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MATH 54
Section 4053
SID: 1545566

Ch 6, 7, 8 Review Problems

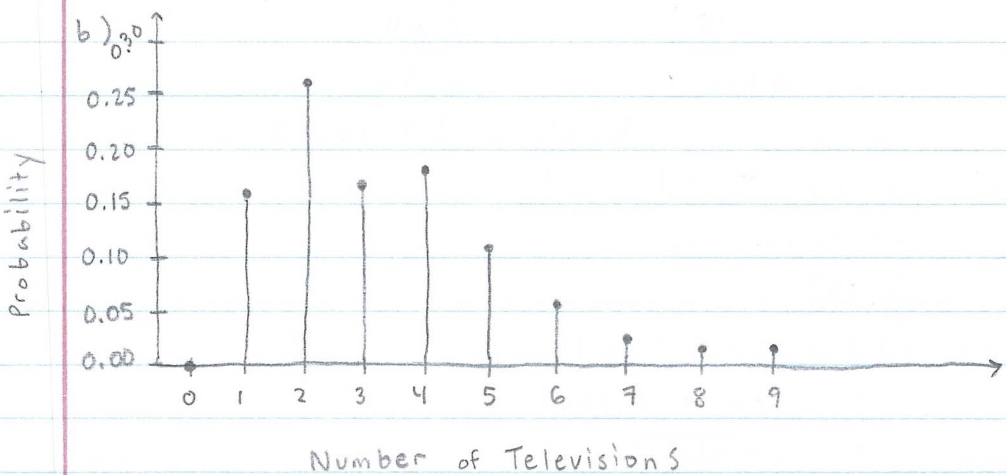
6.1

5. a) discrete $x = 0, 1, \ldots, 20$
   b) continuous $t > 0$
   c) discrete $x = 0, 1, 2, 3, \ldots$
   d) continuous $s \geq 0$

9. Yes, it is a discrete probability distribution b/c (1) $\Sigma P(x) = 1$ and (2) $0 \leq P(x) \leq 1$.

17. a) (1) $\Sigma P(x) = 1$ ✓
   (2) $0 \leq P(x) \leq 1$ ✓

   b)



   - skewed right

   c) $\mu_x = \Sigma\, x \cdot P(x) = 0(0) + 1(0.161) + 2(0.261) + 3(0.176) + 4(0.186) + 5(0.116) + 6(0.055) + 7(0.025) + 8(0.010) + 9(0.010)$
   $= \boxed{3.21}$

   If we surveyed many households, we would expect the mean number of televisions per household to be 3.21.

   $\mu_x \approx 3.2$ televisions

d) $\sigma_x = \sqrt{\Sigma[x^2 P(x)] - \mu_x^2}$

x	$P(x)$	$x^2 \cdot P(x)$
0	0	$(0)^2(0) = 0$
1	0.161	$(1)^2(0.161) = 0.161$
2	0.261	1.044
3	0.176	1.584
4	0.186	2.976
5	0.116	2.9
6	0.055	1.98
7	0.025	1.225
8	0.010	0.64
9	0.010	0.81

$\Sigma[x^2 P(x)] = 13.32$

$\sigma_x = \sqrt{13.32 - (3.21)^2} = \boxed{1.737}$

$\sigma_x \approx 1.7$ televisions

e) $P(3) = \boxed{0.176}$

f) $P(3 \text{ or } 4) = P(3) + P(4)$

$= 0.176 + 0.186$

$= \boxed{0.362}$

g) $P(0) = 0$

No, not impossible.

6.2

17. $n = 10,\ p = 0.4,\ x = 3$

$P(3) = {}_{10}C_3\,(0.4)^3(1-0.4)^{10-3} = \boxed{0.2150}$

19. $n = 40,\ p = 0.99,\ x = 38$

$P(38) = {}_{40}C_{38}\,(0.99)^{38}(1-0.99)^{40-38} = \boxed{0.0532}$

29. $n = 6$, $p = 0.3$

a)

x	$P(x)$	$x^2 \cdot P(x)$
0	binompdf(6, 0.3, 0) = 0.1176	$(0)^2(0.1176) = 0$
1	0.3025	0.3025
2	0.3241	1.2964
3	0.1852	1.6668
4	0.0595	0.9520
5	0.0102	0.2550
6	0.0007	0.0252

b) $\mu_x = \Sigma x \cdot P(x) = 0(0.1176) + 1(0.3025) + 2(0.3241) + 3(0.1852) + 4(0.0595) + 5(0.0102) + 6(0.0007)$

$\mu_x = \boxed{1.7995}$

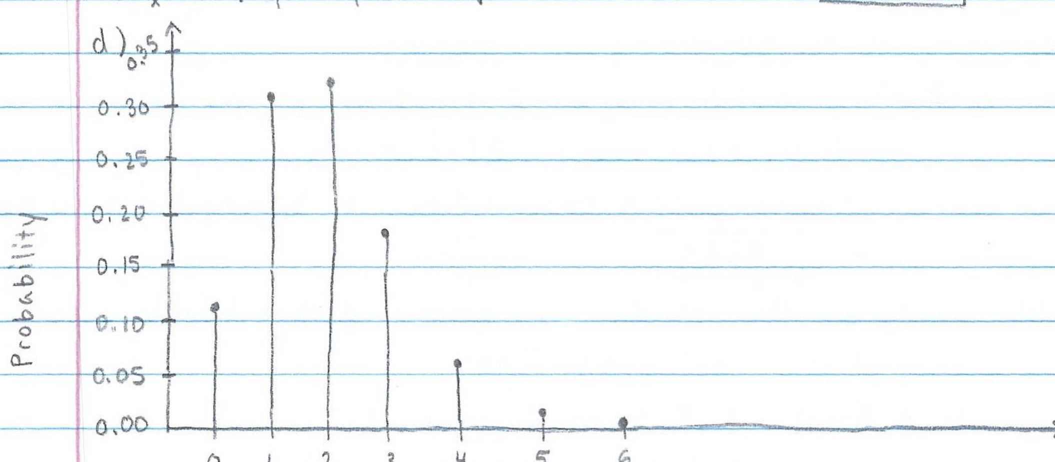
$\Sigma x^2 \cdot P(x) = 4.4979$

$\sigma_x = \sqrt{4.4979 - (1.7995)^2} = \boxed{1.1224}$

c) $\mu_x = np = (6)(0.3) = \boxed{1.8}$

$\sigma_x = \sqrt{np(1-p)} = \sqrt{(6)(0.3)(1-0.3)} = \boxed{1.1225}$

d)



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35. $p = 0.80$, $n = 15$

a) Criteria for a Binomial Probability Experiment:

1. The experiment is performed a fixed number of times (15)
2. The trials are independent
3. For each trial, there are two mutually exclusive (disjoint) outcomes: on time or not on time
4. The probability of success is the same for each trial of the experiment (0.80)

b) $P(10) = {}_{15}C_{10}\,(0.80)^{10}(1-0.80)^{15-10}$

$= \boxed{0.1032}$

c) $P(X < 10) =$

$= P(0 \text{ or } 1 \text{ or } 2 \text{ or } \ldots \text{ or } 9)$

$= P(0) + P(1) + P(2) + \ldots + P(9)$

$= \text{binompdf}(15, 0.80, 0) + \ldots + \text{binompdf}(15, 0.80, 9)$

$= \boxed{0.0611}$

In 100 trials of this study, we expect about 6 to result in fewer than 10 flights being on time.

d) $P(X \geq 10) = P(10 \text{ or } 11 \text{ or } 12 \text{ or } 13 \text{ or } 14 \text{ or } 15)$

$= P(10) + \ldots + P(15)$

$= \text{binompdf}(15, 0.80, 10) + \ldots + \text{binompdf}(15, 0.80, 15)$

$= \boxed{0.9389}$

In 100 trials of this study, we expect about 94 to result in 10 or more flights being on time.

e) $P(8 \leq X \leq 10) = P(8 \text{ or } 9 \text{ or } 10)$

$= P(8) + P(9) + P(10)$

$= \text{binompdf}(15, 0.80, 8) + \text{binompdf}(15, 0.80, 9) + \text{binompdf}(15, 0.80, 10)$

$= \boxed{0.1600}$

In 100 trials of this study, we expect about 16 to result in between 8 and 10 flights being on time.

43. $p = 0.80 \quad n = 100$

a) $\mu_x = np = (100)(0.80) = \boxed{80}$ flights

$\sigma_x = \sqrt{np(1-p)} = \sqrt{(100)(0.80)(1-0.80)} = \boxed{4}$ flights

from Dallas to Chicago

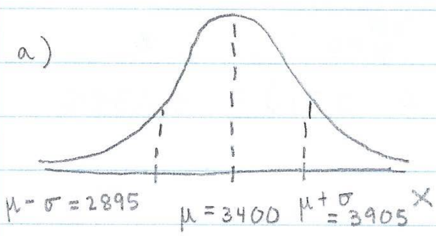
b) We expect that, in a random sample of 100 flights, 80 will be on time.

c) $\mu_x - 2\sigma_x = 80 - 2(4) = 72$

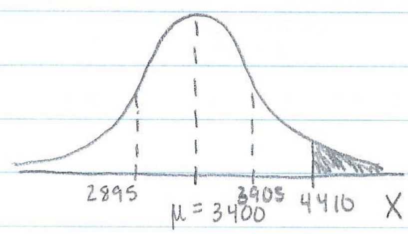
No, this would not be unusual b/c 75 is within 2 standard deviations of the mean.

7.1

33. a)



b)



c) Area = 0.0228

1. The proportion of full-term babies who weigh more than 4410 grams is 0.0228. (or 2.28%)

2. The probabability that a randomly selected full-term baby weighs more than 4410 grams is 0.0228.

35. $\mu = 266 \quad \sigma = 16$

a) 1. The proportion of human pregnancies longer than 280 days is 0.1908.

2. The probability that a randomly selected human pregnancy is longer than 280 days is 0.1908.

b) 1. The proportion of human pregnancies between 230 and 260 days is 0.3416.

2. The probability that a randomly selected human pregnancy is between 230 and 260 days is 0.3416.

7.2

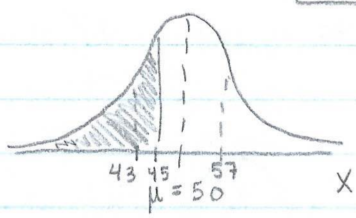
11. a) $0.0228 + 0.0228 = \boxed{0.0456}$

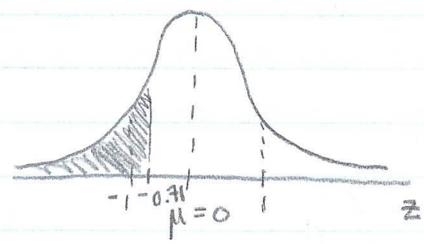
b) $0.0594 + (1 - 0.9948) = \boxed{0.0646}$

c) $0.4052 + (1 - 0.8849) = \boxed{0.5203}$

25. $\mu = 50, \ \sigma = 7$

$P(X \leq 45) = \boxed{0.2389}$

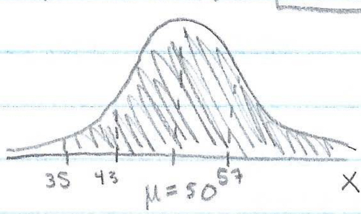


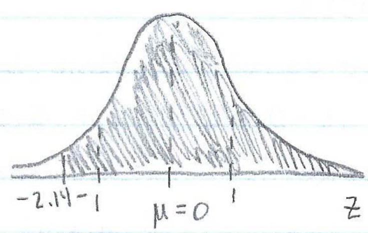


$z = \dfrac{45 - 50}{7} = -0.71$ $\quad P(z \leq -0.71) = 0.2389$

23. $\mu = 50, \ \sigma = 7$

$P(X > 35) = \boxed{0.9838}$





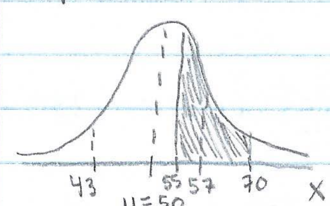
$z = \dfrac{35 - 50}{7} = -2.14$ $\quad P(z > -2.14) = 1 - P(z \leq -2.14)$

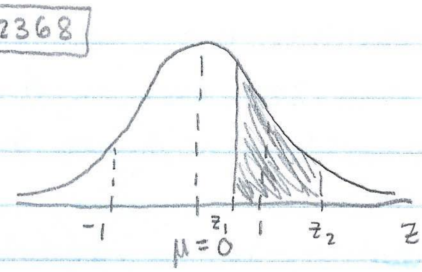
$= 1 - 0.0162$

$= 0.9838$

$\mu = 50, \ \sigma = 7$

29. $P(55 \leq X \leq 70) = \boxed{0.2368}$





$z_1 = \dfrac{55 - 50}{7} = 0.71$ $\quad z_2 = \dfrac{70 - 50}{7} = 2.86$

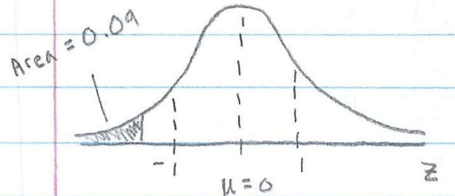
$P(0.71 \leq z \leq 2.86) = P(z \leq 2.86) - P(z \leq 0.71)$

$= 0.9979 - 0.7611$

$= 0.2368$

33. $\mu = 50$, $\sigma = 7$

The 9th percentile

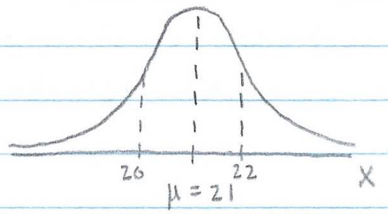


$z = -1.34$

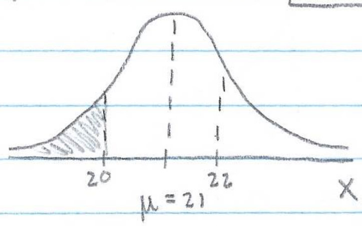
$x = \mu + z\sigma = 50 + (-1.34)(7) = \boxed{40.62}$

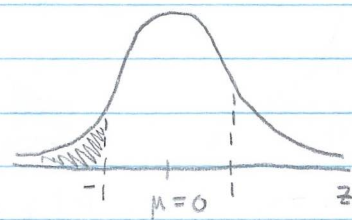
37. $\mu = 21$ $\sigma = 1$

a)



b) $P(X < 20) = \boxed{0.1587}$

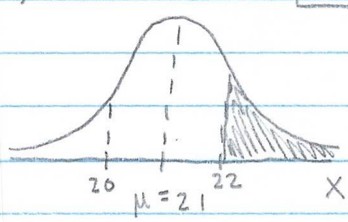


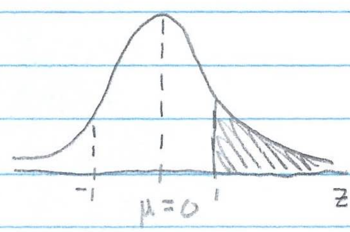


$z = \dfrac{20 - 21}{1} = -1$ $P(z < -1) = 0.1587$

If we randomly selected 100 fertilized chicken eggs, we would expect about 16 of them to hatch in less than 20 days.

c) $P(x > 22) = \boxed{0.1587}$



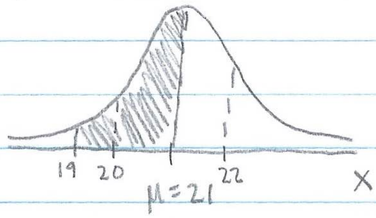


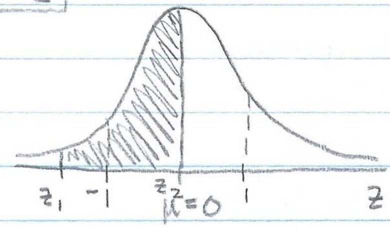
$z = \dfrac{22 - 21}{1} = 1$

$P(z > 1) = 1 - P(z \leq 1)$
$= 1 - 0.8413$
$= 0.1587$

If we randomly selected 100 fertilized chicken eggs, we would expect about 16 to hatch in more than 22 days

d) $P(19 \le X \le 21) = \boxed{0.4772}$



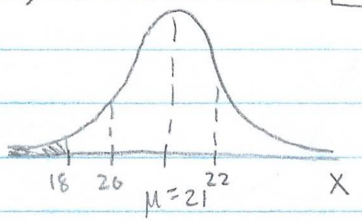


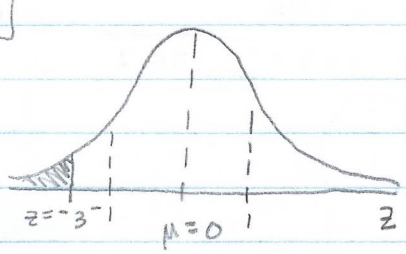
$z_1 = \dfrac{19-21}{1} = -2 \qquad z_2 = \dfrac{21-21}{1} = 0$

$$\begin{aligned} P(-2 \le z \le 0) &= P(z \le 0) - P(z \le -2) \\ &= 0.5000 - 0.0228 \\ &= 0.4772 \end{aligned}$$

If we randomly selected 100 fertilized chicken eggs, we would expect about 48 of them to hatch between 19 and 21 days.

e) $P(X < 18) = \boxed{0.0013}$





$z = \dfrac{18-21}{1} = -3 \qquad P(z \le -3) = 0.0013$

Yes it would be unusual b/c $P(X < 18) = 0.0013 < 0.05$. In other words, if we randomly selected 1000 fertilized chicken eggs, we would expect only about 1 of them to hatch in less than 18 days.

8.1

9. $\mu = 80, \; \sigma = 14, \; n = 49$

$\mu_{\bar{x}} = \mu = \boxed{80}$ $\qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14}{\sqrt{49}} = \frac{14}{7} = \boxed{2}$

13. a) $\boxed{\mu_{\bar{x}} = 500}$

b) $\mu_{\bar{x}} - \sigma_{\bar{x}} = 480$

$500 - \sigma_{\bar{x}} = 480$

$-\sigma_{\bar{x}} = -20$

$\boxed{\sigma_{\bar{x}} = 20}$

□ Clarify why c) The population must be approximately normally distributed.

d) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

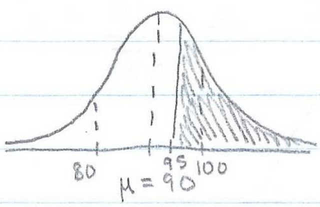
$20 = \frac{\sigma}{\sqrt{16}}$

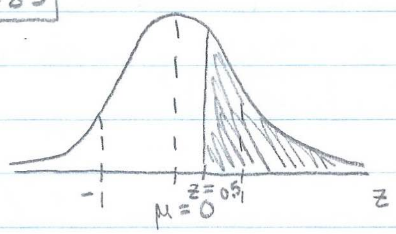
$\sigma = 20 \cdot 4$

$\boxed{\sigma = 80}$

21. $\mu = 90, \; \sigma = 10$

a) $P(X > 95) = \boxed{0.3085}$





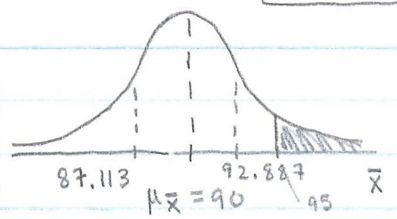
$z = \frac{95 - 90}{10} = 0.5$ $\qquad P(z > 0.5) = 1 - P(z \leq 0.5)$

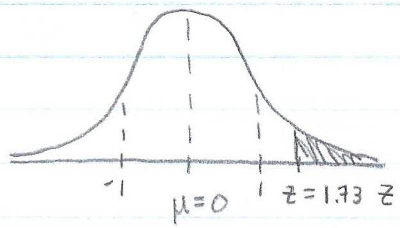
$= 1 - 0.6915$

$= 0.3085$

b) $n = 12$ $\qquad \mu_{\bar{x}} = 90 \qquad \sigma_{\bar{x}} = \frac{10}{\sqrt{12}} = 2.887$

$P(\bar{x} > 95) = \boxed{0.0418}$



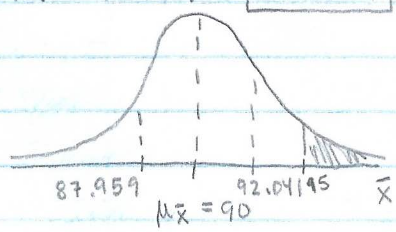


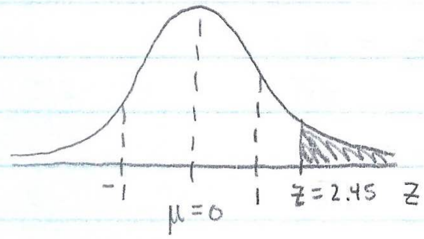
$z = \frac{95 - 90}{2.887} = 1.73$

$P(z > 1.73) = 1 - P(z \leq 1.73)$
$= 1 - 0.9582$
$= 0.0418$

c) $n = 24$ $\qquad \mu_{\bar{x}} = 90 \qquad \sigma_{\bar{x}} = \frac{10}{\sqrt{24}} = 2.041$

$P(\bar{x} > 95) = \boxed{0.0071}$





$z = \frac{95 - 90}{2.041} = 2.45$

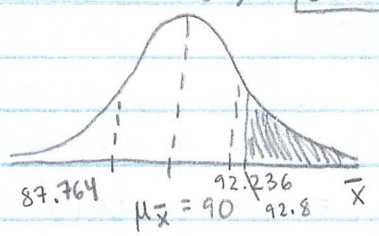
$P(z > 2.45) = 1 - P(z \leq 2.45)$
$= 1 - 0.9929$
$= 0.0071$

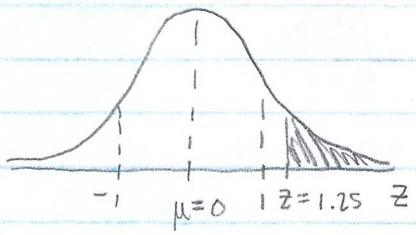
□ Clarify why

d) Increasing the sample size decreases the probability $P(\bar{x} > 95)$ b/c $\sigma_{\bar{x}}$ decreases as n increases.

e) $n = 20 \qquad \bar{x} = 92.8 \qquad \mu_{\bar{x}} = \mu = 90 \qquad \sigma_{\bar{x}} = \frac{10}{\sqrt{20}} = 2.236$

$P(\bar{x} \geq 92.8) = \boxed{0.1056}$



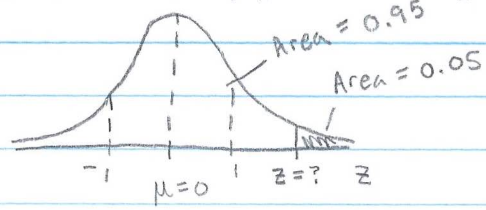


$z = \frac{92.8 - 90}{2.236} = 1.25$

$P(z \geq 1.25) = 1 - P(z \leq 1.25)$
$= 1 - 0.8944$
$= 0.1056$

A mean reading score of 92.8 wpm is not unusual b/c $P(\bar{x} \geq 92.8) = 0.1056 > 0.05$. ∴ The new program is not abundantly better than the old program.

f) $n = 20, \quad \mu_{\bar{x}} = 90, \quad \sigma_{\bar{x}} = 2.236$



$z = 1.645$

$\bar{x} = \mu_{\bar{x}} + z\,\sigma_{\bar{x}} = 90 + (1.645)(2.236)$

$= \boxed{93.68}$

8.2

7. $N = 25000$

$n = 500 \quad p = 0.4$

1) $n \leq 0.05N$

$500 \leq 0.05(25000)$

$500 \leq 1250$

✓

2) $np(1-p) \geq 10$

$(500)(0.4)(1-0.4) \geq 10$

$120 \geq 10$

✓

The sampling distribution of $\hat{p}$ is approximately normally distributed with $\mu_{\hat{p}} = p = 0.4$ and $\sigma_{\hat{p}} = \sqrt{\frac{(0.4)(1-0.4)}{500}} = 0.022$

12. $n = 200 \quad N = 25000 \quad p = 0.65$

☐ Check answers

a) 1) $n \leq 0.05N$

$200 \leq 0.05(25000)$

$200 \leq 1250$

✓

2) $np(1-p) \geq 10$

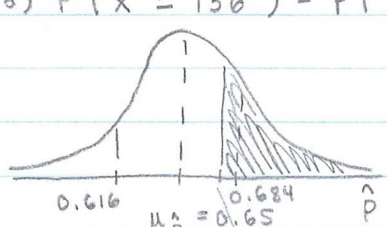
$(200)(0.65)(1-0.65) \geq 10$

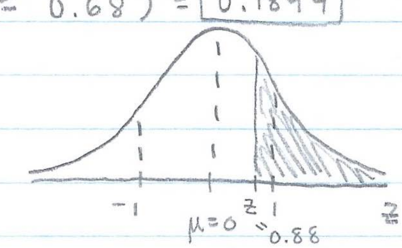
$45.5 \geq 10$

✓

The sampling distribution of $\hat{p}$ is approximately normally distributed with $\mu_{\hat{p}} = p = 0.65$ and $\sigma_{\hat{p}} = \sqrt{\frac{0.65(1-0.65)}{200}} = 0.034$

b) $P(x \geq 136) = P(\hat{p} \geq 0.68) = \boxed{0.1894}$



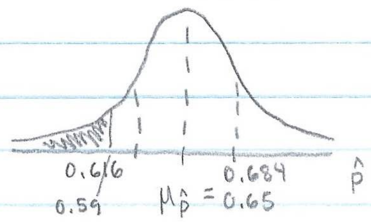


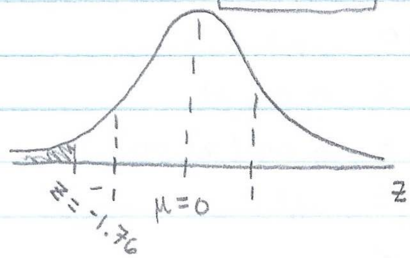
$z = \frac{0.68 - 0.65}{0.034} = 0.88$

$P(z \geq 0.88) = 1 - P(z \leq 0.88)$

$= 1 - 0.8106 = 0.1894$

c) $P(x \le 118) = P(\hat{p} \le 0.59) = \boxed{0.0392}$





$$z = \frac{0.59 - 0.65}{0.034} = -1.76$$

$P(z \le -1.76) = 0.0392$

17. $p = 0.39$

a) $n = 500$

1) $n \le 0.05N$

✓ b/c there are over 250 million adult Americans

2) $np(1-p) \ge 10$

$(500)(0.39)(1-0.39) \ge 10$

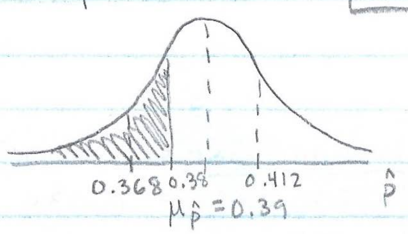
$118.95 \ge 10$

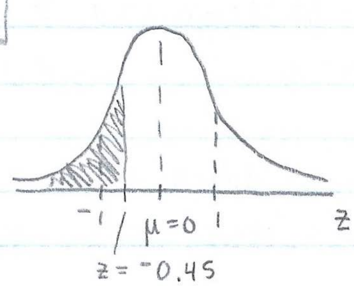
✓

The sampling distribution of $\hat{p}$ is approximately normally distributed with $\mu_{\hat{p}} = p = 0.39$ and $\sigma_{\hat{p}} = \sqrt{\frac{(0.39)(1-0.39)}{500}} = 0.022$

b) $n = 500$

$P(\hat{p} < 0.38) = \boxed{0.3264}$



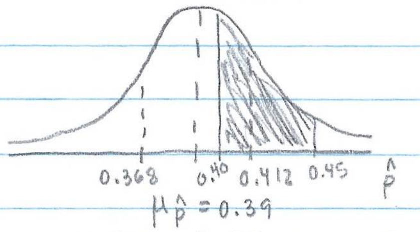


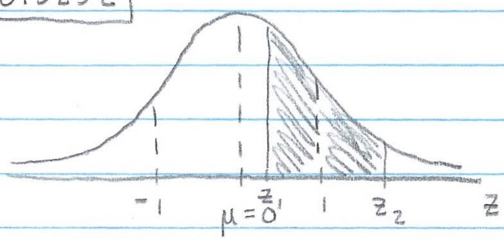
$$z = \frac{0.38 - 0.39}{0.022} = -0.45$$

$P(z < -0.45) = 0.3264$

c) $n = 500$

$P(0.40 \le \hat{p} \le 0.45) = \boxed{0.3232}$





$z_1 = \dfrac{0.40 - 0.39}{0.022} = 0.45$ $\qquad z_2 = \dfrac{0.45 - 0.39}{0.022} = 2.73$

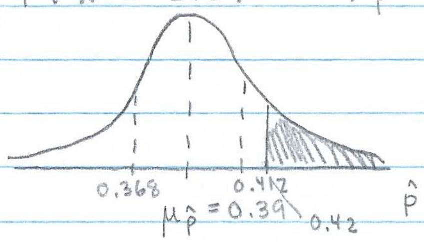
$P(0.45 \le z \le 2.73) = P(z \le 2.73) - P(z \le 0.45)$

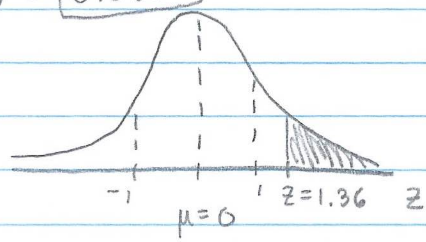
$= 0.9968 - 0.6736$

$= 0.3232$

d) $n = 500$

$P(x \ge 210) = P(\hat{p} \ge 0.42) = \boxed{0.0869}$





$z = \dfrac{0.42 - 0.39}{0.022} = 1.36$ $\qquad P(z \ge 1.36) = 1 - P(z \le 1.36)$

$= 1 - 0.9131$

$= 0.0869$

$\therefore$ No, not unusual b/c $P(\hat{p} \ge 0.42) = 0.0869 > 0.05$.