Dulaney, Stewart
MATH 54
Section 4053
SID: 1545566

10/10

HW 3

6.1

1. A random variable is a numerical measure of the outcome of a probability experiment, so its value is determined by chance.

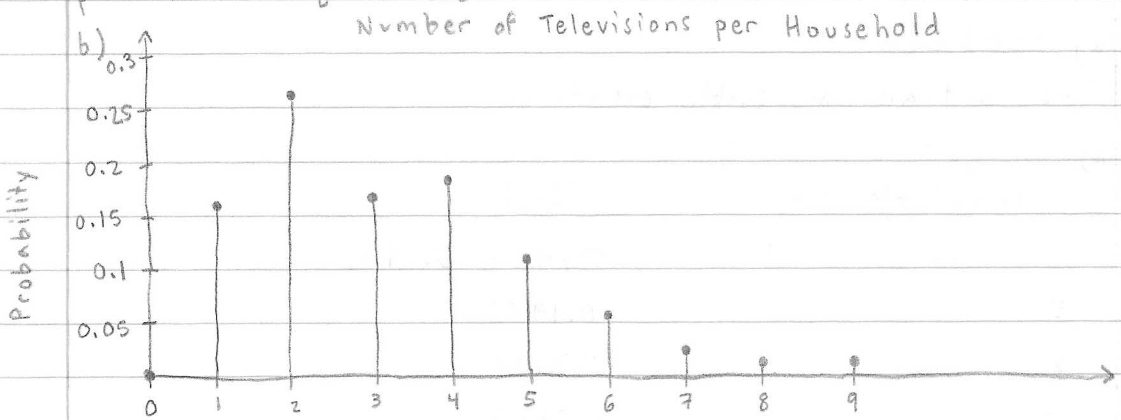
5. a) discrete $0, 1, 2, \ldots, 20$
b) continuous $t > 0$
c) discrete $0, 1, 2, \ldots$
d) continuous $s \geq 0$

9. Yes

13. No, $\Sigma P(x) \neq 1$

17. a) Each probability is between 0 and 1 inclusive, and the sum of the probabilities equals one.

b)



- skewed right

c) $\mu_x = \Sigma x \cdot P(x) = 0(0) + 1(0.161) + 2(0.261) + 3(0.176) + 4(0.186) + 5(0.116) + 6(0.055) + 7(0.025) + 8(0.010) + 9(0.010)$

≈ 3.21

If we surveyed many households, we would expect the mean number of televisions per household to be 3.21.

d)

x	$P(x)$	$(x - \mu_x)^2 \cdot P(x)$
0	0	$(0 - 3.21)^2 \cdot (0) = 0$
1	0.161	$(1 - 3.21)^2 \cdot (0.161) = 0.7863$
2	0.261	0.3821
3	0.176	0.0078
4	0.186	0.1161
5	0.116	0.3717
6	0.055	0.4281
7	0.025	0.3591
8	0.010	0.2294
9	0.010	0.3352

$\sigma_x = \sqrt{\Sigma[(x-\mu_x)^2 \cdot P(x)]} = \sqrt{3.0158} = 1.7366$ televisions

e) $P(3) = 0.176$

f) $P(3 \text{ or } 4) = 0.176 + 0.186 = 0.362$

g) $P(0) = 0$

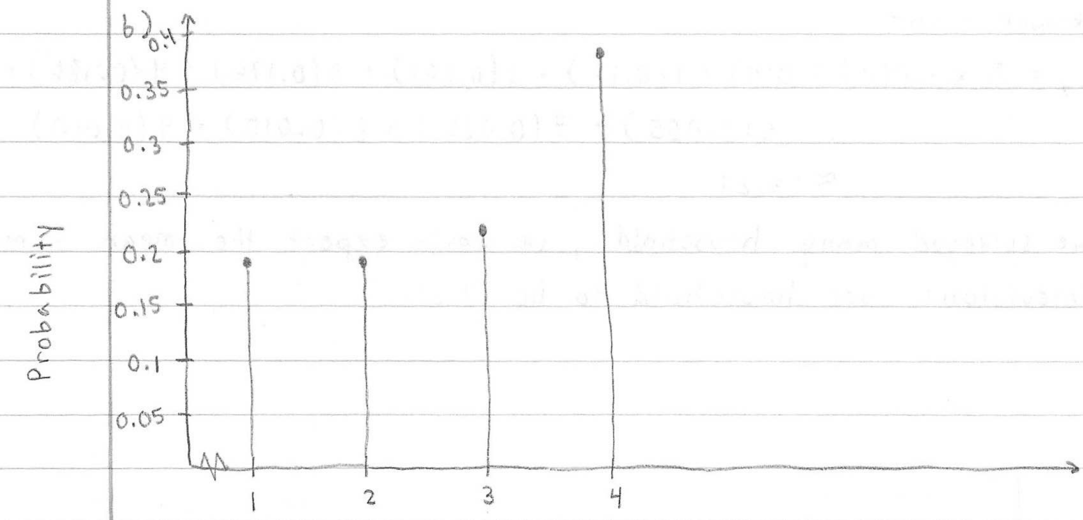
No, not an impossible event

21. a)

total = 91

x (games played)	$P(x)$
4	$18/91 = 0.1978$
5	0.1978
6	0.2198
7	0.3846

b)



c) $\mu_x = 4(0.1978) + 5(0.1978) + 6(0.2198) + 7(0.3846)$

≈ 5.7912

The World Series, if played many times, would be expected to last about 5.7912 games, on average.

d)

x	$P(x)$	$x^2 \cdot P(x)$
4	0.1978	$(4)^2(0.1978) = 3.1648$
5	0.1978	4.945
6	0.2198	7.9128
7	0.3846	18.8454

$\sigma_x = \sqrt{\Sigma[x^2 P(x)] - \mu_x^2} = \sqrt{34.868 - (5.7912)^2} = 1.1533$ games

25. Let the random variable X represent the payout (money lost or gained)

x	$P(x)$
200	0.999544
$200 - 250{,}000 = {}^{-}249{,}800$	0.000456

$E(x) = \mu_x = \Sigma[x \cdot P(x)] = 200(0.999544) + {}^{-}249{,}800(0.000456)$

≈ 86

The insurance company expects to make an average profit of \$86.00 on every 20-year-old female it insures for one year.

29. Let the random variable X represent the payout to the player (money lost or gained)

x	$P(x)$
175	$\frac{1}{38} = 0.0263$
$^{-}5$	$\frac{37}{38} = 0.9737$

$E(x) = \mu_x = 175(0.0263) + {}^{-}5(0.9737) = {}^{-}0.266$

$E(x) = \$0.27$. If you played 1000 times, you would expect to lose about \$270.00.

33. The simulations illustrate the Law of Large Numbers. As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to the probability of the outcome.

6.2

1. trial

5. np

9.
1. fixed # of trials ✓
2. trials independent ✓
3. mutually exclusive outcomes ✓
4. probability of success same for each trial ✓

∴ Binomial

13.
1. ⊘
2. ✓
3. ✓
4. ✓

Not binomial, b/c # of trials is not fixed

17. $P(x) = {}_nC_x \, p^x (1-p)^{n-x}$

$P(3) = {}_{10}C_3 (0.4)^3 (1-0.4)^{10-3} = 0.2150$

21. $P(3) = {}_8C_3 (0.35)^3 (1-0.35)^{8-3} = 0.2786$

25. $P(x \leq 3) = P(0 \text{ or } 1 \text{ or } 2 \text{ or } 3)$

$= P(0) + P(1) + P(2) + P(3)$

$= {}_7C_0 (0.5)^0 (1-0.5)^{7-0} + {}_7C_1 (0.5)^1 (1-0.5)^{7-1} + {}_7C_2 (0.5)^2 (1-0.5)^{7-2} + {}_7C_3 (0.5)^3 (1-0.5)^{7-3}$

$= 0.0078 + 0.0547 + 0.1641 + 0.2734$

$P(x \leq 3) = 0.5$

$P(x > 3) = 1 - P(x \leq 3) = 1 - 0.5 = 0.5$

29. $n = 6$, $p = 0.3$

a)

x	$P(x)$	$(x - \mu_x)^2 \cdot P(x)$ *
0	0.1176	$(0 - 1.7995)^2 (0.1176) = 0.0752$
1	0.3025	0.0122
2	0.3241	0.4671
3	0.1852	0.8968
4	0.0595	0.6095
5	0.0102	0.1800
6	0.0007	0.0189

b) $\mu_x = 0(0.1176) + 1(0.3025) + 2(0.3241) + 3(0.1852) + 4(0.0595) + 5(0.0102) + 6(0.0007)$

≈ 1.7995

$\cancel{\sigma_x = \sqrt{2.2597} = 1.5032}$

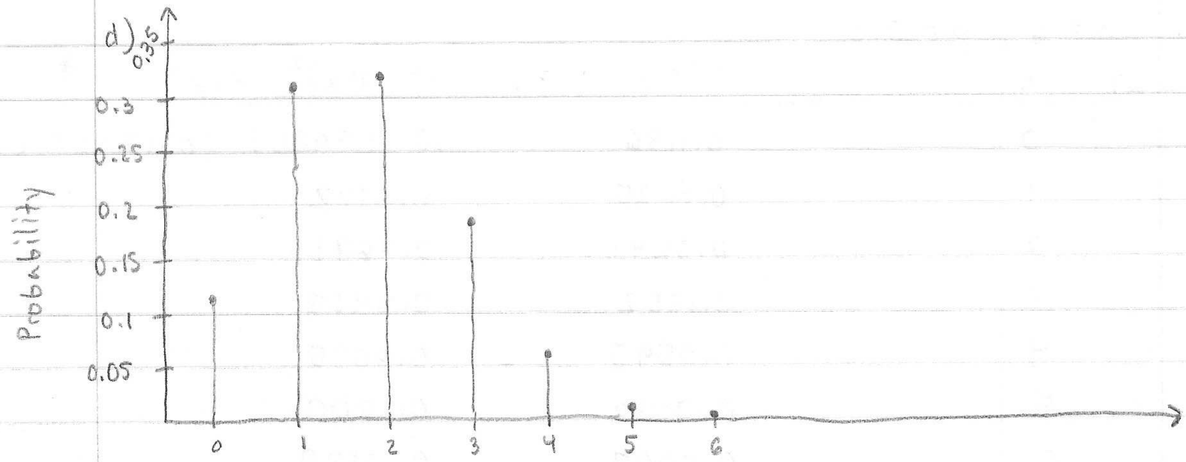
* $x^2 \cdot P(x)$
$(0^2)(0.1176) = 0$
0.3025
1.2964
1.6668
0.952
0.255
0.0252

$\sigma_x = \sqrt{4.4979 - (1.7995)^2} = 1.1224$

c) $\mu_x = np = (6)(0.3) = 1.8$

$\sigma_x = \sqrt{np(1-p)} = \sqrt{(6)(0.3)(1-0.3)} = 1.1225$

d)



- skewed right

33. $n = 10,\ p = 0.5$

a)

x	$P(x)$	$x^2 \cdot P(x)$
0	0.0010	$(0)^2 \cdot (0.0010) = 0$
1	0.0098	0.0098
2	0.0439	0.1756
3	0.1172	1.0548
4	0.2051	3.2816
5	0.2461	6.1525
6	0.2051	7.3836
7	0.1172	5.7428
8	0.0439	2.8096
9	0.0098	0.7938
10	0.0010	0.1

b) $\mu_x = 0(0.0010) + 1(0.0098) + 2(0.0439) + 3(0.1172) + 4(0.2051) + 5(0.2461) + 6(0.2051) + 7(0.1172) + 8(0.0439) + 9(0.0098) + 10(0.0010)$

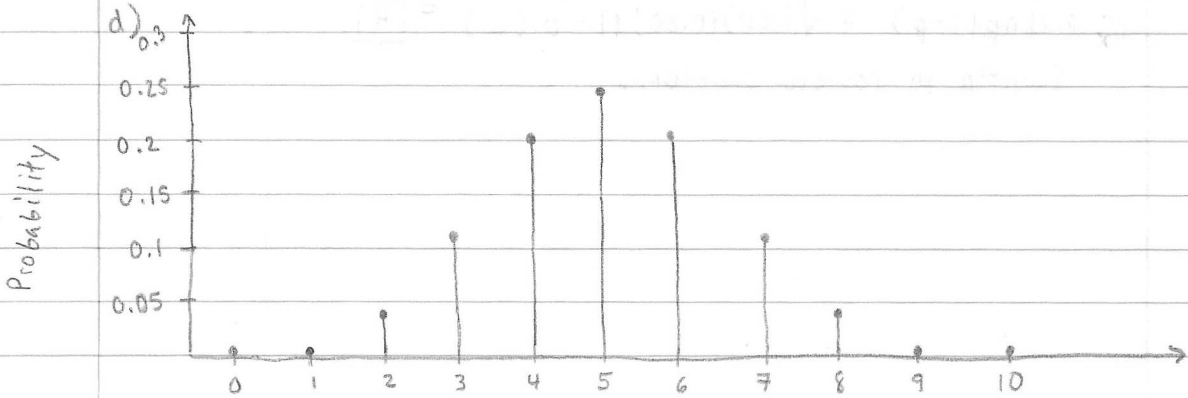
≈ 5.0005

$\sigma_x = \sqrt{27.5041 - (5.0005)^2} = 1.5809$

c) $\mu_x = np = (10)(0.5) = 5$

$\sigma_x = \sqrt{(10)(0.5)(1-0.5)} = 1.5811$

d)



- symmetric

39. $p = 0.267$

a) $n = 10 \quad x = 4$

$P(4) = {}_{10}C_4 \, (0.267)^4 (1-0.267)^{10-4} = \boxed{0.1655}$

b) $n = 10$

$P(x < 3) = P(0 \text{ or } 1 \text{ or } 2)$

$= P(0) + P(1) + P(2)$

$=$ binompdf(10, 0.267, 0) + binompdf(10, 0.267, 1) + binompdf(10, 0.267, 2)

$= \boxed{0.4752}$

c) $n = 10$

$P(x < 5) = P(0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \text{ or } 4)$

$= P(0) + P(1) + P(2) + P(3) + P(4)$

$=$ binompdf(10, 0.267, 0) + ... + binompdf(10, 0.267, 4)

$= \boxed{0.9004}$

No, b/c this result is not unusual b/c $P(x<5) = 0.9004 > 0.05$.

43. $p = 0.80 \quad n = 100$

a) $\mu_x = np = (100)(0.80) = \boxed{80}$

$\sigma_x = \sqrt{np(1-p)} = \sqrt{(100)(0.80)(1-0.80)} = \boxed{4}$

(cont'd in review problems)

7.1

1. probability density function

3. True

5. $\mu - \sigma$, $\mu + \sigma$

7. no, not symmetric

9. no, this graph is not always greater than or equal to zero, it cannot represent a normal <u>density function</u>

11. yes

13. a) $P(5 \le X \le 10) = (\text{width})(\text{height}) = (5)(\frac{1}{30}) = \frac{1}{6}$

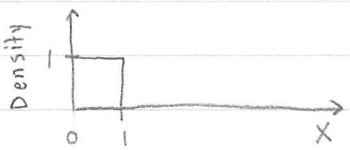
b) $x_0 \cdot \frac{1}{30} = 0.4$

$x_0 = 0.4(30) = 12$

15. $P(X > 20) = P(20 \le X \le 30)$

$= (10)(\frac{1}{30}) = \frac{1}{3}$

17. a)



b) $P(0 \le X \le 0.2) = (0.2)(1) = 0.2$

c) $P(0.25 \le X \le 0.6) = (0.6 - 0.25)(1) = (0.35)(1) = 0.35$

d) $P(0.95 \le X \le 1) = (0.05)(1) = 0.05$

e) $\frac{46}{200} = 0.23$, close to computed probability

19. Normal

21. Not normal

23. Graph A: $\mu = 10$, $\sigma = 2$

Graph B: $\mu = 10$, $\sigma = 3$

The larger the standard deviation, the lower and more spread out the graph is.

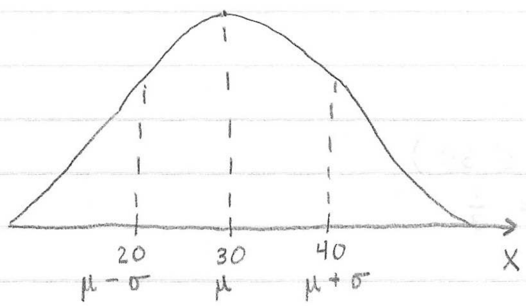
25. $\mu = 2$

$$5 = \mu + \sigma$$
$$5 = 2 + \sigma$$
$$\sigma = 3$$

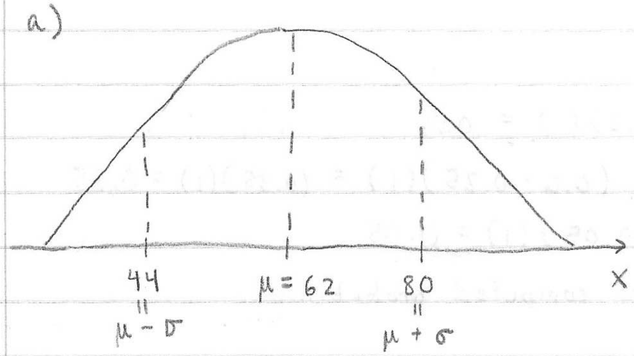
27. $\mu = 100$

$$85 = \mu - \sigma$$
$$85 = 100 - \sigma$$
$$-\sigma = -15$$
$$\sigma = 15$$

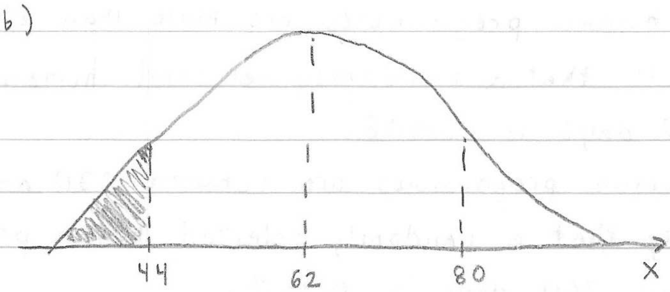
29.



31. a)



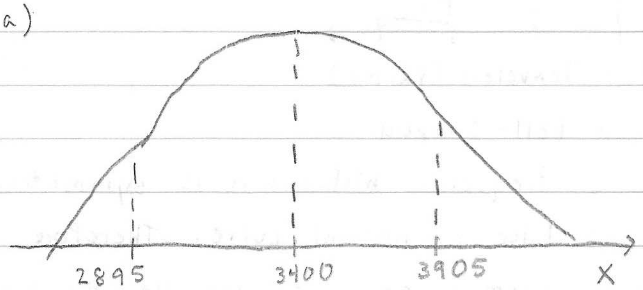
b)



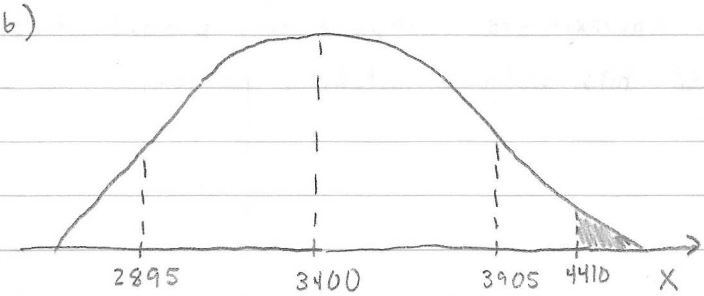
c) (i) 15.87% of the cell phone plans in the United States are less than \$44

(ii) The probability that a randomly selected phone plan in the United States is less than \$44 is 0.1587.

33. a)



b)



c) (i) 2.28% of the birth weights of full-term babies are more than 4410 grams

(ii) The probability that the birth weight of a randomly selected full-term baby is more than 4410 grams is 0.0228

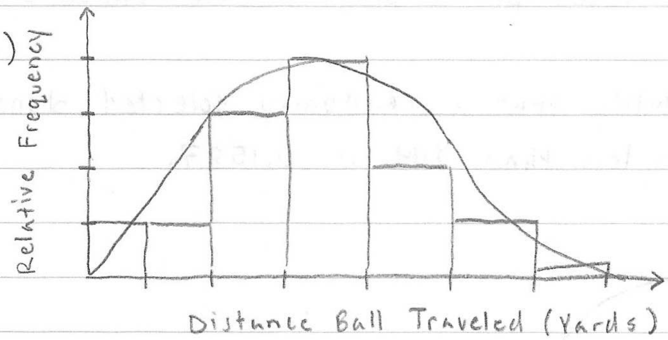
35. a) (i) 19.08% of human pregnancies are more than 280 days

(ii) The probability that a randomly selected human pregnancy is more than 280 days is 0.1908.

b) (i) 34.16% of human pregnancies are between 230 and 260 days

(ii) The probability that a randomly selected human pregnancy is between 230 and 260 days is 0.3416.

37. a)



(TI-84)

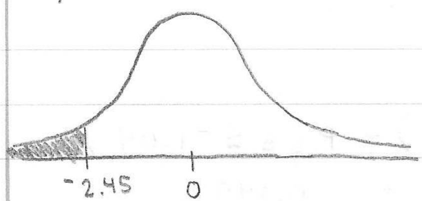
roughly symmetric and bell-shaped

b) Yes b/c the relative frequency histogram is symmetric and bell-shaped and shaped like a normal curve. Therefore the distance is approximately normal and we can use the area under the normal curve to approximate proportions/probabilities to describe the distance Michael hits with a pitching wedge.

7.2

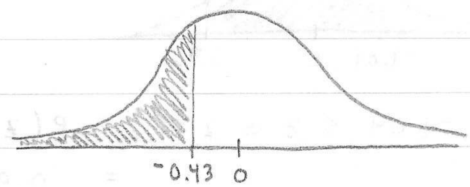
1. standard normal distribution

5. a) $z = -2.45$



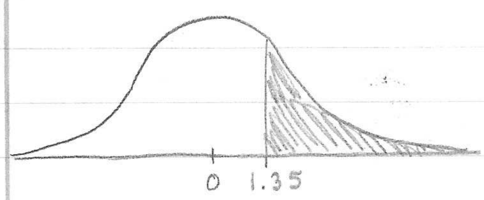
Area = 0.0071

b) $z = -0.43$



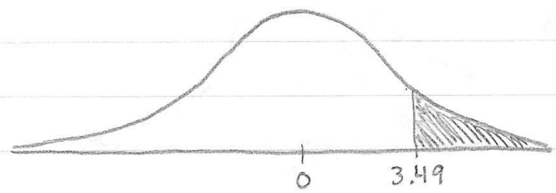
Area = 0.3336

c) $z = 1.35$



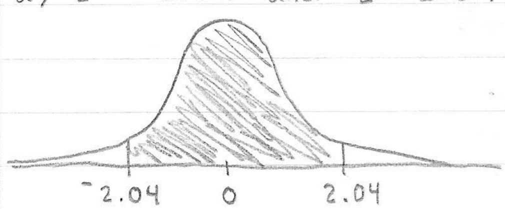
Area = 0.9115

d) $z = 3.49$



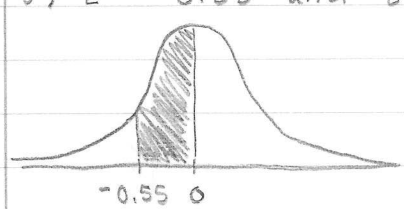
Area = 0.9998

9. a) $z = -2.04$ and $z = 2.04$



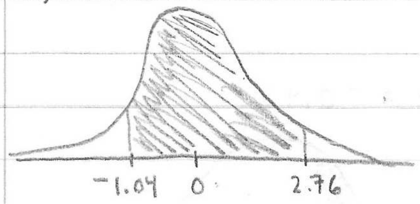
$$P(-2.04 \le z \le 2.04) = P(z \le 2.04) - P(z \le -2.04)$$
$$= 0.9793 - 0.0207$$
$$= 0.9586$$

b) $z = -0.55$ and $z = 0$



$$P(-0.55 \le z \le 0) = P(z \le 0) - P(z \le -0.55)$$
$$= 0.5000 - 0.2912$$
$$= 0.2088$$

c) $z = {}^{-}1.04$ and $z = 2.76$

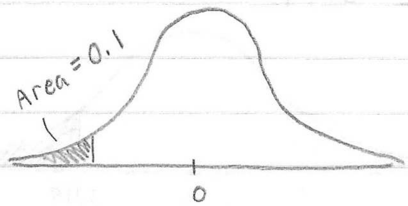


$P(^{-}1.04 \le z \le 2.76) = P(z \le 2.76) - P(z \le {}^{-}1.04)$

$= 0.9971 - 0.1492$

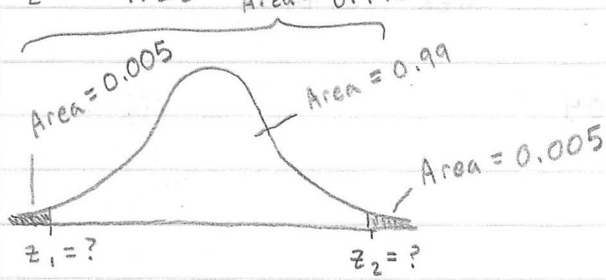
$= 0.8479$

13.



$z = {}^{-}1.28$

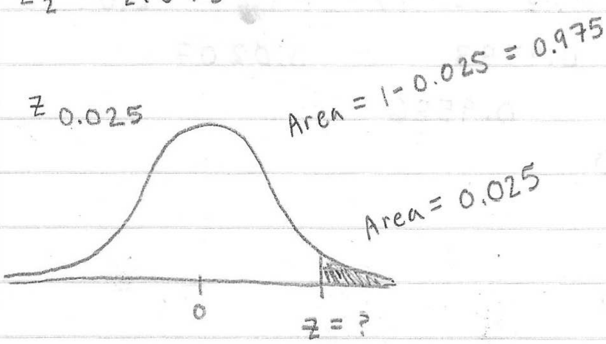
17.



$z_1 = {}^{-}2.575$

$z_2 = 2.575$

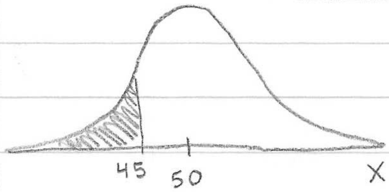
21. $z_{0.025}$

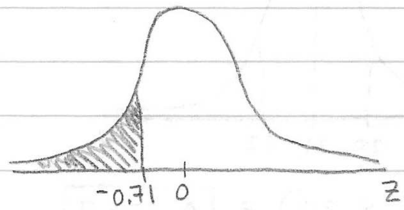


$z = 1.96$

25. $\mu = 50 \quad \sigma = 7$

$P(X \leq 45) = \boxed{0.2389}$

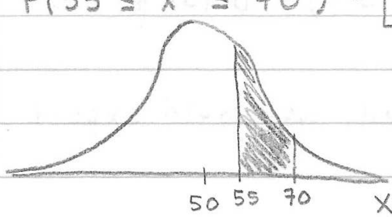


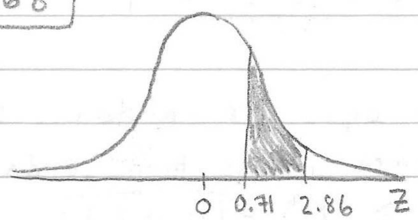


$z = \frac{x - \mu}{\sigma} = \frac{45 - 50}{7} = -0.71$

29. $\mu = 50 \quad \sigma = 7$

$P(55 \leq X \leq 70) = \boxed{0.2368}$

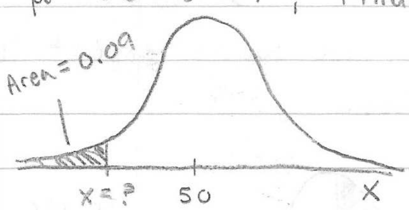


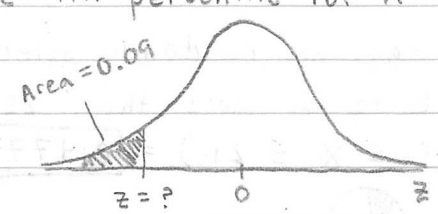


$z_1 = \frac{55 - 50}{7} = 0.71 \qquad z_2 = \frac{70 - 50}{7} = 2.86$

$$\begin{aligned} P(0.71 \leq z \leq 2.86) &= P(z \leq 2.86) - P(z \leq 0.71) \\ &= 0.9979 - 0.7611 \\ &= 0.2368 \end{aligned}$$

33. $\mu = 50 \quad \sigma = 7$, Find the 9th percentile for X



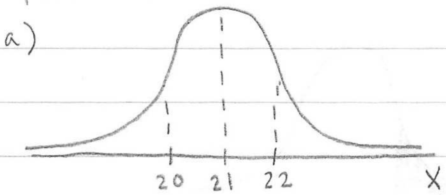


$x = \mu + z\sigma = 50 + (-1.34)(7) \qquad z = -1.34$

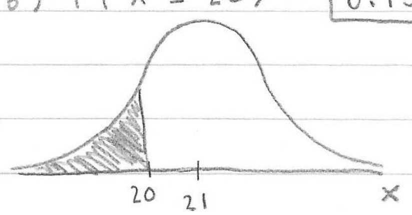
$= \boxed{40.62}$

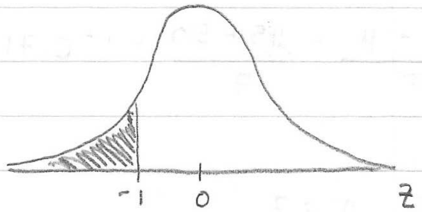
37. $\mu = 21 \quad \sigma = 1$

a)



b) $P(X \leq 20) = \boxed{0.1587}$

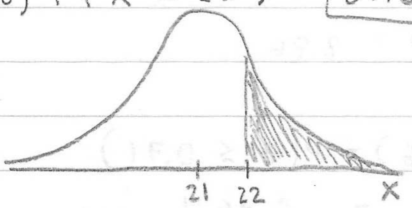


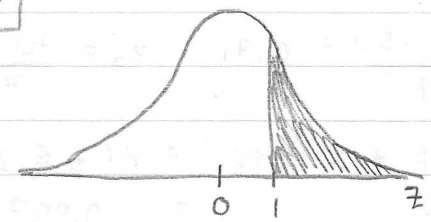


$z = \dfrac{X - \mu}{\sigma} = \dfrac{20-21}{1} = -1 \qquad P(z \leq -1) = 0.1587$

If 100 eggs are randomly selected, we would expect 16 to incubate in less than 20 days.

c) $P(X \geq 22) = \boxed{0.1587}$





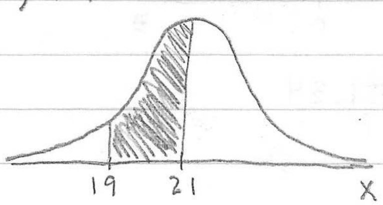
$z = \dfrac{X - \mu}{\sigma} = \dfrac{22-21}{1} = 1 \qquad P(z \geq 1) = 1 - P(z \leq 1)$

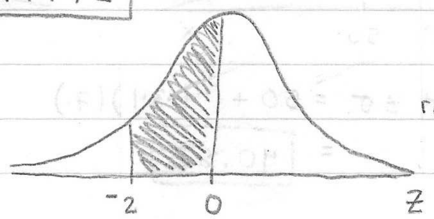
$= 1 - 0.8413$

$= 0.1587$

If 100 eggs are randomly selected, we would expelt 16 to incubate in more than 22 days.

d) $P(19 \leq X \leq 21) = \boxed{0.4772}$





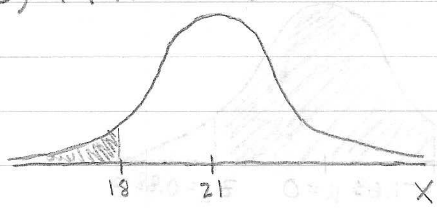
If 100 eggs are randomly selected, we would expect 48 to incubate between 19 and 21 days.

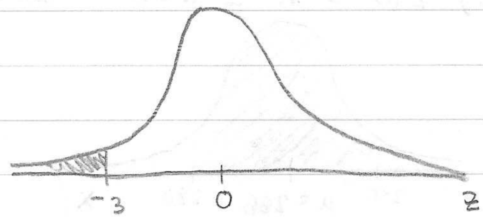
$z_1 = \dfrac{19-21}{1} = -2 \qquad z_2 = \dfrac{21-21}{1} = 0$

$P(-2 \leq z \leq 0) = P(z \leq 0) - P(z \leq -2)$

$= 0.5000 - 0.0228 = 0.4772$

e) $P(X \le 18) = 0.0013$



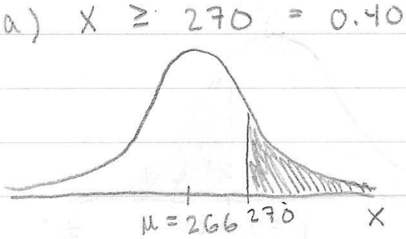


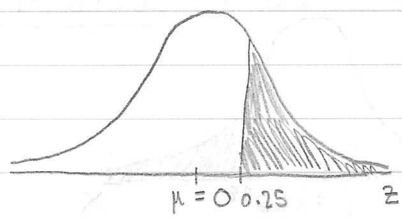
$z = \dfrac{18-21}{1} = {}^{-}3$ $\qquad P(z \le {}^{-}3) = 0.0013$

Yes b/c $P(X \le 18) = 0.0013 \le 0.05$. We would expect only 1 egg in 1000 to incubate in less than 18 days.

41. $\mu = 266 \quad \sigma = 16$

a) $X \ge 270 = 0.4013$





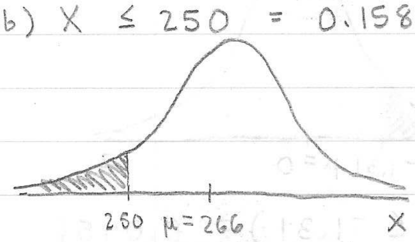
$z = \dfrac{270-266}{16} = 0.25$ $\qquad z \ge 0.25 = 1 - z \le 0.25$

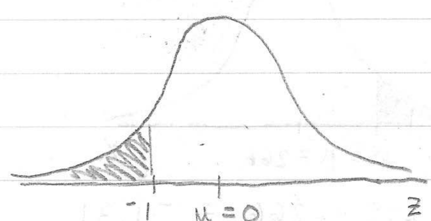
$= 1 - 0.5987$

$= 0.4013$

0.4013 of pregnancies last more than 270 days

b) $X \le 250 = 0.1587$

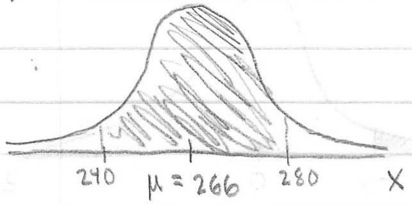


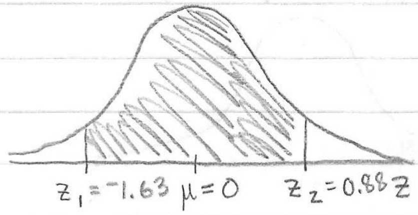


$z = \dfrac{250-266}{16} = {}^{-}1$ $\qquad z \le {}^{-}1 = 0.1587$

0.1587 of pregnancies last less than 250 days

c) $240 \le X \le 280 = 0.7590$





$z_1 = \frac{240 - 266}{16} = {}^{-}1.63 \quad z_2 = \frac{280 - 266}{16} = 0.88$

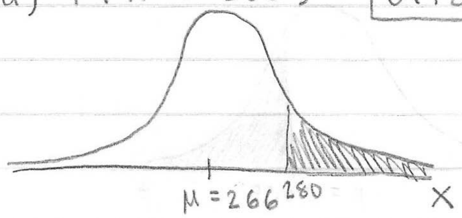
$^{-}1.63 \le z \le 0.88 = z \le 0.88 - z \le {}^{-}1.63$

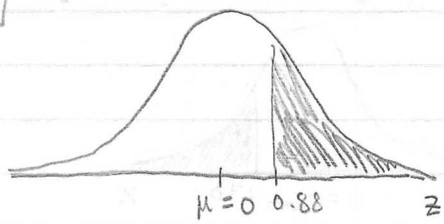
$= 0.8106 - 0.0516$

$= 0.7590$

0.7590 of pregnancies are between 240 and 280 days

d) $P(X > 280) = \boxed{0.1894}$



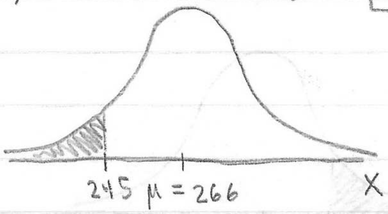


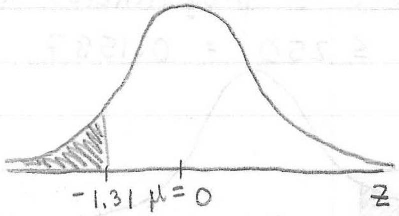
$z = \frac{280 - 266}{16} = 0.88 \qquad P(z > 0.88) = 1 - P(z \le 0.88)$

$= 1 - 0.8106$

$= 0.1894$

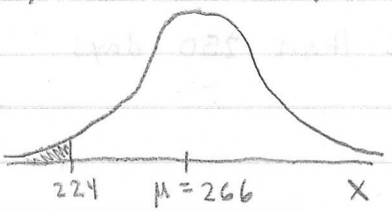
e) $P(X \le 245) = \boxed{0.0951}$

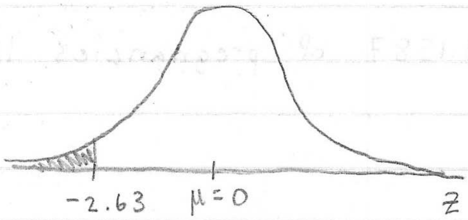




$z = \frac{245 - 266}{16} = {}^{-}1.31 \qquad P(z \le {}^{-}1.31) = 0.0951$

f) $P(X < 224)$





$z = \frac{224 - 266}{16} = {}^{-}2.63 \qquad P(z < {}^{-}2.63) = 0.0043$

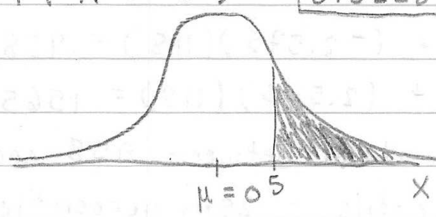
Yes unusual b/c $P(X < 224) = 0.0043 \le 0.05$. We would expect only 4 pregnancies in 1000 to be less than 224 days.

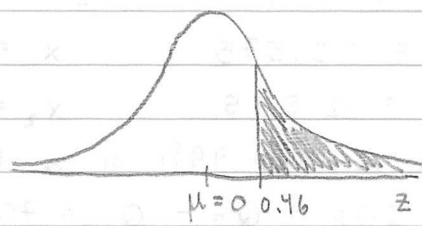
45. a) The favored team is equally likely to win or lose relative to the spread. Yes, a mean of 0 implies the spreads are accurate.

☐ Why?

b) $\mu = 0 \quad \sigma = 10.9$

$P(X \geq 5) = \boxed{0.3228}$

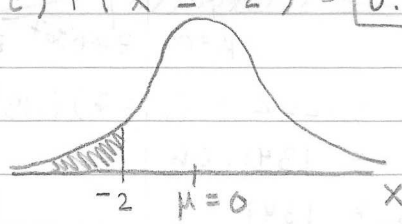


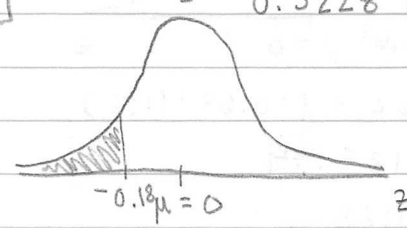


$z = \dfrac{5 - 0}{10.9} = 0.46$

$P(z \geq 0.46) = 1 - P(z \leq 0.46)$
$= 1 - 0.6772$
$= 0.3228$

c) $P(X \leq {}^{-}2) = \boxed{0.4286}$



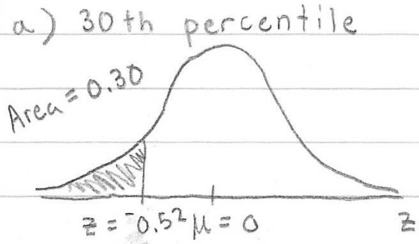


$z = \dfrac{{}^{-}2 - 0}{10.9} = {}^{-}0.18$

$P(z \leq {}^{-}0.18) = 0.4286$

49. $\mu = 1262 \quad \sigma = 118$

a) 30th percentile

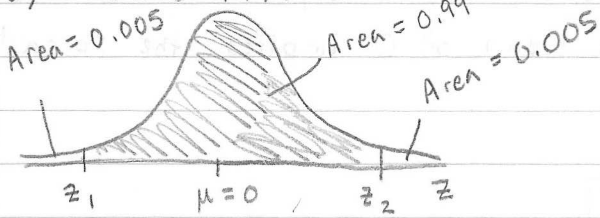


$x = \mu + z\sigma = 1262 + ({}^{-}0.52)(118)$
$= 1200.64$
$\approx \boxed{1201 \text{ chips}}$

The 30th percentile for the number of chips in an 18 oz bag is 1201 chips.

Area = 0.995

b) middle 99%

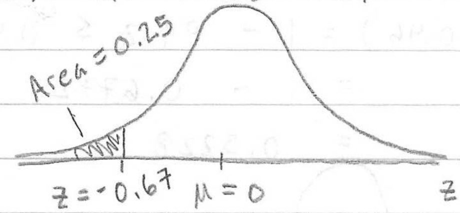


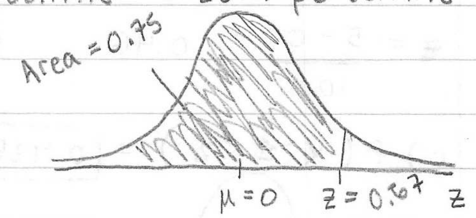
$z_1 = -2.575 \qquad x_1 = 1262 + (-2.575)(118) = 958.15 \approx 958$

$z_2 = 2.575 \qquad x_2 = 1262 + (2.575)(118) = 1565.85 \approx 1566$

The middle 99% of the bags contain between 958 and 1566 chips

c) $IQR = Q_3 - Q_1$ = 75th percentile − 25th percentile





$x = 1262 + (-0.67)(118) \qquad x = 1262 + (0.67)(118)$

$= 1182.94 \qquad = 1341.06$

$Q_1 \approx 1183 \qquad Q_3 \approx 1341$

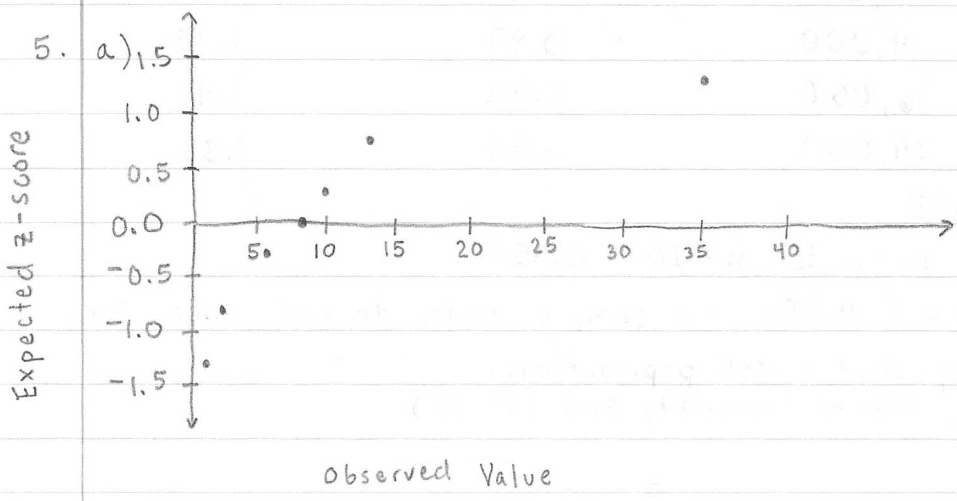
$IQR = 1341 - 1183 = \boxed{158 \text{ chips}}$

53. The area under a normal curve can be interpreted as a proportion, probability, or percentile.

7.3

1. normal probability plot

5. a)



b) $r = 0.873$

c) critical value for $n = 7$: 0.898

∴ the sample data do not come from a population that is normally distributed b/c $0.873 < 0.898$

9.

Index, i	Observed Value	f_i	Expected z-score
1	750	$\frac{1-0.375}{20+0.25} = 0.03$	-1.88
2	1,000	0.08	-1.41
3	1,400	0.13	-1.13
4	1,800	0.18	-0.92
5	2,000	0.23	-0.74
6	2,200	0.28	-0.58
7	2,200	0.33	-0.44
8	2,500	0.38	-0.31
9	2,500	0.43	-0.18
10	2,800	0.48	-0.05
11	3,800	0.52	0.05
12	5,000	0.57	0.18
13	6,200	0.62	0.31
14	9,100	0.67	0.44
15	10,100	0.72	0.58

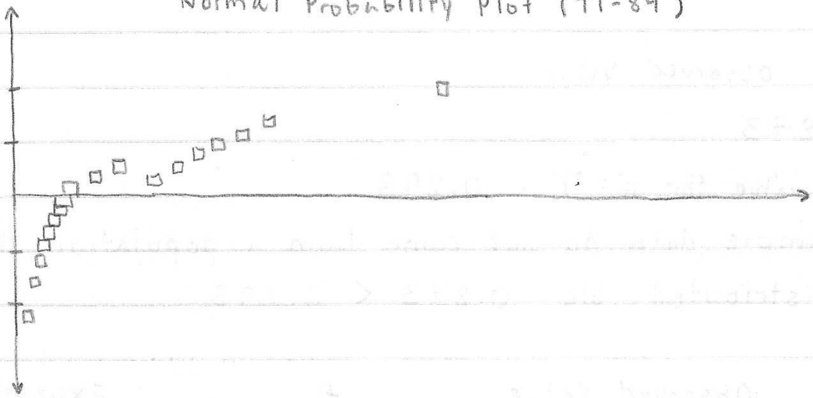
16	12,000	0.77	0.74
17	13,200	0.82	0.92
18	14,000	0.87	1.13
19	16,000	0.92	1.41
20	29,000	0.97	1.88

r = 0.883

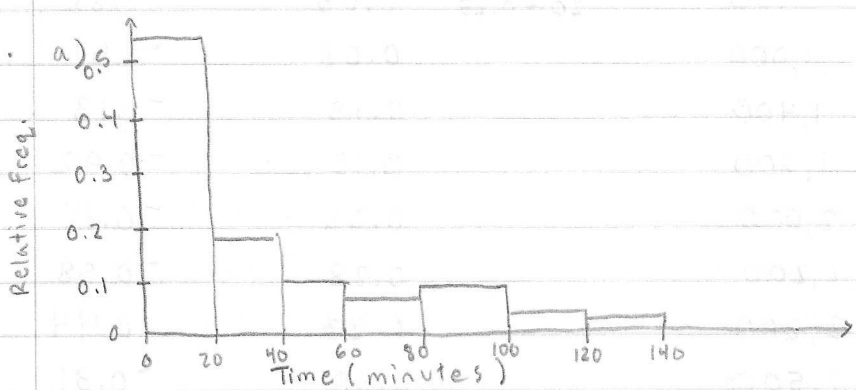
critical value for n = 20 : 0.951

B/c 0.883 < 0.951 the sample data do not come from a normally distributed population.

Normal Probability Plot (TI-84)

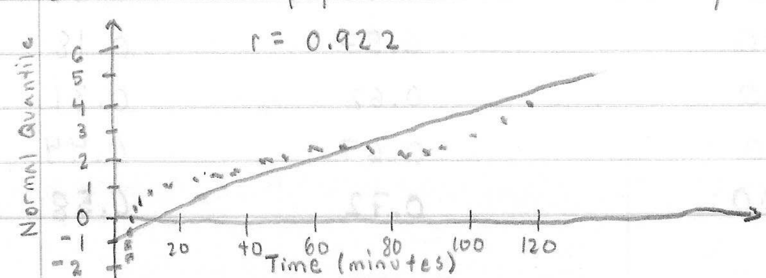


13. a)



b) skewed right

c) b/c 0.922 < 0.960 (Table VI w/ n = 30), the data do not come from a population that is normally distributed.



8.1

1. sampling distribution

5. False

9. $\mu = 80,\ \sigma = 14,\ n = 49$

$\mu_{\bar{x}} = \mu = \boxed{80}$ $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14}{\sqrt{49}} = \frac{14}{7} = \boxed{2}$

13. a) $\mu_{\bar{x}} = \boxed{500}$

b) $520 = \mu_{\bar{x}} + \sigma_{\bar{x}}$

$520 = 500 + \sigma_{\bar{x}}$

$\sigma_{\bar{x}} = \boxed{20}$

c) The population must be normally distributed

d) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$20 = \frac{\sigma}{\sqrt{16}}$

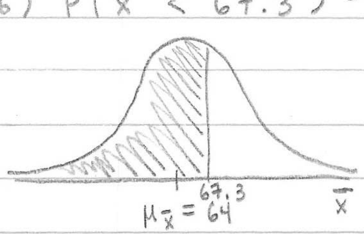
$\boxed{80} = \sigma$

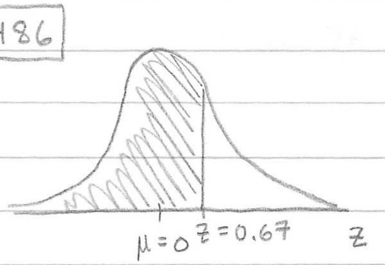
17. $n = 12,\ \mu = 64,\ \sigma = 17$

a) The population must be normally distributed in order to use the normal model to compute probabilities involving the sample mean. If the population is normally distributed, then the sampling distribution of $\bar{x}$ is also normally distributed with:

$\mu_{\bar{x}} = \mu = 64$ and $\sigma_{\bar{x}} = \frac{17}{\sqrt{12}} = 4.907$

b) $P(\bar{x} < 67.3) = \boxed{0.7486}$

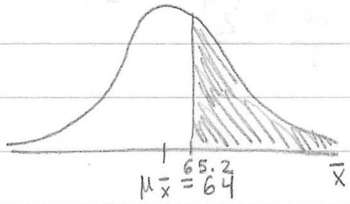


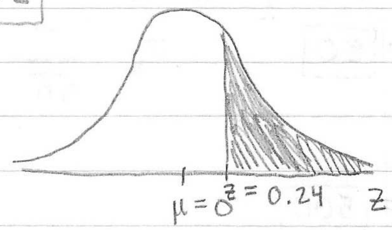


$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{67.3 - 64}{4.907} = 0.67$ $\quad P(z \le 0.67) = 0.7486$

If we take 100 simple random samples of size $n = 12$ from a population that is normally distributed with $\mu = 64$ and $\sigma = 17$, then about 75 of the samples will result in a mean that is less than 67.3.

c) $P(\bar{x} \geq 65.2) = \boxed{0.4052}$





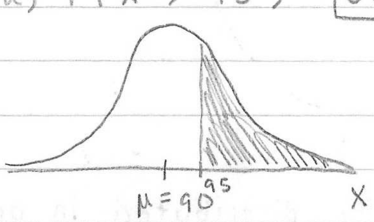
$z = \dfrac{65.2 - 64}{4.907} = 0.24$

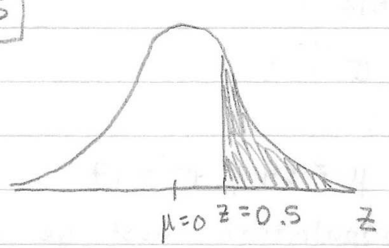
$P(z \geq 0.24) = 1 - P(z \leq 0.24)$
$= 1 - 0.5948$
$= 0.4052$

" , then about 40 or 41 of the samples will result in a mean that is greater than or equal 65.2.

21. $\mu = 90$, $\sigma = 10$

a) $P(X > 95) = \boxed{0.3085}$





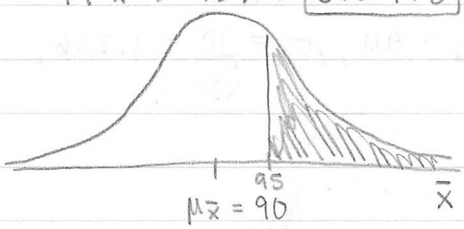
$z = \dfrac{95 - 90}{10} = 0.5$

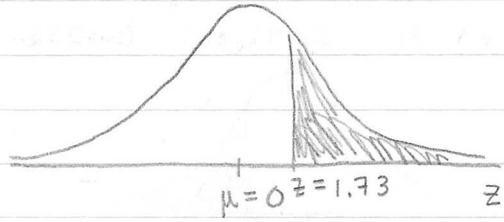
$P(z > 0.5) = 1 - P(z \leq 0.5)$
$= 1 - 0.6915$
$= 0.3085$

If we select a simple random sample of $n = 100$ second grade students, then about 31 of the students will read more than 95 wpm.

b) $n = 12$ $\mu_{\bar{x}} = \mu = 90$ $\sigma_{\bar{x}} = \frac{10}{\sqrt{12}} = 2.887$

$P(\bar{x} > 95) = \boxed{0.0418}$





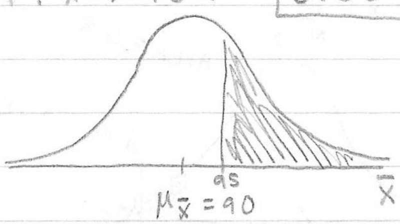
$z = \frac{95 - 90}{2.887} = 1.73$

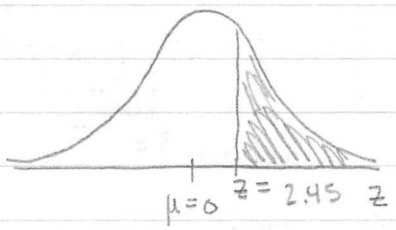
$P(z > 1.73) = 1 - P(z \leq 1.73)$
$= 1 - 0.9582$
$= 0.0418$

If we take 100 simple random samples of size $n = 12$ second grade students, then about 4 of the samples will result in a mean reading rate that is more than 95 wpm.

c) $n = 24$ $\mu_{\bar{x}} = \mu = 90$ $\sigma_{\bar{x}} = \frac{10}{\sqrt{24}} = 2.041$

$P(\bar{x} > 95) = \boxed{0.0071}$





$z = \frac{95 - 90}{2.041} = 2.45$

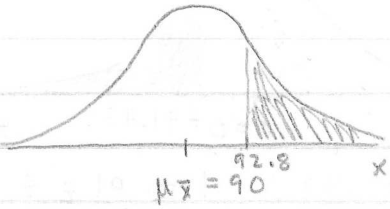
$P(z > 2.45) = 1 - P(z \leq 2.45)$
$= 1 - 0.9929$
$= 0.0071$

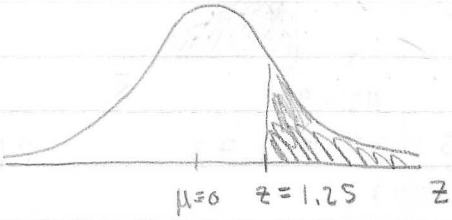
If we take 1000 simple random samples of size $n = 24$ second grade students, then about 7 of the samples will result in a mean reading rate that is more than 95 wpm.

□ Clarify why

d) Increasing the sample size decreases $P(\bar{x} > 95)$. This happens b/c $\sigma_{\bar{x}}$ decreases as n increases.

e) $P(\bar{x} \geq 92.8) = 0.1056$ $\mu_{\bar{x}} = 90$, $\sigma_{\bar{x}} = \frac{10}{\sqrt{20}} = 2.236$, $n = 20$



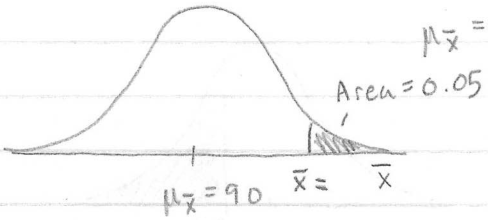


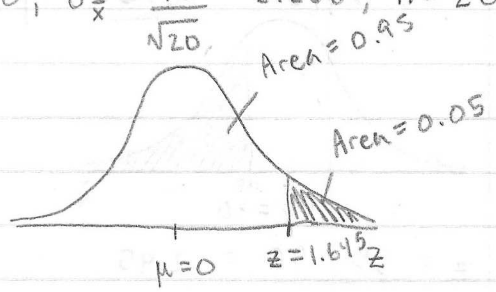
$z = \frac{92.8 - 90}{2.236} = 1.25$

$P(z \geq 1.25) = 1 - P(z \leq 1.25)$
$= 1 - 0.8944$
$= 0.1056$

A mean reading rate is not unusual since $P(\bar{x} \geq 92.8) = 0.1056$. This means that the new reading program is not abundantly more effective than the old program.

f) $\mu_{\bar{x}} = 90$, $\sigma_{\bar{x}} = \frac{10}{\sqrt{20}} = 2.236$, $n = 20$





$\bar{x} = \mu_{\bar{x}} + z\,\sigma_{\bar{x}}$
$= 90 + (1.645)(2.236)$
$= \boxed{93.7}$

$z = 1.645$

There is a 5% chance that the mean reading speed of a random sample of 20 second grade students will exceed 93.7 wpm.

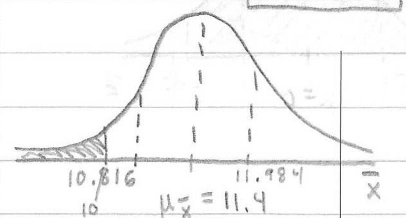
25. $\mu = 11.4 \quad \sigma = 3.2$

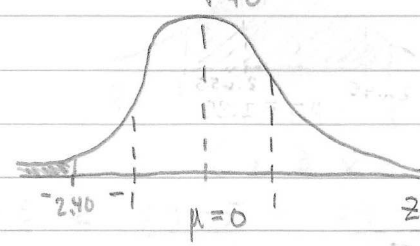
a) A sample of size 30 would be required (CLT)

b) $n = 40 \qquad \mu_{\bar{x}} = \mu = 11.4 \qquad \sigma_{\bar{x}} = \dfrac{3.2}{\sqrt{40}} = 0.584$

$P(\bar{x} < 10) = \boxed{0.0082}$

(A = 0.0028 if don't round σ)





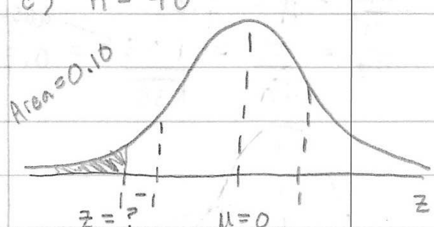
$P(\bar{x} < 10)$

$P\left(\dfrac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} < \dfrac{10 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$

$P\left(z < \dfrac{10 - 11.4}{0.584}\right)$

$P(z < -2.40) = 0.0082$

c) $n = 40$



$z = -1.28$

$\bar{x} = \mu_{\bar{x}} + z\sigma_{\bar{x}} = (11.4) + (-1.28)(0.584)$

$= \boxed{10.65}$

(A = 10.8 ")

Why? 29. a) No, the variable "weekly time spent watching television" is likely skewed right.

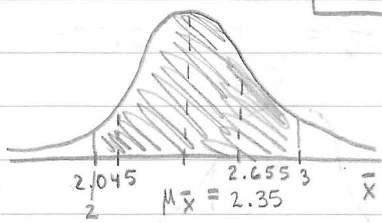
b) $\mu = 2.35 \quad \sigma = 1.93 \quad n = 40$

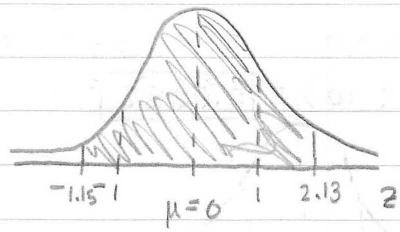
$n = 40 > 30 \therefore$ approximately normally distributed by CLT with

$\mu_{\bar{x}} = \mu = 2.35$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{1.93}{\sqrt{40}} = 0.305$

c) $n = 40$

$P(2 \le \bar{x} \le 3) = \boxed{0.8583}$





$P(2 \le \bar{x} \le 3)$

$$P\left(\frac{2 - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \le \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \le \frac{3 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$

$$P\left(\frac{2 - 2.35}{0.305} \le z \le \frac{3 - 2.35}{0.305}\right)$$

$$P(-1.15 \le z \le 2.13) = P(z \le 2.13) - P(z \le -1.15)$$

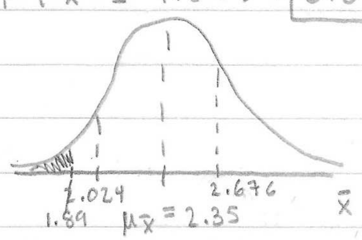
$$= 0.9834 - 0.1251$$

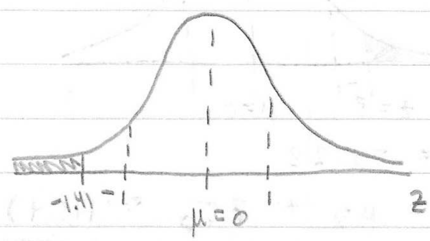
$$= 0.8583$$

d) $n = 35$ $\quad \bar{x} = 1.89$ $\quad \mu = 2.35$ $\quad \mu_{\bar{x}} = \mu = 2.35$

$P(\bar{x} \le 1.89) = \boxed{0.0793}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.93}{\sqrt{35}} = 0.326$ assumed





$P(\bar{x} \le 1.89)$

$$P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \le \frac{1.89 - 2.35}{0.326}\right)$$

$$P(z \le -1.41) = 0.0793$$

This result is not unusual b/c $P(\bar{x} \le 1.89) = 0.0793 > 0.05$.

$\therefore$ We cannot conclude avid internet users watch less television.

33. a)

x	$P(x)$	$x^2 \cdot P(x)$
35	$\frac{1}{38} = 0.0263$	32.2175
-1	$\frac{37}{38} = 0.9737$	0.9737

b) $\mu_x = \Sigma\, x \cdot P(x) = 35(0.0263) + {-1}(0.9737)$

$= \boxed{-0.05}$

$\sigma_x = \sqrt{\Sigma[x^2 P(x)] - \mu_x^2}$ $\qquad \Sigma[x^2 P(x)] = 33.1912$

$= \sqrt{33.1912 - (-0.0532)^2}$

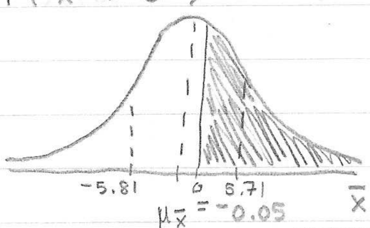
$= \boxed{5.76}$

c) $n = 100$

B/c $n = 100 > 30$, by the CLT the sampling distribution of $\bar{x}$ is approx. normally distributed with $\mu_{\bar{x}} = \mu = -0.05$ and $\sigma_{\bar{x}} = \frac{5.76}{\sqrt{100}} = 0.576$

d) $n = 100$

$P(\bar{x} > 0)$



$P(\bar{x} > 0) = \boxed{0.4641}$

$P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} > \frac{0 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$

$P\left(z > \frac{0 - (-0.05)}{0.576}\right)$

$P(z > 0.09) = 1 - P(z \le 0.09)$

$= 1 - 0.5359$

$= 0.4641$

e) $n = 200$, $P(\bar{x} > 0) = 0.4522$

f) $n = 1000$, $P(\bar{x} > 0) = 0.3936$

☐ Clarify why

g) The probability of being ahead decreases as the number of games played increases.

□ Clarify why

37. (b) b/c for a larger sample size n, $\sigma_{\bar{x}}$ is smaller and so $P(90 \le \bar{x} \le 110)$ is higher

41. a) population: full-time college students at your college
sample: 10 students in the simple random sample
b) b/c different people sleep different amounts
c) statistic b/c it is a numerical summary (mean) of a sample
d) The sample mean from part (c) is a random variable b/c its value will change depending on the 10 individuals in the sample. In this study, there are two sources of variation. First, there is person-to-person variability since different people have different sleeping habits. Second, there is variability of sleeping habits of an individual day-to-day (40(d)).

8.2

1. $p = \frac{X}{N} = \frac{220}{500} = \boxed{0.44}$

5. The shape of the sampling distribution of $\hat{p}$ is approximately normal provided (1) the sample size is less than 5% of the population size $(n \le 0.05N)$ and $np(1-p) \ge 10$.

9. $N = 25{,}000, \quad n = 1{,}000, \quad p = 0.103$

$\frac{n}{N} = \frac{1000}{25000} = 0.04 < 0.05 \qquad np(1-p) = (1000)(0.103)(1-0.103)$
$= 92.39 \ge 10$

$\therefore$ The sampling distribution of $\hat{p}$ is approximately normal with
$\mu_{\hat{p}} = p = 0.103$ and $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.103)(1-0.103)}{1000}} = 0.010$

13. $n = 1000 \quad N = 1{,}000{,}000 \quad p = 0.35$

a) (1) $n \le 0.05N$

$1000 \le 0.05(1{,}000{,}000)$

$1000 \le 50{,}000$

✓

(2) $np(1-p) \ge 10$

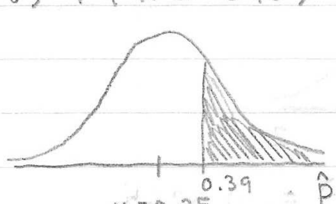
$(1000)(0.35)(1-0.35) \ge 10$

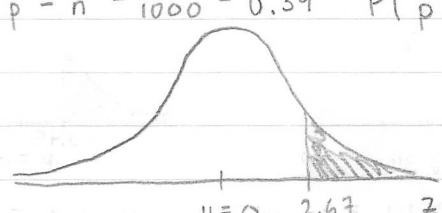
$227.5 \ge 10$

✓

$\therefore$ The sampling distribution of $\hat{p}$ is approximately normal with
$\mu_{\hat{p}} = p = 0.35$ and $\sigma_{\hat{p}} = \sqrt{\frac{(0.35)(1-0.35)}{1000}} = 0.015$

b) $P(x \ge 390)$ $\qquad \hat{p} = \frac{x}{n} = \frac{390}{1000} = 0.39 \qquad P(\hat{p} \ge 0.39) = \boxed{0.0038}$



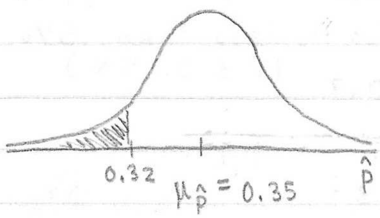


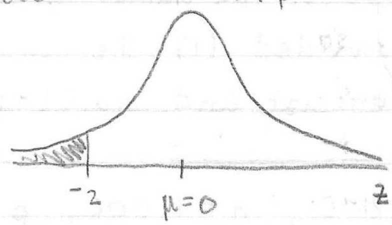
$P(z \ge 2.67)$
$= 1 - P(z \le 2.67)$
$= 1 - 0.9962$
$= 0.0038$

$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.39 - 0.35}{0.015} = 2.67$

About 4 out of 1000 random samples of size $n = 1000$ will result in 390 or more individuals (that is, 39% or more) with the characteristic.

c) $P(x \leq 320)$ $\hat{p} = \frac{x}{n} = \frac{320}{1000} = 0.32$ $P(\hat{p} \leq 0.32) = 0.0228$





☐ Textbook answers off?
↓
confirmed this is due to rounding the standard deviation

$z = \frac{0.32 - 0.35}{0.015} = -2$ $P(z \leq -2) = 0.0228$

About 2 out of 100 random samples of size $n = 1000$ will result in 320 or fewer individuals (that is, 32% or less) with the characteristic.

17. $p = 0.39$

a) $n = 500$

1) $n \leq 0.05N$

✓ b/c over 250 million adult Americans

2) $np(1-p) \geq 10$

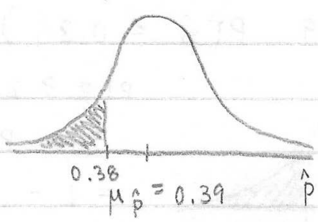
$(500)(0.39)(1-0.39) \geq 10$

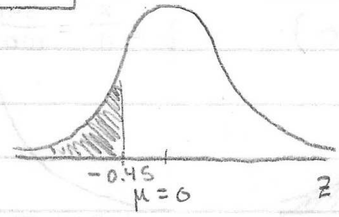
$118.95 \geq 10$

✓

∴ The sampling distribution of $\hat{p}$ is approximately normal with $\mu_{\hat{p}} = 0.39$ and $\sigma_{\hat{p}} = \sqrt{\frac{(0.39)(1-0.39)}{500}} = 0.022$

b) $P(\hat{p} < 0.38) = \boxed{0.3264}$

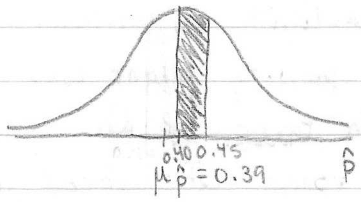


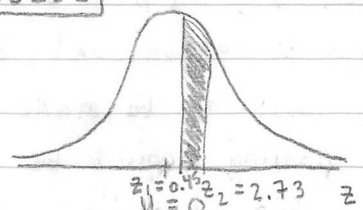


$z = \frac{0.38 - 0.39}{0.022} = -0.45$ $P(z < -0.45) = 0.3264$

About 32 out of 100 random samples of size $n = 500$ adult Americans will result in fewer than 190 individuals (that is, less than 38%) who believe that marriage is obsolete.

c) $P(0.40 \le \hat{p} \le 0.45) = \boxed{0.3232}$



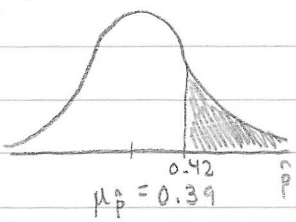


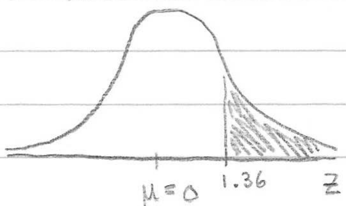
$z_1 = \frac{0.40 - 0.39}{0.022} = 0.45 \qquad z_2 = \frac{0.45 - 0.39}{0.022} = 2.73$

$$\begin{aligned} P(0.45 \le z \le 2.73) &= P(z \le 2.73) - P(z \le 0.45) \\ &= 0.9968 - 0.6736 \\ &= 0.3232 \end{aligned}$$

About 32 out of 100 random samples of size $n = 500$ adult Americans will result in between 200 and 225 individuals (that is, between 40% and 45%) who believe that marriage is obsolete.

d) $P(x \ge 210) \quad \hat{p} = \frac{x}{n} = \frac{210}{500} = 0.42 \quad P(\hat{p} \ge 0.42) = \boxed{0.0869}$





$z = \frac{0.42 - 0.39}{0.022} = 1.36$

$$\begin{aligned} P(z \ge 1.36) &= 1 - P(z \le 1.36) \\ &= 1 - 0.9131 \\ &= 0.0869 \end{aligned}$$

About 9 out of 100 random samples of size $n = 500$ adult Americans will result in 210 or more individuals (that is, 42%) who believe that marriage is obsolete. This result is not unusual b/c $0.0869 > 0.05$.

21. $N = 100000+$

$n = 310$ $\quad x = 164$ $\quad p = 0.49$

$\hat{p} = 0.53$

$P(X \geq 164) = P(\hat{p} \geq 0.529) = 0.0853$

This result is not unusual, so it wouldn't be unusual for a wrong call to be made if exit polling alone was considered. The exit polling would be biased in favor of an increase in funding for education, since a voter who voted against it in privacy might not want to admit it.