Multi risk factor model
Minimum variance optimization under constraints
Market neutrality
130/30 long short portfolio

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Plan

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1. Factor model and stochastic processes

We have $N \in \mathbb{N}$ stocks. Their returns $R_i, \forall i \in \{1, \cdots, N\}$ are modeled by N random processes χ_i corresponding to idiosyncratic risk together with K random processes f_A :

$$R_i = \chi_i + \sum_{A=1}^{K} \widetilde{\Omega}_{iA} f_A \tag{1}$$

$$\langle \chi_i, \chi_j \rangle = \xi_{ij}^2 \delta_{ij} \tag{2}$$

$$\langle \chi_i, f_A \rangle = 0 \tag{3}$$

$$\langle f_A, f_B \rangle = \Phi_{AB} \tag{4}$$

$$\langle R_i, R_j \rangle = \Theta_{ij} \tag{5}$$

2. Covariance matrix

The R_i covariance matrix can be written as :

$$\Theta = \Xi + \widetilde{\Omega}\Phi\widetilde{\Omega}^{\mathsf{T}} \tag{6}$$

where $\Xi_{ij} = \xi_{ij}^2\delta_{ij}$ is the diagonal idiosyncratic part of the variance and Φ the $K \times K$ factor covariance matrix, $\widetilde{\Omega}$ is the $K \times K$ factor loading matrix. Note that $K << N$ gives more out of samples stability. Through Cholesky :

$$\Theta = \Xi + \Omega\Omega^{\mathsf{T}} \tag{7}$$

where $\Omega = \widetilde{\Omega}\widetilde{\Phi}$ and $\widetilde{\Phi}\widetilde{\Phi}^{\mathsf{T}} = \Phi$

Note that the standard variance for a portfolio with weights ω decomposes as :

$$\omega^{\mathsf{T}}\Omega\omega = \sum_{i=1}^{N} \xi_{ij}^2 + f^{\mathsf{T}}\Phi f \tag{8}$$

where $f = \Omega\omega$ is the factor values for our portfolio with weights ω

3. Regressing returns over factor loading/exposure

At each observation time we have to regress our stock returns to our stock factors :

$$R(t) = \sum_{k=1}^{K} \beta_k(t) \times F_k(t) + \epsilon(t) \qquad (9)$$

$$R(t) = \sum_{k=1}^{K} f_k(t) \times \Omega_k(t) + \epsilon(t) \qquad (10)$$

Ω_i, F_i is the factor exposure or loading for the stock i and β_i, f_i is the factor return or factor itself.

The variance of $\epsilon(t)$ is the idiosyncratic risk for stock i

The covariance of $(f_k(t))_{k\in\{1,\cdots,K\}}$ is the factor covariance matrix.

Here a proprietary strategy is to be defined to compute the factor matrix : a rolling standard deviation or garch volatility estimation for idiosyncratic variance and a rolling Ledoit-Wolf covariance matrix estimation

4. Minimum variance optimization rebalancement at each date

$$\operatorname{argmin} \quad \{\omega\Sigma\omega^\intercal - q\times\mu^\intercal\omega\}\,,\ becomes$$
$$\begin{cases} \omega\in\mathbb{R}^N, \omega_i > 0, \omega_i < 1 \\ \sum_{i=1}^N \omega_i = 1 \\ \sum_{i=1}^n \omega_i\beta_i = \sum_{i=1}^n \omega_i \frac{\mathrm{Cov}(R_i, R_{market})}{\mathrm{Var}(R_i)} = 1 \end{cases} \tag{11}$$

$$\operatorname{argmin} \quad \{xQx^\intercal - q\times\widetilde{\mu}^\intercal x\} \tag{12}$$
$$\begin{cases} x = (f,\omega)\in\mathbb{R}^{(N+K)}, \omega_i > 0, \omega_i < 1 \\ \sum_{i=1}^N \omega_i = 1 \\ \sum_{i=1}^n \omega_i\beta_i = \sum_{i=1}^n \omega_i \frac{\mathrm{Cov}(R_i, R_{market})}{\mathrm{Var}(R_i)} = 1 \\ \Omega\omega = f \end{cases}$$

where :

$$Q = \left(\begin{array}{c|ccc} \Phi & & 0\cdots 0 & \\ \hline & \xi_{11} & & \mathbf{0} \\ & & \ddots & \\ 0 & & & \\ & & & \ddots \\ \vdots & \mathbf{0} & & \xi_{nn} \\ 0 & & & \end{array}\right), \widetilde{\mu} = \begin{pmatrix} 0 \\ \mu \end{pmatrix} \qquad (13)$$

5. Long/Short 130/30 Market Neutral portfolio

$$\operatorname{argmin} \quad \{xQx^\intercal - q \times \widetilde{\mu}^\intercal x\} \tag{14}$$

$$\begin{cases} x = (f, \omega) \in \mathbb{R}^{(N+K)}, \omega_i > 0, \omega_i < 1 \\ \sum_{i=1}^{N} \omega_i = 1 \\ \sum_{i=1}^{n} \omega_i \beta_i = \sum_{i=1}^{n} \omega_i \frac{\mathrm{Cov}(R_i, R_{market})}{\mathrm{Var}(R_i)} = 1 \\ \Omega\omega = f \end{cases}$$

where :

$$Q = \left(\begin{array}{c|ccc} \Phi & & 0 \cdots 0 & \\ \hline & \xi_{11} & & \mathbf{0} \\ & & \ddots & \\ 0 & & & \\ & & & \ddots \\ \vdots & \mathbf{0} & & & \xi_{nn} \\ 0 & & & \end{array} \right), \widetilde{\mu} = \begin{pmatrix} 0 \\ \mu \end{pmatrix} \tag{15}$$

6. 130/30 long/short portfolio