1 Pretest

1. Given the equation $x^2 - 4x + 3 = 0$,

   (a) Write the solution set using set-builder notation

   Using the quadratic equation we quickly get $x = \frac{4 \pm \sqrt{16-12}}{2} = 1, 3$ thus, our solutions are $-1, -3$. Putting this into set-builder notation we have: $\{x | x = -1, -3\}$.

   (b) Write the solution set in roster notation

   Simply applying the answer from the previous problem into roster notation: $\{-3, -1\}$.

2. (a) Show that $(A \cap B) \cup C$ and $A \cap (B \cup C)$ need not be the same set.

   Writing this out formally: $\forall a$ s.t. $a \in A$ and $a \in B$ we result in a new set, say, D; then let $(A \cap B) \cup C = E$, then $E = a \in D$ or $x \in C$. In the case that $A \cap (B \cup C)$: $\forall b$ s. t. $b \in B$ and $b \in C$ which we let form into a new set D; let $A \cap (B \cup C) = E$, then $E = b \in D$ or $x \in A$. An example to demonstrate this is as follows: $E_1 = (\{1,2,3,4\} \cap \{2,3,4,5\}) \cup \{3,4,5,6\} = \{2,3,4,5,6\}$ and $E_2 = \{1,2,3,4\} \cap (\{2,3,4,5\} \cup \{3,4,5,6\}) = \{1,2,3,4,5\}$. Notice that $E_1 \neq E_2$.

   (b) What is the most general case in which $(A \cap B) \cup C$ and $A \cap (B \cup C)$ are the same?

   When they are all the empty set.

   (c) What is wrong with the expression $A \cap B \cup C$?

   It is not well defined as it is unclear which operation takes precedence.

3. Let $A = \{1, 2, 3\}$

   (a) How many elements belong to the set $\{f | f : A \to A\}$?

   $\mathscr{P}(f) = 2^3 = 8$.

   (b) How many of these elements are one-to-one?

   All of them, in fact, they are also onto and all bijective.

   (c) If $f(1) = 2, f(2) = 3$, and $f(3) = 1$, describe $f^{-1} : A \to A$.

   The inverse function can be described by the transformations $2 \mapsto 1$ and $3 \mapsto 2$ and $1 \mapsto 3$. Alternatively: $f^{-1}(2) = 1, f^{-1}(3) = 2$, and $f^{-1}(1) = 3$.

4. Define f by

$$f(x) = \begin{cases} 2 - x, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}.$$

   (a) $f(-x)$

   (b) $-f(x)$

   (c) $f(x+3)$

   (d) $f(2x+3)$

   These are far too trivial, so I will not be doing them.

5. Let f and g be defined by:

$$f(x) = \frac{x^2 - 9}{x - 3}$$
$$g(x) = x + 3.$$

(a) How do f and g differ?

f has a point of removable discontinuity at $x = 3$; g does not.

(b) Does $\lim_{x \to} \frac{x^2-9}{x-3} = \lim_{x \to 3}(x + 3)$?

Yes, it does. As said, the point of discontinuity is removable, observe:

$$f(x) = \frac{x^2 - 9}{x - 3} = \frac{(x + 3)(x - 3)}{(x - 3)} = (x + 3).$$