5

$50^{+1}/50$

Summer 2018
Math 54: Test 4

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First Name Stewart

For each of the following show all necessary work on the space provided.

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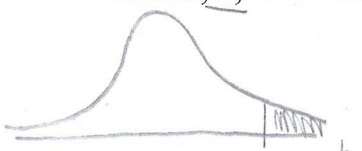
5

(9.2) **1a. [5 points]** A simple random sample of size 35 is drawn. The sample mean $\bar{x}$ is found to be 18.4, and the sample standard deviation, s, is found to be 4.5. Construct a 95% confidence interval for μ.



Step 1) $\bar{x} = 18.4$

Step 2) Conditions:

① simple random sample ✓ (given)

② $n \geq 30$ ✓ $(n = 35)$

③ $n \leq 0.05N$ ✓ (assumed)

Step 3) $\alpha = 1 - 0.95 = 0.05$

$t_{\alpha/2} = t_{0.05/2} = t_{0.025} = 2.032$

$df = 35 - 1 = 34$

Step 4) LB: $18.4 - (2.032)\left(\frac{4.5}{\sqrt{35}}\right) = 16.8544$

UB: $18.4 + (2.032)\left(\frac{4.5}{\sqrt{35}}\right) = 19.9456$

Step 5) $\boxed{\text{We are 95\% confident the population mean is between 16.8544 and 19.9456.}}$

(9.1) b. **[5 points]** Determine the point estimate of the population proportion and the margin of error for the given confidence interval
Lower bound: 0.201, upper bound: 0.249

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$LB = \hat{p} - E = 0.201$

$UB = \hat{p} + E = 0.249$

$\hat{p} - E = 0.201$

$+\ \hat{p} + E = 0.249$

$2\hat{p} = 0.201 + 0.249$

$\hat{p} = \frac{0.45}{2}$

$\boxed{\hat{p} = 0.225}$

$LB = \hat{p} - E$

$0.201 = 0.225 - E$

$E = 0.225 - 0.201$

$\boxed{E = 0.024}$

Total for the page → 10

Show all necessary work

(9.1) 2.[**10 POINTS**] A survey of 2306 adult Americans aged 18 and older conducted by Harris Interactive found that 417 have donated blood in the past two years.

Step 1) a. Obtain a point estimate for the population proportions of adults Americans aged 18 and older who have donated blood in the past two years.

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$n = 2306 \quad x = 417$

$\hat{p} = \frac{x}{n} = \frac{417}{2306} = \boxed{0.1808}$

Step 2) b. Verify the conditions of constructing confidence interval

2

① simple random sample ✓

② $n\hat{p}(1-\hat{p}) \geq 10$

$(2306)(0.1808)(1-0.1808) \geq 10$

$341.54 \geq 10$ ✓

③ $n \leq 0.05N$

✓ b/c there are at least 1 million Americans aged 18 and older

c. Constrict a 90% confidence interval for the population proportion of of adults Americans aged 18 and older who have donated blood in the past two years.

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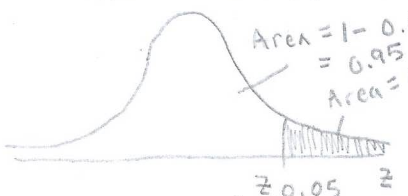
Step 3) $\alpha = 1 - 0.90 = 0.10$

$z_{\alpha/2} = z_{0.10/2} = z_{0.05} = 1.645$



Step 4) LB: $0.1808 - (1.645)\sqrt{\frac{0.1808(1-0.1808)}{2306}}$

$= 0.1676$

UB: $0.1808 + (1.645)\sqrt{\frac{0.1808(1-0.1808)}{2306}}$

$= 0.1940$

d. Interpret the interval

2

Step 5) We are 90% confident the population proportion of adult Americans aged 18 and older who have donated blood in the past two years is between 0.1676 and 0.1940

If we take many simple random samples of size n = 2306 adult Americans aged 18 and older, we would expect the population proportion who have donated blood in the past two years to be between 0.1676 and 0.1940 about 90% of the time. In other words, we are 90% confident in the confidence interval method.

Total for the page → 10

Show all necessary work

$x = 256$, $n = 800$ $\hat{p} = \frac{256}{800} = 0.32$

(10.2) 3. **[10 points]**In 1994, 52% of parents of children in high school felt it was a serious problem that a high school students were not being taught enough math and science. A recent survey found that 256 out of 800 parents of children in high school felt it was a serious problem that high school students were not being taught enough math and science. Do parents feel differently today than they did in 1994?

Step 0) Verify the conditions

① simple random sample ✓

② 2 $n\hat{p}(1-\hat{p}) \geq 10$ → $(800)(0.32)(1-0.32) \geq 10$

$174.08 \geq 10$ ✓

③ $n \leq 0.05N$

✓ b/c at least 100,000 parents of students in high school

Test the claim. Use $\alpha = 0.05$ level of significance. Use **Classical** Method.

Step 1: state the hypothesis.

$H_0: p = 0.52$

2 $H_1: p \neq 0.52$ (two-tailed) ✓

Step2: Select Significance level

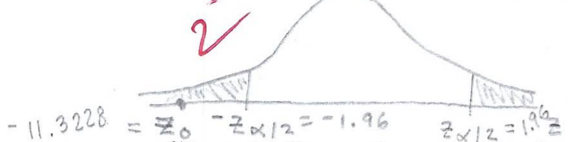
1 $\alpha = 0.05$ ✓

Step 3: Compute the test statistic.

2 $z_0 = \dfrac{0.32 - 0.52}{\sqrt{\dfrac{0.52(1-0.52)}{800}}} = \boxed{-11.3228}$ ✓

Step 3: Sketch the distribution used and show the critical level

2 ✓



$z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$ ✓

$-z_{\alpha/2} = -1.96$

Step 4: State the conclusion.

$z_0 < -z_{\alpha/2}$? ✓

2 $-11.3228 < -1.96$

∴ reject the null hypothesis

Step 5) There is sufficient evidence to conclude parents feel differently today than they did in 1994 ($p \neq 0.52$).

Total for the page ⟶ 11 ✓ **Show all necessary work**

$n = 20, \ \bar{x} = 6.3, \ s = 2.1$

(10.3) **4.[10 points]** A local retailer claims that the mean waiting time is less than 8 minutes. A random sample of 20 waiting times has a mean of 6.3 minutes with a standard deviation of 2.1 minutes. At $\alpha = 0.01$, test the retailer's claim. Assume the distribution is normally distributed. Round the test statistic to the nearest thousandth. Use **classical** method.

Step 0) **Verify the conditions.** ① simple random sample ✓ (given) ② distribution normally distributed ✓ (given) ③ $n \le 0.05N$ ✓ b/c, at least 500 customers

Step 1: state the hypothesis

$H_0: \mu = 8$

$H_1: \mu < 8$ (left-tailed)

Step2: Select Significance level

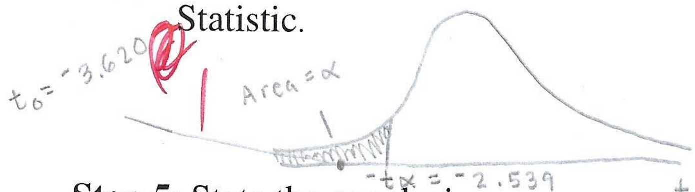
$\alpha = 0.01$

Step 3: Compute the test statistic

$$t_0 = \frac{6.3 - 8}{2.1/\sqrt{20}} = -3.620$$

Step 4: Sketch the distribution curve and indicate critical and test Statistic.

$-t_\alpha = -t_{0.01} = -2.539$ $\quad df = 20 - 1 = 19$



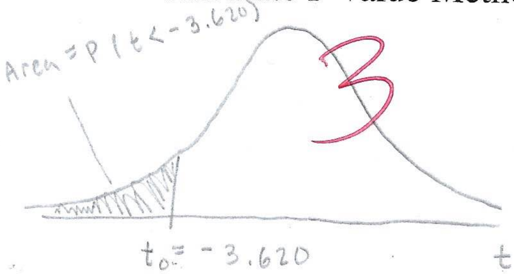
Step 5: State the conclusion.

$t_0 < -t_\alpha$ ∴ reject null hypothesis

$-3.620 < -2.539$

There is sufficient evidence to conclude the mean waiting time is less than 8 minutes.

Calculate P-value Method and compare the result.



$P\text{-value} = P(t < t_0) = P(t < -3.620)$

$P\text{-value} = P(t > 3.620)$ (by symmetry of t-Distribution)

$3.579 < t_0 < 3.883$

$0.0005 < P\text{-value} < 0.001$

Step 4) P-value < α ?

Max(P-value) = 0.001 < 0.01

∴ reject null hypothesis

Step 5) There is sufficient evidence to conclude mean waiting time is less than 8 minutes.

Total for the page → 10

Show all necessary work

∴ Same result

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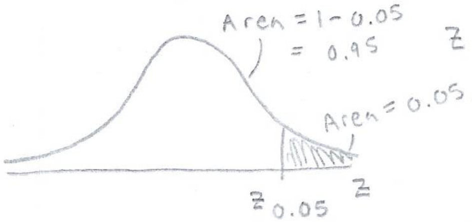
(9.2) 5.a [5 points] Determine the sample size required to estimate the mean score on a standardized test within 4 percentage of the true mean with 90% confidence. Assume that s = 15 based on earlier studies. (round up)

$\alpha = 1 - 0.90 = 0.10$

$s = 15$, $E = 0.04$



$z_{\alpha/2} = z_{0.10/2} = z_{0.05} = 1.645$

$$n = \left(\frac{1.645 \cdot 15}{0.04}\right)^2 = 380534.77 \approx \boxed{380,535}$$

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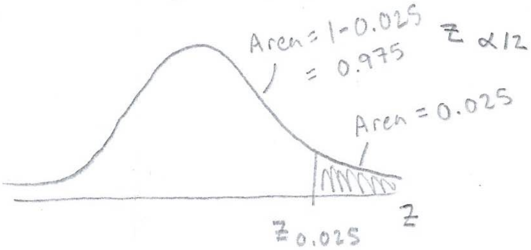
(9.1) b. [5 points] A physical therapist want to determine the difference in the proportion of men and woman who participated in regular, sustained physical activity. What sample size should be obtained if she wishes the estimate to be with in 3 percent margin of error and 95% confidence level, assuming that she does not use any prior estimate? (round up)

$\alpha = 1 - 0.95 = 0.05$

$E = 0.03$



$z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$

$$n = 0.25 \left(\frac{1.96}{0.03}\right)^2 = 1067.11 \approx \boxed{1,068}$$

Total for the page → 10

Show all necessary work