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MAE-598/494 - HW-4

(3) max. $f = x_1 x_2 + x_2 x_3 + x_1 x_3$

s.to: $h = x_1 + x_2 + x_3 - 3 = 0$

Using Lagrangian Method -

$$L = -(x_1 x_2 + x_2 x_3 + x_1 x_3) + \lambda(x_1 + x_2 + x_3 - 3)$$

$$\therefore \frac{\partial L}{\partial x} = \begin{pmatrix} \frac{\partial L}{\partial x_1} \\ \frac{\partial L}{\partial x_2} \\ \frac{\partial L}{\partial x_3} \end{pmatrix} = \begin{pmatrix} -x_2 - x_3 + \lambda \\ -x_1 - x_3 + \lambda \\ -x_2 - x_1 + \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + x_3 - 3 = 0$$

Solving above system of 4-linear eq$^n$ using calculator we obtain

$$\boxed{x_1 = x_2 = x_3 = 1 \quad \text{and} \quad \lambda = -2.}$$

Now,

$$L_{xx} = \begin{pmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \frac{\partial^2 L}{\partial x_1 \partial x_3} \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_2 \partial x_3} \\ \frac{\partial^2 L}{\partial x_3 \partial x_1} & \frac{\partial^2 L}{\partial x_2 \partial x_3} & \frac{\partial^2 L}{\partial x_3^2} \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$$

$$\frac{\partial h}{\partial x} = \begin{pmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

then $\frac{\partial h}{\partial x} \cdot dx = 0 \Rightarrow \begin{pmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix} = 0$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix} = 0$$

$$\Rightarrow \boxed{dx_1 + dx_2 + dx_3 = 0}$$

$$\Rightarrow \boxed{dx_1 = -dx_2 - dx_3} \quad (1)$$

Also,

$$dx^T L_{xx} dx = \begin{pmatrix} dx_1 & dx_2 & dx_3 \end{pmatrix} \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix}$$

$$\Rightarrow \boxed{dx^T L_{xx} dx = -2dx_1 dx_2 - 2dx_1 dx_3 - 2dx_2 dx_3} \quad (2)$$

Plugging eq^n (1) in (2),

$$\Rightarrow dx^T . L_{xx} . dx = -2(-dx_2 - dx_3)dx_2 - 2(-dx_2 - dx_3)dx_3 - 2dx_2 dx_3$$

$$= 2dx_2^2 + 2dx_2 dx_3 + \cancel{2dx_2 dx_3} + 2dx_3^2 - \cancel{2dx_2 dx_3}$$

$$= 2(dx_2^2 + dx_2 dx_3 + dx_3^2)$$

$$\Rightarrow dx^T . L_{xx} . dx = 2\left(\underbrace{\left(dx_2 + \frac{1}{2} dx_3\right)^2}_{\geq 0} + \underbrace{\frac{3}{4} dx_3^2}_{\geq 0}\right)$$

Since RHS all terms are sq. so they are $\geq 0$.

If $dx_3 = 0$ & $dx_2 = 0$ then from eq^n - (1) $dx_1 = 0$ → which means No Perturbation takes place

therefore

~~RHS in eq^n (3)~~

$$2\left(\left(dx_2 + \frac{1}{2} dx_3\right)^2 + \frac{3}{4} dx_3^2\right) > 0, \forall dx_1, dx_2$$

Therefore at $x_1 = x_2 = x_3 = 1$ and $\lambda = -2$ our

Ans) objective funcⁿ $(f) = x_1 x_2 + x_2 x_3 + x_1 x_3$

has a maxⁿ. value i.e. $= 1 + 1 + 1 = 3$.