\left(\frac{x-\mu}{\sigma} < \frac{100-\mu}{\sigma}\right) = P\left(\frac{x-125}{24} < \frac{100-125}{24}\right) = P(z < -1.04) = \boxed{0.1492}$$ ✓

c. **[4 points]** Find and interpret the probability that randomly selected sixth-grade students reads more than 140 words per minute

$P(x > 140)$ 4



$$P(x > 140) = P\left(\frac{x-\mu}{\sigma} > \frac{140-\mu}{\sigma}\right) = P\left(\frac{x-125}{24} > \frac{140-125}{24}\right) = P(z > 0.63)$$

$$= 1 - P(z \le 0.63)$$

$$= 1 - 0.7357$$

$$= \boxed{0.2643}$$

If we randomly selected 100 sixth-grade students, we would expect about 26 of them to read more than 140 wpm.

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6. A Simple random sample of n = 75 is obtained from a population whose size is N = 10,000 and whose population proportion with a specified characteristic is $p = 0.8$

Criteria for using Normal Model:

1) $n \le 0.05N$

$75 \le 0.05(10000)$

$75 \le 500$ ✓

2) $np(1-p) \ge 10$

$(75)(0.8)(1-0.8) \ge 10$

$12 \ge 10$ ✓

$\therefore$ sampling distribution of $\hat{p}$ is approximately normally distributed

a. Describe the sampling distribution of $\hat{p}$

i. **[1 points]** Find $\mu_{\hat{p}}$

$$\mu_{\hat{p}} = p = \boxed{0.8}$$

ii. **[1 points]** Find $\sigma_{\hat{p}}$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.8(1-0.8)}{75}} = \boxed{0.046}$$

b. **[3 points]** What is the probability of obtaining $x = 63$ or more individuals with a specified characteristic? That is, what is $P(\hat{p} \ge 0.84)$.

3

$$P(x \ge 63) = P(\hat{p} \ge 0.84) = P\left(\frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} \ge \frac{0.84 - \mu_{\hat{p}}}{\sigma_{\hat{p}}}\right)$$

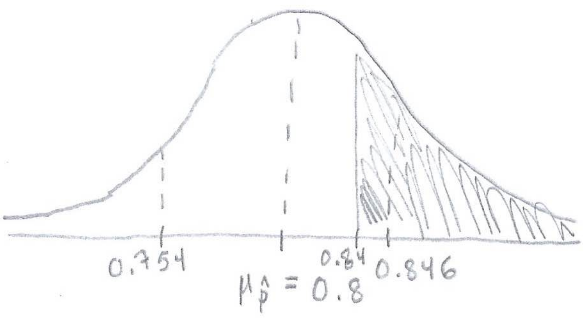
$$= P\left(\frac{\hat{p} - 0.8}{0.046} \ge \frac{0.84 - 0.8}{0.046}\right)$$

$$= P(z \ge 0.87)$$

$$= 1 - P(z \le 0.87)$$

$$= 1 - 0.8078$$

$$= \boxed{0.1922}$$



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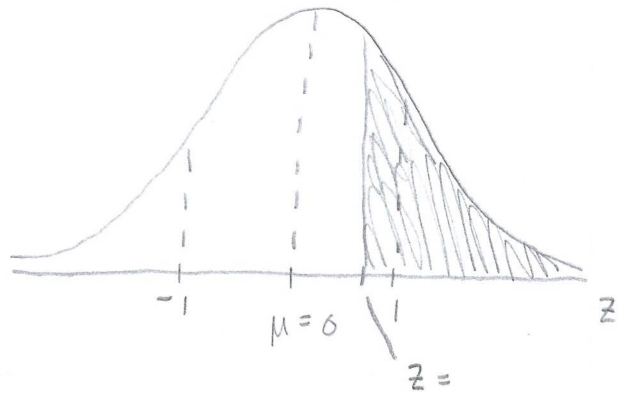

Inflection Points:

$\mu + \sigma = 0.8 + 0.046 = 0.846$

$\mu - \sigma = 0.8 - 0.046 = 0.754$

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