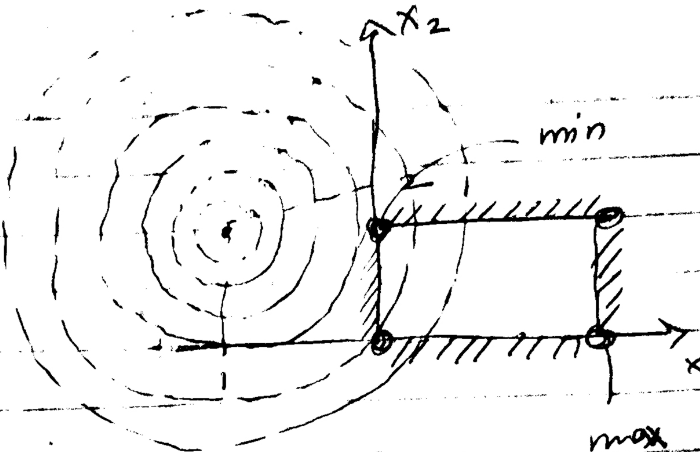
Name - SHUBHIVORATA DUTTA

ASU - ID - 1223486849

DOP - HW - 4



(1) $\min_{x_1, x_2} (x_1+1)^2 + (x_2-2)^2$

s.t. $\underbrace{-x_1 \le 0}_{g_1}$, $\underbrace{-x_2 \le 0}_{g_2}$

$\underbrace{x_1 \le 2}_{g_3}$, $\underbrace{x_2 \le 1}_{g_4}$

$$L = (x_1+1)^2 + (x_2-2)^2 + \mu_1(-x_1) + \mu_2(-x_2) + \mu_3(x_1-2) + \mu_4(x_2-1)$$

$$\nabla_x L = \begin{pmatrix} 2(x_1+1) - \mu_1 + \mu_3 \\ 2(x_2-2) - \mu_2 + \mu_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

* if $-x_1 = 0$	then	$\mu_1 > 0$	if $-x_1 < 0$	then	$\mu_1 = 0$
if $-x_2 = 0$	then	$\mu_2 > 0$	" $-x_2 < 0$	"	$\mu_2 = 0$
" $x_1 - 2 = 0$	"	$\mu_3 > 0$	" $x_1 - 2 < 0$	"	$\mu_3 = 0$
" $x_2 - 1 = 0$	"	$\mu_4 > 0$	" $x_2 - 1 < 0$	"	$\mu_4 = 0$

Checking diffn cases -

(1) when $\mu_2 > 0, \mu_3 > 0$ and $\mu_1 = \mu_4 = 0$, then

$x_1 = 0$ $\quad \mu_3 = 2 > 0$

$x_2 = 1$ $\quad \mu_2 = 2 > 0$

Since the objective funcn is a convex funcn and feasible domain is convex set too, the problem is convex problem. Therefore, the soln is

$x_1 = 0$, $x_2 = 1$, $\mu_1 = 0$ $\quad \mu_2 = 2$

$\mu_3 = 2$ $\quad \mu_4 = 0$

and min. value of objective funcn is 1. (Ans)

(Case-2) when $\mu_1 > 0$, $\mu_4 > 0$, $\mu_2 = \mu_3 = 0$, then

$$x_1 = 2, \quad x_2 = 0,$$

$$\mu_1 = -6 < 0 \; ; \; \mu_4 = -4 < 0$$

As we observe μ_1 and μ_4 value above contradicts our assumption so $x_1 = 2$ and $x_2 = 0$ is not a min. sol^n. From figure we can observe this point is max. point.

(Case-3) when $\mu_1 > 0$, $\mu_2 > 0$ and $\mu_3 = \mu_4 = 0$, then

$$x_1 = 2, \quad x_2 = 1,$$

$$\mu_1 = -6 < 0 \leftarrow \text{contradicts}$$

$$\mu_2 = 2 > 0$$

So $x_1 = 2$, $x_2 = 1 \rightarrow$ not min. sol^n.

(Case-4) when $\mu_3 > 0$, $\mu_4 > 0$ and $\mu_1 = \mu_2 = 0$ then

$$x_1 = 0 \quad x_2 = 0$$

$$\mu_3 = 2 > 0 \text{ and } \mu_4 = -4 < 0 \leftarrow \text{contradiction}$$

So $x_1 = 0$, $x_2 = 0 \rightarrow$ not a min. sol^n

(Ans) Therefore, min. value of given objective funct^n is 1 when $x_1 = 0$, and $x_2 = 1$

(2)

$$\min \; -x_1$$

$$\text{s.t.} \quad x_2 - (1 - x_1)^3 \le 0 \leftarrow g_1$$

$$x_2 \ge 0 \Rightarrow -x_2 \le 0 \leftarrow g_2$$

$$\therefore L = -x_1 + \mu_1 \left[x_2 - (1 - x_1)^3\right] + \mu_2 (-x_2)$$

$$\nabla_x L = \begin{pmatrix} -1 + 3(1-x_1)^2 \mu_1 \\ \mu_1 - \mu_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

if $x_2 - (1-x_1)^3 = 0 \rightarrow \mu_1 > 0$; if $x_2 - (1-x_1)^3 < 0 \rightarrow \mu_1 = 0$

" $-x_2 = 0 \rightarrow \mu_2 > 0$; " $-x_2 < 0 \rightarrow \mu_2 = 0$.

Case-1) when $\mu_1 > 0$ and $\mu_2 > 0$, then

$x_2 - (1-x_1)^3 = 0 \Rightarrow x_1 = 1$

$-x_2 = 0 \Rightarrow x_2 = 0$

Substituting in $\nabla_x L$, we get $\boxed{-1 = 0}$ → Not possible

Hence case-1 not valid.

Case-2) when $\mu_1 > 0$ and $\mu_2 = 0$, then

$x_2 - (1-x_1)^3 = 0$

$-x_2 < 0 \Rightarrow \mu_2 = 0$

Again checking in $\nabla_x L$, $\mu_1 - \mu_2 = 0 \Rightarrow \boxed{\mu_1 = 0}$ → Not possible (contradict)

Case-3) when $\mu_1 = 0$ and $\mu_2 > 0$, then

$x_2 - (1-x_1)^3 < 0$ and $-x_2 = 0$

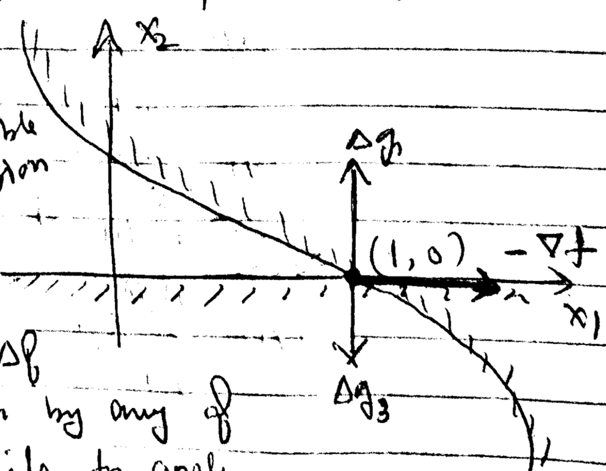
Checking in $\nabla_x L$, $\mu_1 - \mu_2 = 0 \Rightarrow \boxed{\mu_2 = 0}$ → contradict (Reject)

Case-4) when $\mu_1 = 0$ and $\mu_2 = 0$, then

from $\nabla_x L$ → $\boxed{-1 = 0}$ → not possible (reject)

$\nabla f + \mu_1 \nabla g_1 + \mu_2 \Delta g_3 = 0$

$\Rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \mu_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mu_2 \begin{pmatrix} 0 \\ -1 \end{pmatrix} = 0$



As seen in figure $\Delta_x g_1$ and $\Delta_x g_2$ are linearly independent and $-\Delta f$ does not lie in the plane span by any of these gradient. KKT cond^n fails to apply. Therefore, no optimal sol^n exist.

⑤ Let c_{ij} = cost to move from edge $i \to j$

$$x_{ij} = \begin{cases} 0 & \text{for } i \not\to j \\ 1 & \text{for } i \to j \end{cases}$$

$\mathcal{V} = N$ = Node domain.

m = Steps req. ~~after~~ to return to Station-0 from final destination.

$s \to$ starting point (Station-0)

$j \to$ any mid node

$t \to$ target node

Problem

Objective : $\min\limits_{\{x_{ij}\}} \sum\limits_{ij \in \mathcal{V}} c_{ij} x_{ij}$

s. to. (i) $\sum\limits_{j=1}^{N} x_{sj} = 1$ (start to adjacent node)

(ii) $\sum\limits_{j=1}^{N} x_{js} = 0$

(iii) $\sum\limits_{j=(N+1)}^{N+M} x_{js} = 1$

(iv) $\sum\limits_{j=1}^{N} x_{jt} = 1$

(v) $\sum\limits_{j=N+1}^{N+M} x_{tj} = 1$

(vi) $\sum\limits_{j=1}^{N+M} x_{ij} = \sum\limits_{j=1}^{N+M} x_{ji}$, $\forall\ i \in \mathcal{V}$ / except start and end point.

domain of nodes