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MAE-598/494 - HW-4

3 max. $f = x_1 x_2 + x_2 x_3 + x_1 x_3$

s.to: $h = x_1 + x_2 + x_3 - 3 = 0$

Using Lagrangian Method -

$$L = -(x_1 x_2 + x_2 x_3 + x_1 x_3) + \lambda (x_1 + x_2 + x_3 - 3)$$

$$\therefore \frac{\partial L}{\partial x} = \begin{pmatrix} \frac{\partial L}{\partial x_1} \\ \frac{\partial L}{\partial x_2} \\ \frac{\partial L}{\partial x_3} \end{pmatrix} = \begin{pmatrix} -x_2 - x_3 + \lambda \\ -x_1 - x_3 + \lambda \\ -x_2 - x_1 + \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{\partial L}{\partial \lambda} = x_1 + x_2 + x_3 - 3 = 0$$

Solving above system of 4-linear eq^n using calculator we obtain

$$\boxed{x_1 = x_2 = x_3 = 1 \text{ and } \lambda = -2.}$$

Now,

$$L_{xx} = \begin{pmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \frac{\partial^2 L}{\partial x_1 \partial x_3} \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_2 \partial x_3} \\ \frac{\partial^2 L}{\partial x_3 \partial x_1} & \frac{\partial^2 L}{\partial x_2 \partial x_3} & \frac{\partial^2 L}{\partial x_3^2} \end{pmatrix} = \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$$

$$\frac{\partial h}{\partial x} = \begin{pmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

then $\frac{\partial h}{\partial x} . dx = 0 \Rightarrow \begin{pmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix} = 0$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix} = 0$$

$$\Rightarrow \boxed{dx_1 + dx_2 + dx_3 = 0}$$

$$\Rightarrow \boxed{dx_1 = -dx_2 - dx_3} \quad (1)$$

Also,

$$dx^T L_{xx} dx = (dx_1 \; dx_2 \; dx_3) \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix}$$

$$\Rightarrow \boxed{dx^T L_{xx} dx = -2dx_1 dx_2 - 2dx_1 dx_3 - 2dx_2 dx_3} \quad (2)$$

Plugging eq^n (1) in (2),

$$\Rightarrow dx^T . L_{xx} . dx = -2(-dx_2 - dx_3)dx_2 - 2(-dx_2 - dx_3)dx_3 - 2dx_2 dx_3$$

$$= 2dx_2^2 + 2dx_2 dx_3 + \cancel{2dx_2 dx_3} + 2dx_3^2 - \cancel{2dx_2 dx_3}$$

$$= 2(dx_2^2 + dx_2 dx_3 + dx_3^2)$$

$$\Rightarrow dx^T . L_{xx} . dx = 2\left(\underbrace{\left(dx_2 + \frac{1}{2} dx_3\right)^2}_{\geq 0} + \underbrace{\frac{3}{4} dx_3^2}_{\geq 0}\right)$$

Since RHS all terms are sq. so they are ≥ 0.

If $dx_3 = 0$ & $dx_2 = 0$ then from eq^n - (1)
$dx_1 = 0$ $\rightarrow$ which means No Perturbation takes place

therefore

~~RHS in eq^n (3)~~

$$2\left(\left(dx_2 + \frac{1}{2} dx_3\right)^2 + \frac{3}{4} dx_3^2\right) > 0, \; \forall \; dx_1, dx_2$$

Therefore at $x_1 = x_2 = x_3 = 1$ and $\lambda = -2$ our

Ans) objective funcⁿ $(f) = x_1 x_2 + x_2 x_3 + x_1 x_3$

has a max.ⁿ value i.e. $= 1 + 1 + 1 = 3$.

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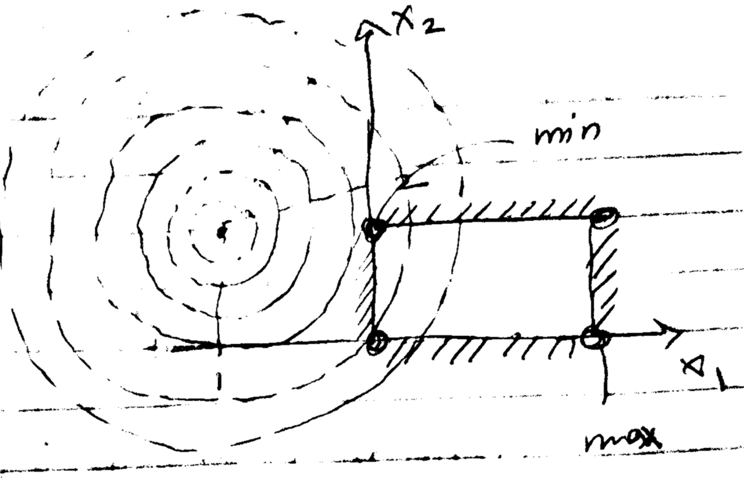


(1) $\min_{x_1, x_2} (x_1+1)^2 + (x_2-2)^2$

s.t. $\underbrace{-x_1 \le 0}_{g_1}$, $\underbrace{-x_2 \le 0}_{g_2}$

$\underbrace{x_1 \le 2}_{g_3}$, $\underbrace{x_2 \le 1}_{g_4}$

$$L = (x_1+1)^2 + (x_2-2)^2 + \mu_1(-x_1) + \mu_2(-x_2) + \mu_3(x_1-2) + \mu_4(x_2-1)$$

$$\nabla_x L = \begin{pmatrix} 2(x_1+1) - \mu_1 + \mu_3 \\ 2(x_2-2) - \mu_2 + \mu_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

* if $-x_1 = 0$	then	$\mu_1 > 0$	if $-x_1 < 0$	then	$\mu_1 = 0$
if $-x_2 = 0$	then	$\mu_2 > 0$	" $-x_2 < 0$	"	$\mu_2 = 0$
" $x_1 - 2 = 0$	"	$\mu_3 > 0$	" $x_1 - 2 < 0$	"	$\mu_3 = 0$
" $x_2 - 1 = 0$	"	$\mu_4 > 0$	" $x_2 - 1 < 0$	"	$\mu_4 = 0$

Checking diff.^{n} cases -

(1) when $\mu_2 > 0, \mu_3 > 0$ and $\mu_1 = \mu_4 = 0$, then

$x_1 = 0$ $\quad \mu_3 = 2 > 0$

$x_2 = 1$ $\quad \mu_2 = 2 > 0$

Since the objective func^n is a convex func^n and feasible domain is convex set too, the ~~pro~~ problem is convex problem. Therefore, the sol^n is

$x_1 = 0$, $x_2 = 1$, $\mu_1 = 0$, $\mu_2 = 2$

$\mu_3 = 2$, $\mu_4 = 0$

and ~~the~~ min. value of objective func^n is 1. (Ans)

(Case-2) when $\mu_1 > 0$, $\mu_4 > 0$, $\mu_2 = \mu_3 = 0$, then

$x_1 = 2$, $x_2 = 0$,

$\mu_1 = -6 < 0$; $\mu_4 = -4 < 0$

As we observe μ_1 and μ_4 value above contradicts our assumption so $x_1 = 2$ and $x_2 = 0$ is not a min. solⁿ. From figure we can observe this point is max. point.

(Case-3) when $\mu_1 > 0$, $\mu_2 > 0$ and $\mu_3 = \mu_4 = 0$, then

$x_1 = 2$, $x_1 = 1$.

$\mu_1 = -6 < 0$ ← contradicts

$\mu_2 = 2 > 0$

So $x_1 = 2$, $x_2 = 1$ → not min. solⁿ.

(Case-4) when $\mu_3 > 0$, $\mu_4 > 0$ and $\mu_1 = \mu_2 = 0$ then

$x_1 = 0$ $x_2 = 0$

$\mu_3 = 2 > 0$ and $\mu_4 = -4 < 0$ ← contradiction

So $x_1 = 0$, $x_2 = 0$ → not a min. solⁿ

(Ans) Therefore, min. value of given objective funcⁿ is 1 when $x_1 = 0$, and $x_2 = 1$

(2) min. $-x_1$

s.t. $x_2 - (1 - x_1)^3 \leq 0$ ← g_1

$x_2 \geq 0 \Rightarrow -x_2 \leq 0$ ← g_2

$\therefore L = -x_1 + \mu_1 [x_2 - (1-x_1)^3] + \mu_2 (-x_2)$

$$\nabla_x L = \begin{pmatrix} -1 + 3(1-x_1)^2 \mu_1 \\ \mu_1 - \mu_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

if $x_2 - (1-x_1)^3 = 0 \rightarrow \mu_1 > 0$; if $x_2 - (1-x_1)^3 < 0 \rightarrow \mu_1 = 0$

" $-x_2 = 0 \rightarrow \mu_2 > 0$; " $-x_2 < 0 \rightarrow \mu_2 = 0$.

(Case-1) when $\mu_1 > 0$ and $\mu_2 > 0$, then

$x_2 - (1-x_1)^3 = 0 \Rightarrow x_1 = 1$

$-x_2 = 0 \Rightarrow x_2 = 0$

Substituting in $\nabla_x L$, we get $\boxed{-1 = 0}$ → Not possible

Hence case-1 not valid.

(Case-2) when $\mu_1 > 0$ and $\mu_2 = 0$, then

$x_2 - (1-x_1)^3 = 0$

$-x_2 < 0 \Rightarrow \mu_2 = 0$

Again checking in $\nabla_x L$, $\mu_1 - \mu_2 = 0 \Rightarrow \boxed{\mu_1 = 0}$ → Not possible (contradict)

(Case-3) when $\mu_1 = 0$ and $\mu_2 > 0$, then

$x_2 - (1-x_1)^3 < 0$ and $-x_2 = 0$

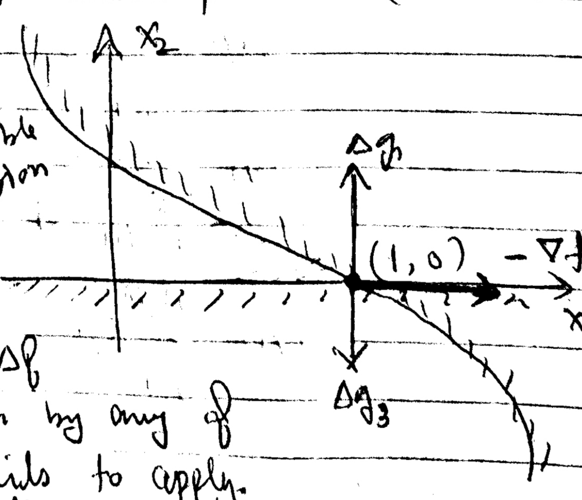
Checking in $\nabla_x L$, $\mu_1 - \mu_2 = 0 \Rightarrow \boxed{\mu_2 = 0}$ → contradict (Reject)

(Case-4) when $\mu_1 = 0$ and $\mu_2 = 0$, then

from $\nabla_x L$ → $\boxed{-1 = 0}$ → not possible (reject)

$\nabla f + \mu_1 \nabla g_1 + \mu_2 \nabla g_3 = 0$

$\Rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \mu_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mu_2 \begin{pmatrix} 0 \\ -1 \end{pmatrix} = 0$



As seen in figure $\nabla_x g_1$ and $\nabla_x g_2$ are linearly independent and $-\nabla f$ does not lie in the plane span by any of these gradient. KKT condn fails to apply.

Therefore, no optimal soln exist.

⑤ Let c_{ij} = cost to move from edge $i \to j$

$$x_{ij} = \begin{cases} 0 & \text{for } i \not\to j \\ 1 & \text{for } i \to j \end{cases}$$

$\mathcal{V} = N$ = Node domain.

m = Steps req. ~~after~~ to return to Station-0 from final destination.

$S \to$ starting point (Station-0)

$j \to$ any mid node

$t \to$ target node

Problem

Objective : $\min\limits_{\{x_{ij}\}} \sum\limits_{ij \in \mathcal{V}} c_{ij} x_{ij}$

S. to.

(i) $\sum\limits_{j=1}^{N} x_{Sj} = 1$ (start to adjacent node)

(ii) $\sum\limits_{j=1}^{N} x_{jS} = 0$

(iii) $\sum\limits_{j=(N+1)}^{N+M} x_{jS} = 1$

(iv) $\sum\limits_{j=1}^{N} x_{jt} = 1$

(v) $\sum\limits_{j=N+1}^{N+M} x_{tj} = 1$

(vi) $\sum\limits_{j=1}^{N+M} x_{ij} = \sum\limits_{j=1}^{N+M} x_{ji}$, $\forall\, i \in \mathcal{V}$ / except start and end point.

domain of nodes