moon lander state $\rightarrow (h, v, m)^T$

$\dot{h}(t) = v(t)$

$\dot{v}(t) = -g + \frac{a(t)}{m(t)}$

$\dot{m}(t) = -K\, a(t)$

h = altitutde

v = velocity

m = mass of the lander

$a(t) \in [0,1] \rightarrow$ thrust

$K \rightarrow$ const. fuel burning rate

initial state $\rightarrow (h_0, v_0, m_0)^T$

Target $\Rightarrow h(t^*) = 0$ and $v(t^*) = 0$

$t \rightarrow$ terminal time

Objective $\Rightarrow \min\limits_{\alpha(t)} P(\alpha) = \int_0^{\tau} \alpha(t)\, dt$

s.t. $h(\tau) = 0$

$v(\tau) = 0$

$$f = \begin{pmatrix} v \\ -g + \frac{\alpha}{m} \\ -k\alpha \end{pmatrix}, \quad l = \alpha$$

$$H = -l + \lambda^t f = -\alpha + \lambda_1 \cdot v + \lambda_2\left(-g + \frac{\alpha}{m}\right) + \lambda_3 \cdot (-k\alpha)$$

$$\alpha^* = \underset{\alpha \in [0,1]}{\operatorname{argmax}} H = \underset{\alpha \in [0,1]}{\operatorname{argmax}} \underbrace{\left(-1 + \frac{\lambda_2}{m} - \lambda_3 k\right)}_{=b}\alpha + \lambda_1 v - \lambda_2 g$$

let $b = \left(-1 + \frac{\lambda_2}{m} - \lambda_3 k\right)$

$$\alpha^* = \begin{cases} 0 & ; \ b \leq 0 \\ 1 & ; \ b > 0 \end{cases}$$

when 'b' $\rightarrow$ changes monotonically

So thrust value changes by following policy :-

Policy : $\alpha^* = 0 \quad t \in [0, t^*]$

$\alpha^* = 1 \quad t \in [t^*, \tau]$

$$t \in [0, t^*] \qquad f = \begin{pmatrix} v \\ -g \\ 0 \end{pmatrix} \Rightarrow \begin{cases} \dot{h} = v \\ \dot{v} = -g \\ \dot{m} = 0 \end{cases}$$

$$t \in [t^*, \tau] \qquad f = \begin{pmatrix} v \\ -g + \frac{\alpha}{m} \\ -k\alpha \end{pmatrix} \Rightarrow \begin{cases} \dot{h} = v \\ \dot{v} = -g + \frac{1}{m} \\ \dot{m} = -K \end{cases}$$