

# Optimal stable coin folding strategy

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**Abstract**—A basic 'Lend/Borrow' strategy on the Aave protocol is one of the best and safest way to generate yield on your stable coins.

Those strategies are perfect fits for multiple 'Lend/Borrow' iterations or 'Folding' due to the inherent pegging of stable coins and the low liquidation risk incurred.

The question of the optimal *AUM* allocation between multiple pools is a complex one.

APY dilution generate non linear effects, which can be solved through advanced non linear optimization algorithms/solvers, but those solvers are computationally intensive and won't transport easily into solidity code and limited computations due to high gas prices.

We here propose a linearization which give a simpler constrained quadratic optimization problem, whose solution can be expressed analytically through Lagrange multiplier.

That analytical solution can be seamlessly computed in solidity and live on a blockchain even with high gas prices.

## I. FOLDING STRATEGY

Iteration	Supply	Borrow
0	A	A*X
1	A*X	A*X*X
...		
n	A*X <sup>n</sup>	A*X <sup>n+1</sup>

The total amount supplied after  $n$  iterations is:

$$S = \sum_{i=0}^n A * X^i = A \frac{1 - X^{n+1}}{1 - X} \quad (1)$$

$$S \approx \sum_{i=0}^{+\infty} A * X^i = A \frac{1}{1 - X}$$

The total amount borrowed after  $n$  iterations is:

$$B = \sum_{i=0}^n A * X^{i+1} = A * X \frac{1 - X^{n+1}}{1 - X} \quad (2)$$

$$B \approx \sum_{i=0}^{+\infty} A * X^{i+1} = A \frac{X}{1 - X}$$

By defining the total APY on the supply side as the sum of the base APY and the liquidity incentive APY, here comes :

$$sAPY = \text{supply APY} + \text{incentive supply APY} \quad (3)$$

$$bAPY = \text{borrow APY} - \text{incentive borrow APY} \quad (4)$$

$$\text{total APY} = \frac{A}{1 - X} * sAPY - \frac{A * X}{1 - X} * bAPY \quad (5)$$

This total APY is obviously strictly increasing as a function of the percentage ratio  $X$ .

$$\frac{d}{dX} (\text{total APY}) = A \frac{(sAPY - bAPY)}{(1 - X)^2} > 0 \quad (6)$$

The upper bound for  $X$  is only set by type of pools according to the incurred liquidation threshold.

For stable coin pools with historicaly proved pegging,  $X$  can be chosen as high as 72%.

## II. ALLOCATING BETWEEN POOLS : AN OPTIMIZATION PROBLEM

*A. Aave protocol : a trade off to maximize utilization/assured withdrawal*

Aave protocol implements at its core a regulating mechanism which aims at maximizing profits for liquidity supplier while keeping them safe and always able to withdraw their liquidity.

The utilization rate, which is just the ratio of the total borrowed divided by the total supplied:

$$U = \frac{B_t}{L_t} \quad (7)$$

is algorithmically targeted to stay close to an optimal  $U_{\text{optimal}}$  fixed by Aave governance per different pools.

The targeting mechanism can be summed up to the definition of two different borrow rate growth regime according to the value of  $U$  under or above  $U_{\text{optimal}}$ .

$$R_b = \begin{cases} R_{b0} + \frac{U}{U_{\text{optimal}}} * R_{\text{slope1}}, & \text{if } U \leq U_{\text{optimal}} \\ R_{b0} + R_{\text{slope1}} + \frac{U - U_{\text{optimal}}}{1 - U_{\text{optimal}}} * R_{\text{slope2}}, & U \geq U_{\text{optimal}} \end{cases}$$

$R_b$  is the borrow rate, which will rise sharply after  $U > U_{\text{optimal}}$  to prevent the ratio from being too close to one, which could keep suppliers from withdrawing their fund.  $R_{b0}$  is the constant base variable borrow rate.

The supply rate is then defined as the borrow rate multiplied by the utilization ratio.

$$R_s = R_b * U \quad (8)$$

The greater the utilization ratio, the more yield suppliers will get.

$$R_s - R_b = R_b * (U - 1) < 0 \quad (9)$$

### B. Stable coin pool parameters

Aave protocol governance has assigned the following parameters all common across stable pool reserves :

Coin	$U_{optimal}$	base	slope1	slope2
DAI	90%	0%	4%	60%
USDC	90%	0%	4%	60%
USDT	90%	0%	4%	60%

### C. Liquidity incentives APR

1) *Definition*: The definition of the liquidity incentives can be found in Aave V2 protocol white paper :

$$isAPR = 100 * \frac{aEmissionPerYear * rewardPrice}{totalATokenSupply * tokenPrice} \quad (10)$$

where the parameters match the following in the smart contract are detailed in the solidity section

$$ibAPR = 100 * \frac{vEmissionPerYear * rewardPrice}{totalCurrentVariableDebt * tokenPrice} \quad (11)$$

where the parameters match the following in the smart contract are detailed in the solidity section

We see that from the assumption of a fixed reward price and emission rate, the borrow APR depends on the total current debt :

$$APR_b = \frac{cte_b}{B_t} \quad (12)$$

And the supply APR depends on the total liquidity:

$$APR_s = \frac{cte_s}{L_t} \quad (13)$$

2) *Compounding*: We will here assume that those incentive annual APR are indeed harvested and reinvested in the strategy leading to an annualized compounded APY:

$$APY = (1 + \frac{APR}{secondsPerYear})^{secondsPerYear} - 1 \quad (14)$$

The reverse equation is :

$$APR = ((1 + APY)^{\frac{1}{secondsPerYear}} - 1) * secondsPerYear \quad (15)$$

### D. Dilution effect

When a proportion  $X$  of an additional amount  $AUM$  is dispatched to a specific liquidity pool. This pool will incur a dilution effect on its liquidity.

$$L_t \leftarrow L_t + X * AUM$$

### E. First order approximation

We can simplify the non linear term under the assumption:

$$\frac{1}{L_t + X * AUM} = \frac{1}{L_t} * \frac{1}{1 + \frac{X}{\frac{L_t}{AUM}}} = \frac{1}{L_t} * \sum_{n=0}^{n=+\infty} (-1)^n \frac{X^n}{\frac{L_t}{AUM}^n}$$

If  $X \ll \frac{L_t}{AUM}$ , as a first order approximation:

$$\frac{1}{L_t + X * AUM} = \frac{1}{L_t} - \frac{AUM}{L_t^2} * X$$

Here comes the new diluted utilization ratio :

$$U(X) = \frac{B_t}{L_t + X * AUM} = \frac{B_t}{L_t} * \frac{1}{1 + \frac{X}{\frac{L_t}{AUM}}} \approx \frac{B_t}{L_t} * (1 - \frac{X}{\frac{L_t}{AUM}}) \quad (16)$$

Here comes the new diluted incentive APYs:

$$APR_s(X) = \frac{cte_s}{L_t + X * AUM} \approx cte_s * (\frac{1}{L_t} - \frac{AUM}{L_t^2} * X) \quad (17)$$

After multiple folding and , we then linearize as in 17

It comes after linearization :

$$\begin{aligned} APR(X) &= APR_s(X) + APR_b(X) \\ &= \frac{cte_s}{L_t + \frac{X * AUM}{1 - X_{leverage}}} + \frac{cte_b}{B_t + \frac{X_{leverage} * X * AUM}{1 - X_{leverage}}} \\ &\approx cte_s * (\frac{1}{L_t} - \frac{AUM}{L_t^2} * X) \\ APR(X) &\approx A_0 + A_1 * X \end{aligned} \quad (18)$$

where

$$A_0 = \frac{cte_s}{L_t} + \frac{cte_b}{B_t} \quad (19)$$

$$A_1 = -\frac{cte_s * AUM}{L_t^2 * (1 - X_{leverage})} - \frac{cte_b * AUM * X_{leverage}}{B_t^2 * (1 - X_{leverage})} \quad (20)$$

### F. Optimization problem

When having to allocate an amount  $AUM$  between  $n$  pools, one will look at optimizing the total generated yield :

$$(\omega_1, \dots, \omega_n) = \arg \max_{\sum_{i=1}^n \omega_i = 1, 0 \leq \omega_i \leq 1} \sum_{i=1}^n \omega_i \times tAPY_i(\omega_i * AUM) \quad (21)$$

where the total APYs  $APY_i$  for each pool  $i$  are defined in equations 3 and 4.

The 14 equation shows that the liquidity incentive APYs on Aave are actually the main prominent reason for choosing a specific pool.

We can in a first approximation try to solve the simpler problem :

$$(\omega_1, \dots, \omega_n) = \arg \max_{\sum_{i=1}^n \omega_i = 1, 0 \leq \omega_i \leq 1} \sum_{i=1}^n \omega_i \times APR_i(\omega_i * AUM) \quad (22)$$

### G. Constrained quadratic optimization for the linearized problem

Under the conditions  $X \ll \frac{L_t}{AUM}$  and  $U < U_{optimal}$ , by using the first order approximations 18 :

$$(\omega_1, \dots, \omega_n) = \arg \max_{\sum_{i=1}^n \omega_i = 1, 0 \leq \omega_i \leq 1} \sum_{i=1}^n \omega_i \times (A_{0i} + A_{1i} * \omega_i)$$

where  $A_{0i}$ ,  $A_{1i}$  are the constants for the pool  $i$ .  
By calling

$$J(x) = - \sum_{i=1}^n x_i \times (A_{0i} + A_{1i} * x_i)$$

The optimization problem becomes a constrained quadratic minimization problem

$$\min_{\sum_{i=1}^n x_i = 1, 0 \leq x_i \leq 1} J(x)$$

### H. Analytical solution from Lagrange multiplier

By rewriting the constraint as :

$$g(x) = \sum_{i=1}^n x_i$$

We can compute the optimal solution using the Lagrange multiplier and the modified functional taking into the equality constraints and the inequality constraints using KKT conditions and slack variables:

$$\begin{aligned} J'(x) &= J(x) + \lambda (g(x) - 1) \\ &+ \sum_i \theta_i * (x_i - s_i^2) + \sum_i \gamma_i * (1 - t_i^2 - x_i) \end{aligned}$$

We solve the Lagrangian equations :

$$\begin{cases} \nabla_x J'(x) = 0 \\ \nabla_\lambda J'(x) = 0 \\ \nabla_{\theta} J'(x) = 0 \end{cases} \quad (23)$$

The  $\nabla_x$  conditions give the following :

$$\begin{pmatrix} A_{01} + 2 * A_{11} * x_1 + \lambda + \theta_1 - \gamma_1 \\ \vdots \\ A_{0i} + 2 * A_{1i} * x_i + \lambda + \theta_i - \gamma_i \\ \vdots \\ A_{0n} + 2 * A_{1n} * x_n + \lambda + \theta_n - \gamma_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

First case, if all  $\theta_i$  and  $\gamma_i$  are null, it means that all slack variables  $s_i^2$  and  $t_i^2$  are positive and the constraints are not active.

The  $x$  solution reads :

$$\forall i \in [1, \dots, n], x_i = - \frac{\lambda + A_{0i}}{2 * A_{1i}} \quad (24)$$

and the Lagrange multiplier can then be quantified through the constraint :

$$\lambda = \frac{-1 - \sum_{i=1}^n \frac{A_{0i}}{2 * A_{1i}}}{\sum_{i=1}^n \frac{1}{2 * A_{1i}}} \quad (25)$$

In the case where on one  $x_i$  does not respect the inequalities, it means that either  $x_i = 1$  or  $0$  depending on if  $\theta_i$  or  $\gamma_i$  is not null. The solution is then obtained recursively by reiterating the 24 and 25 with the remaining  $x_i$ s until all are found respecting the inequalities.

### III. ALLOCATING BETWEEN POOLS : AN OPTIMIZATION PROBLEM

We here present optimization results for the following snapshot parameter for a three pools (DAI, USDC, USDT) allocation on Aave Avalanche.

DAI (DAI.e)	1,1B	762,7M	2,17 % ± 0,07 % (1H)	3,51 % ± 0,07 % (1H)
USDT Coin (USDT.e)	282,24M	150,45M	1,29 % ± 0,78 % (1H)	2,70 % ± 0,48 % (1H)
USD Coin (USDC.e)	1,36B	775,97M	1,47 % ± 0,46 % (1H)	2,89 % ± 0,48 % (1H)

Fig. 1. Optimization parameters

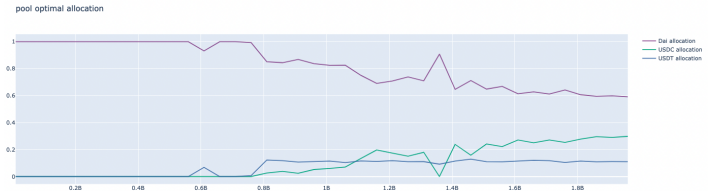


Fig. 2. Non linearized exact solution

A.

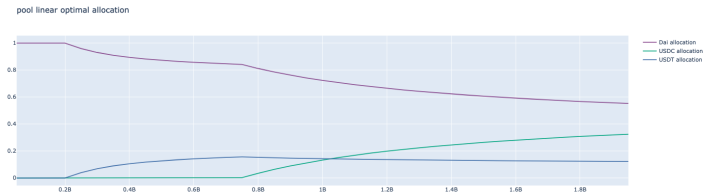


Fig. 3. Approximative linearized solution

### IV. SOLIDITY CODE FOR OPTIMIZATION

The new diluted incentive APYs expression 17 and both equations 27 and 10 give us the expression for our  $A_{0i}$  and  $A_{1i}$

$$isAPR = 100 * \frac{a * r * w}{L_t * t * d} = \frac{cte_s}{L_t} \quad (26)$$

where  $cte_s = 100 * \frac{a * r * w}{t * d}$  the parameters match the followings in the smart contract :

$$\left\{ \begin{array}{l} isAPR = incentiveDepositAPRPercent \\ a = aEmissionPerYear \\ r = REWARDPRICEETH \\ w = WEIDECIMALS \\ t = TOKENPRICEETH \\ d = TOKENDECIMALS \\ L_t = totalATokenSupply \end{array} \right.$$

$$ibAPR = 100 * \frac{v * r * w}{B_t * t * d} = \frac{cte_b}{B_t} \quad (27)$$

where  $cte_b = 100 * \frac{v * r * w}{t * d}$  the parameters match the followings in the smart contract :

$$\left\{ \begin{array}{l} ibAPR = incentiveBorrowAPRPercent \\ v = vEmissionPerYear \\ r = REWARDPRICEETH \\ w = WEIDECIMALS \\ B_t = totalCurrentVariableDebt \\ t = TOKENPRICEETH \\ d = TOKENDECIMALS \end{array} \right.$$

By using equations 20 and 19, we can compute  $A_{0i}$  and  $A_{1i}$  for each pool.

Equations 25 and ?? give us then the optimal pool allocation.

## V. MULTI-PROTOCOL OPTIMAL ALLOCATION

### A. Global optimization

We here make the assumption that each protocol yield is shared with each participant according to the pro rata of their liquidity participation.

The APY of protocol  $i$  at time  $t$  can be computed as :

$$APY_i(t) = \frac{fees_i(t)}{L_i(t)} \quad (28)$$

We forgot the  $t$  parameter as protocol yield and liquidity as those are fixed when we rebalance at a specific time  $t$ . When distributing our liquidity  $AUM$  over  $i \in (1, \dots, n)$ , each protocol APY  $i$  can be computed as :

$$APY_i = \frac{fees_i}{L_i + \omega_i * AUM} \quad (29)$$

where  $(\omega_1, \dots, \omega_n)$  are the liquidity proportions attributed to each protocol.

We rewrite 22 as one more general of a multi-protocol allocation constrained optimization problem :

$$(\omega_1, \dots, \omega_n) = \arg \max_{\sum_{i=1}^n \omega_i = 1, 0 \leq \omega_i \leq 1} \sum_{i=1}^n \omega_i \times APY_i(\omega_i * AUM) \quad (30)$$

### B. Risk consideration

The constraint on weights :

$$0 \leq \omega_i \leq 1, \forall i \in (1, \dots, n) \quad (31)$$

can be tailored by tighter bounds which result from a protocol risk assessing methodology :

$$l_i \leq \omega_i \leq u_i, \forall i \in (1, \dots, n) \quad (32)$$

where  $u_i$  is the maximal proportion of liquidity we can afford for this protocol after a careful risk assessment and  $l_i$  a diversification bound which can be null or common for all protocols  $i$ .

## VI. CONCLUSION

We have managed to simplify the optimization problem and transport it to EVM like blockchain languages. The solution can scale up to at least ten pools without being too computationally intensive.

## REFERENCES

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