1 Naïve Set Theory

1. Consider the sentence not($x \in x$), i.e., the subset by way of axiom of specification $B = \{x \in A : x \notin x\}$. Is it the case that $B \in A$?

   No. It is not the case that $B \in A$. If $B \in A$, then either $B \in B$, $B = \{B \in A : B \notin A\}$, but B cannot be in A by definition. Similarly, if $B \in A$ AND $B \notin B$, then by definition B is contained within the subset B, which once again, is a contradiction.

2. Given some set $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \ldots\} = \mathfrak{E}$ prove or disprove that all sets in $\mathfrak{E}$ obtained this way are unique.

   For all $x \in \mathfrak{E}$ there exists another element, $(y \neq \emptyset) \in \mathfrak{E}$, such that $x \subset y$. We show that $x \neq y$. For contradiction, we say $y \subset x$. Then $x = \{\}$ then $y \subset \emptyset$; this cannot be, however, since the only subset of the empty set is the empty set, and, since $y \neq \emptyset$ this is a contradiction.

   Let $n, k \in \mathbb{N}$. Given $A_0 = \emptyset$ and $A_{n+1} = \{A_n\}$. We show that $A_n \neq A_k$ if $n \neq k$. Suppose then, for contradiction, when 5