Numerical Analysis of a Morphing Wing

Master of Aerospace Engineering, S3 Project Report

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***Abstract*—The paper deals with the documentation of numerical analysis of a morphing wing. The paper will follow the guidelines of development of a tool to run the numerical analysis over a non-linear elastic structure like a beam or a morphing wing shaped to an airfoil. The tool allows to calculate the potential energy of the body and minimise it. The software used here is MATLAB. The system's potential energy has been derived and further calculations takes place to get the derivatives of those variables.The paper will also cover the previous work on meshless methods in order to understand the numerical analysis approach on a complex geometries.**

***Index Terms*—Numerical Analysis, Morphing wing, potential energy, fmincon function**

I. Introduction

Numerical analysis deals with the approximation based machine learning algorithms. Numerical models are build with the help of these algorithms. Numerical analysis has been considered for computing the solutions for differential equations. The mathematical analysis is being carried out with the help of algorithms using numerical approximation. There are multiple solvers used for the analysis and in this thesis, we will be using MATLAB for our progress. The researchers has been deploying numerical analysis in various domains and applications. One of the most interesting application at present is morphing wings.

Morphing Wing is a simplified structure that not just reduces the fuel efficiency by amplifying the aerodynamics of the wing, but also simplifying the manufacturing process. A morphing wing is a light weight but high strength structure which alloys the production to be composite based. They are reliable in reduction of the aerodynamic drag and vibration control. The morphing wings are designed in a manner to exhibit strength of sustaining loads and also the stiffness to sustain external loads. In this project, a tool is being designed to which any application could be derived and solved including the morphing wings.

The mesh based method are used dominantly in the field of engineering. The most known method, Finite Element Method (FEM) has been used widely for the purpose of numerical analysis. The FEM technique is based on the division of the domain into finite number of elements specified with a finite number of parameters. The solution of the problem is obtained on solving the equations at the nodes of the elements. The discretisation process under finite element is its foremost advantage. With time, the application of mesh based method expanded and hence put the major drawbacks into light.In case of mesh based methods, the data is transferred from the physical domain to the computational domain for all the calculations are performed in the computational domain. Once the calculations are complete, mapping is done and the entire data is again transferred back to the physical domain. In case of a linear structural problem, the method of modeling and solutions are well establishes but when it comes to Non-linear problem, the solutions process and modeling depends highly on the characteristics of the problem. They are more complicated than the linear problem and hence useful for structures like morphing wings. Using MATLAB allowed to understand the working of non-linear solutions using practical computer programs. The basic approach in this report includes the solving of stress and strain and formulation of structural equilibrium using the energy principle.

The basic principle on which the tool is made is to minimize the potential/strain energy of the system. The system's potential energy has been derived and further calculations takes place to get the derivatives of those variables. A structural system is said to be conservative if the system has some potential energy and therefore its equilibrium is considered only when the energy is minimum. As the potential energy of many structural problems is the positive definite quadratic function of a state variable, the condition will yield a unique global minimum solution. As in the field of mechanics, many governing equations of structural mechanics includes the derivatives of the variable with respect to spatial coordinates. The subsequent derivatives of the field variable in this particular case has also been analysed in order to attain verified results. The tool has been initially designed taking into account a beam/a basic geometry. Later the results of complex geometries like Morphing are taken into account.

II. Previous Work

Earlier during the research on numerical analysis, the principle need of the hour was to understand the implementations of various methods in computational platforms like MATLAB. The work was carried out on Overvelde's research thesis on numerical analysis of a cantilever beam. But before starting with the problem solving technique, the requirement of literature survey was top requirement. The literature survey included the study of various applications of meshless method through research paper. The research papers provide a broad domain of the working of various meshless methods including thin plates[2] and shells[3], transient heat conduction problems[5], elasticity and fracture problems[7],etc. These papers allowed better understanding of various methods that can be opted to numerically analyse a problem. Along with the various meshless methods, the study of kernel functions[1][8] used in Gaussian process are also very important as changing the kernels will help us change the error terms in the problem.

The major outcome during the previous session was the implementation of the MATLAB code provided for a structural problem in order to observe the working of kernel functions in MATLAB and the way they can be altered in order to achieve different results. The main aspect of the session was to understand a significant test case and observe the alterations that can be readily taken into consideration in future while working on a particular application.

A. *Literature Review*

The project work began with the literature survey of the various components of the project. The fact that there were no subsequent prerequisite available for our project and hence I have to start by studying the most basic part of the project. After the allocation of the project, the major work relied upon learning the Surrogates models and the Gaussian process. The project demanded a strong hold on the basics, so a significant amount of time has been allotted on the literature survey.

During the S2 section of the project, a test case has been studied in order to understand the working of the conventional kernel functions employed in the code and then change the kernel function in order to observe the effectiveness of the results. The test case is based upon the field of topology optimization wherein an optimal material layout based upon specific performance targets is the desired output.[1] The use of mesh less methods has been deployed for the optimization and henceforth all the equations are constructed on the nodes.

B. *Mesh Less methods*

In the case of Mesh less Methods, all the calculations are performed in the physical domain. The interpolation in this method is free from mesh and hence the calculations takes place at the nodes, therefore discarding the possibility of re-meshing and distortion.[4] The mesh less methods works upon the technique of interaction of the nodes with their neighbours. The extensive properties are not assigned to the mesh but rather to every single nodes in the domain. One of the oldest mesh free method is Smoothed Particle Hydrodynamics (SPH); in 1977 discovered by Gingold, Monaghan and Lucy; which initially dealt with the astronomical problems.[4] Later on, Libersky applied the SPH in solid mechanics.

C. *Moving Least Square approximation*

With time, substantial improvements took place in the field of mesh less methods and then Moving Least Square approximation came into use. The Moving least square is a method to reconstruct a function from a set of non-sampled points by calculating the weighted least square. This approximation played a vital role in the establishment of various other mesh less methods. In the previous work, the method taken into major consideration was Element Free Galerkin (EFG) method which is being studied and applied on thin plates deflection[2] and shells earlier.[3] [4]

1) Comparison of methods: During the previous semester, the focus was upon the EFG and MLPG methods as these two are the most used ones. Hence one of the major objective turns out to be the selection of the methods for different analysis purposes. The difference between the two methods allowed to select a particular one for a very specific analysis.

D. *Error norm and Compliance*

The error norm is a scalar quantity calculated by the relative error between the elastic energy of the approximated solution and analytical solution whereas the compliance is the inverse of the global stiffness of the structure. The error norm and compliance is calculated in both the methods and are compared. It is observed with the distinctive values of these parameters that which method is more reliable and accurate in getting results. Moreover the value of error norm varies with change in parameters like number of nodes, domain size and number of integration points. Compliance can be very helpful in understanding the nodal distribution and its largely focused to minimize the compliance for topology optimization.

E. *Predefined Parameters*

As the MATLAB code for the problem of cantilever beam has already been provided to us,[1] the aim as mentioned above was to observe the outcomes while changing the kernels. The cantilever beam has been made initially with the boundary conditions wherein the left edge has been fixed and a transverse load is applied on the right edge as shown in the figure 1 below.

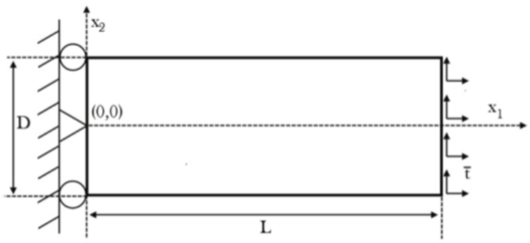

Fig. 1. A cantilever beam of length L and height D is subjected to traction force on right and fixed condition on left. *Cited in [1]*

1) Meshless methods EFG and MLPG: The Element Free Galerkin (EFG) method uses the shape functions of moving least square as their test function by employing a global weak formulation of the model. Due to lack of the Kronecker delta criteria, the displacement vector is expressed in terms of the virtual nodal displacement and hence it prevents the application of essential boundary conditions. Hence the boundary conditions in EFG method is enforced using the Lagrange multiplier. In order to calculate the integrals, Gauss quadrature scheme is used which can numerically find an approximation of the integral by assembling the weighted value at the integration points marked at every integration cell.

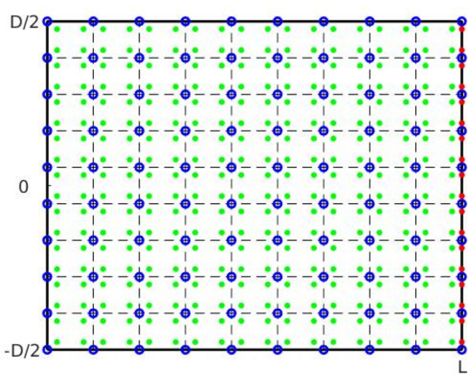

Fig. 2. The discretized domain and the boundary condition on a cantilever beam using EFG method. *Cited in [1]*

The Meshless Local Petrov-Galerkin(MLPG), which is based on a local weak form also works on the MLS approximation. In the case of MLPG mixed collocation method[17], no such background mesh is created and all the integration is done by nodal integration i.e. the equations are solved at the nodes. The integrals are present in local weak form and are evaluated on domains.

The one major difference between EFG and MLPG was derived during the nodal distribution in both the cases. Although both the cases are of mesh less methods, the EFG technique still uses a background mesh or integration cells where the integration of the equations is evaluated.[16] The integration cells consists of integration points and there are 4 defined integration points in every cell.

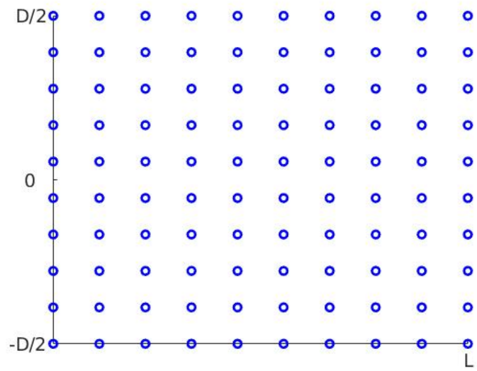

Fig. 3. The nodes created on a cantilever beam using MLPG method. *Cited in [1]*

2) Kernel Functions: The two methods employed to solve the problem were EFG and MLPG. In both the methods, the linear equations are discretized by MLS approximation approach. A MLS approximation has been defined which contains the kernel functions for the analysis. There are many kernel functions that can be employed to get accurate results. A cubic spline weight function has been used initially by the author[1] to compute the results.

$$W(xx^I, d) = W(r) = \alpha(\frac{2}{3}4r^2 + 4r^3), \text{if} 0 \leq r \leq \frac{1}{2} \qquad (1)$$

$$W(r) = \alpha(\frac{4}{3}4r + 4r^2\frac{4}{3}r^3), \text{if}\frac{1}{2} \leq r \leq 1 \qquad (2)$$

$$W(r) = \alpha(0)\text{otherwise} \qquad (3)$$

F. Changes in Kernels

Under this section, the kernel functions has been changed in the code in order to understand the analysis of the structure problem. The different types of kernel used during the computation were-

- **Radial Basis Function (RBF)** which has a value depending upon the distance from the origin or some particular point

$$K(x, x') = exp\frac{-||x - x'||^2}{2\sigma^2} \qquad (4)$$

- **Gaussian kernel** is a type of radial base function where

$$W(r) = \frac{1}{\sqrt{2\pi\sigma}}exp(\frac{-(x-\bar{x})^2}{2\sigma^2}) \qquad (5)$$

- **Laplace Radial Basis Function kernel** is a general purpose kernel which is used when there is no prior knowledge of the data.

$$W(r) = exp(\frac{-x-\bar{x}}{2\sigma}) \qquad (6)$$

G. *Results obtained*

The analysis provided the displacement and the stress on the cantilever beam due to traction force. The aim has also been to keep the error norm minimum which is the relative error between the elastic energy of the approximate and the analytical solution. Other than this, the compliance is also kept minimum. Initial results were obtained for both EFG and MLPG by using the user-defined spline function.

The results were obtained with various different kernels and for every condition, the error norms and the compliance were compared. Furthermore, the number of nodes has been changed in order to achieve minimal norm in all the cases. It is also been observed that with the increase in the number of nodes in the structural problem, the number of oscillations has reduced.

- **RBF Kernel** -

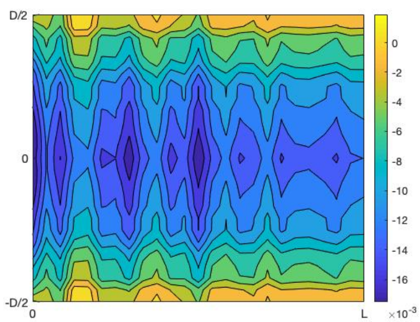

Fig. 4. Representation of shear stress on the beam using RBF kernel.

The error norm achieved for this particular case has been 7.18%.

- **Gaussian Kernel** -

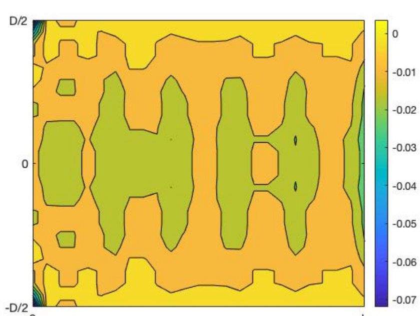

Fig. 5. The oscillations observed in the shear stress representation using Gaussian kernels.

In this case, the error norm has been achieved as 27.13 %, which is very high than compared to the RBF kernel.

- **Laplace Kernel** -

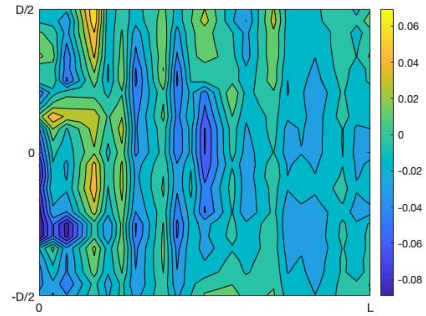

Fig. 6. The oscillations observed in the shear stress representation using Laplace kernels.

The error norm achieved at this kernel is as high as 41.69 % which means that out of all the proposed kernels, the RBF kernel produced the least error norm for this cantilever problem.

With the above formulation and computation, the importance of kernels and the way of implementing them into MATLAB codes has been learnt. Te usage of different kernels over a same set of problem also helped to understand the suitability of the kernel in various applications. A deeper look to the numerical analysis took place with the above work and the way to implement any structural problem, mostly elastic were well understood. With the conclusion of the previous work, the project had been headed towards a very different application for numerical analysis. There have been many interesting applications and morphing wings has been one of those.

H. *Learning Points*

The project was scoped to work on the mesh less methods on various applications like elastic body, thermal transfer,etc. With the equivalence to the previous work done on this topic, the project was set to move in the direction of mesh less analysis of morphing wings. Though with the project done during last semester, it was observed that meshless methods takes a lot of computational time and is very complex for complicated geometries. Even while bringing in the changes in kernel functions in Overvelde's research work, each kernel would take time to implement. The program itself took time to be understood by the user due to its complexity.

Hence with the end of the previous semester, the project that was aimed to work on the meshless analysis of the morphing wings underwent a slight change to produce results timely and the analysis scope was widened. Mesh based methods on the other were not complicated and it become easy to develop a tool for the numerical analysis approach. In this paper, a tool is being designed on MATLAB to solve geometrical elastic problems. A non linear elastic structure is used at the beginning to validate the tool. Hence to save time, this project has been transposed by widening our spectrum.

III. Work in 3rd Semester

During this semester, the entire work has been started off with the introduction of morphing wings and numerical analysis in general. The S3 project started with the study of the morphing wings and to observe the ways it was different from the conventional wings. The fact that the Wright brothers had pulled off their first flight by using wires and pulleys[10] that can bent and twist the wings. This system used during the time of early flights are not the same as of now which uses ailerons and flaps. But, currently in MIT and NASA, the research on ”morphing” had seen a boom and the modern aircraft research is headed towards the time where it begun. A new wing is designed in order to sustain fuel less consumption and simplification of the manufacturing process. The basic aim towards the development of the morphing is to improve the aerodynamics of the wing and also its agility.[11]

Fig. 7. A test version of the deformable wing designed by the MIT and NASA researchers is shown undergoing its twisting motions, which could replace the need for separate, hinged panels for controlling a plane's motion. (Kenneth Cheung/NASA) *Cited in [10]*

The research is on-going and different models are being tested to check upon the efficiency of the wings. The major problem being faced by the researchers in morphing was the deformation of the wing was carried out by mechanical structures within the wing that increased the weight of the structure and hence reduces the efficiency. The research is been carried out on a basic principle of using an array of small light weighted structural pieces which can be assembled into infinite variety of shapes. These pieces should be strong and stiff but the dimensions and the material will determine the flexibility of the final shape. It is studied that morphing wing will provide great efficiency and fuel reduction. The morphing wing not just provide aerodynamic advancements but also enhances the structural performances. [10]

The development of this tool takes place taking into account various functions that are often used during the analysis of elastic non-linear structure. As mentioned above also, the basic principle of the analysis will be the minimization of the potential energy of the system. The initial calculations are done in order to attain the various values of energy derivatives in order to store their values for further calculations.

The basic platform to gain substantial knowledge upon this topic came in from a paper based upon topology optimization of the morphing wing.[13] The main objective of this project was to design a feasible shape morphing wing, considering the feasibility of the entire structure. The idea of using non-linear elastic material has been originated while studying this paper. This paper not just undergo with finite element analysis but also the topology optimization. The author, along with the constraints on the displacement has also considered the constraint related to the maximum stress.

A feasibility constraint is joined in this paper by the author. The finite element analysis is treated as a minimization problem and the function here to minimize is the total energy of the system given by both internal and external forces along with the displacement constraints. The author has defined the objective function by deriving it from Principle of Virtual Works, as:

$$E_{tot} = \int \phi d\Omega - F_{ext} U_a \tag{7}$$

Also, in order to make the optimization more efficient in MATLAB, the author has provided the gradient and the hessian of the total energy. The same method has been deployed in the current project for the analysis. The author has the gradient as the residual R which is:

$$R = F_{int} - F_{ext} \tag{8}$$

where;

$$F_{int} = \int \frac{d\phi}{dq^T} d\Omega \tag{9}$$

where q will be the vector of the nodal displacement. The hessian is the tangent stiffness matrix.

The tool ’fmincon’ is studied and as per the author is considered to be the most suitable instrument to perform finite element analysis as it treats the problem as a constrained minimization problem. The tool contains the stable implementation of the Newton Raphson Method which makes the tool fast and reliable solver for such non-linear mechanical problems.

The author has considered the method proposed by Bhattacharyya[14]. The objective ans constraints functions depends upon density distribution and nodal displacement, an adjoint sensitivity analysis is performed by the researcher for numerical analysis where they have used Lagrangian L which is defined as:

$$L = f + \lambda^T R \tag{10}$$

where f is any function, objective and λ is the Lagrange multiplier and R is the residual force.

The density filtering has been adopted by the author for optimization by a method suggested by Bruns and Tortorelli;

$$\rho_j = \frac{\Sigma_i w_{ij} x_i}{\Sigma_i w_{ij}} \tag{11}$$

where w_{ij} is the weight associated with the i^{th} element within the prescribed neighbourhood of the j_{th} one whereas,

$$w_{ij} = max(0, R_{min} - r_{ji}) \tag{12}$$

Upon studying the research work in depth, the optimizer used for the topology is MMA as the number of design variables are large and this optimizer can accelerate convergence.

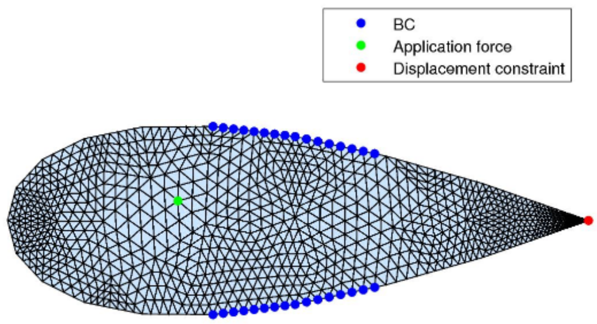

Fig. 8. The initial design and domain description of a morphing.(Cited in [13]).

In the figure above, the author has meshed the entire structure with the triangular mesh and the position of the boundary conditions has been allotted. The application force is placed at a particular point along with the displacement constrained applied at the trailing edge. This helped in the present project by signifying the boundary conditions that can be allocated in our research. Moreover, the idea of using triangular mesh for our research has been derived from the above figure in order to simplify the calculations.

The result in the paper has been very crucial for the study. The author has divided the structure into four sections.

- **Front sector**- This is the frontal section of the airfoil where the stress redistribution takes place.
- **Central Chain**- It acts like a transmitter of forces from the application point to the rear part.
- **Inferior Chain**- This acts as a link between the inferior boundary conditions and the central chain.
- **Rear Chain**- This part is considered only to transmit the displacement to the trailing edge. It doesn't carry any stresses.

These sections are helpful to the author in providing the idea of volume density variation throughout the structure. One of the most influential application that was felt while studying the sections was the positioning of the boundary condition ranges. The inferior chain plays a major role in distributing the stress along the lower part of the airfoil and hence the different range of boundary conditions in such circumstances while provide striking results.

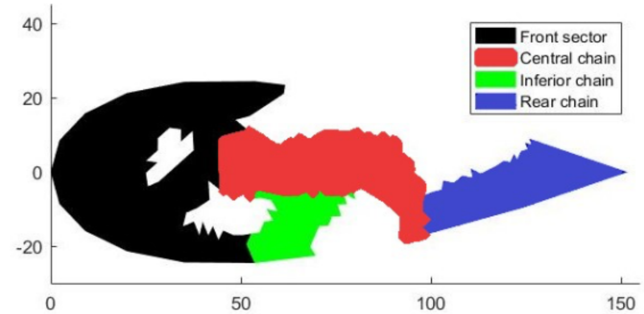

Fig. 9. Positioning of various section of the morphing airfoil.(Cited in [13]).

Influences of various parameters has been studied in this case. The major objective was to understand the changes occurring in the airfoil with the change in boundary conditions and the position of the force.

When the boundary conditions are considered, the analysis is run with various ranges. The BC's range has been set in between 30 to 70% of the chord and the analysis takes place. The effect can be noticed on the terminal chain and stress distribution in the airfoil. The volume fraction is also varying with different ranges. These changes in the boundary conditions are incurred in order to check the feasibility of the analysis to have a definite topology optimization. The case above validates the range from 30 to 65% of the chord which is been selected by the author as the reliable optimised structure.

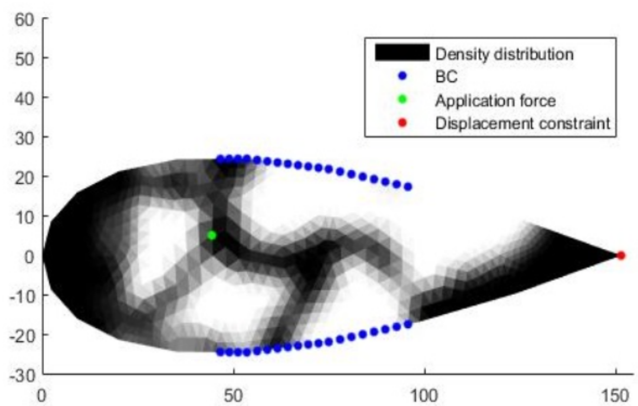

Fig. 10. Topology optimization for boundary condition ranging from 30 to 65% of the chord.(Cited in [13]).

Furthermore, certain variations has been done in the boundary condition range in order to validate and have a proper understanding of the selection. The present work would focus on reducing the volume fraction furthermore in order to have more accurate results of the topology. Upon changing the range to 35 to 65% of the chord, it is observed that the time of convergence has increased a lot and the favourable results are obtained after many iterations. In this case, the convergence takes place in 575 iterations. Also, the optimization achieved in this scenario was not reliable enough to be carried out for further analysis.

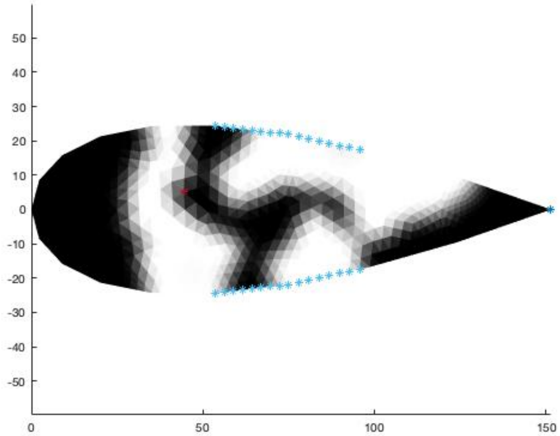

Fig. 11. Topology optimization for boundary condition ranging from 35 to 65% of the chord.

Similar run has been tried with 40 to 70% of the chord length. The iteration number exceeds than expected as even with more than 1000 iterations it didn't converge. This allowed not to take the boundary range from 40% as the analysis has been taking a lot of computational time and the convergence was no where close to be achieved.

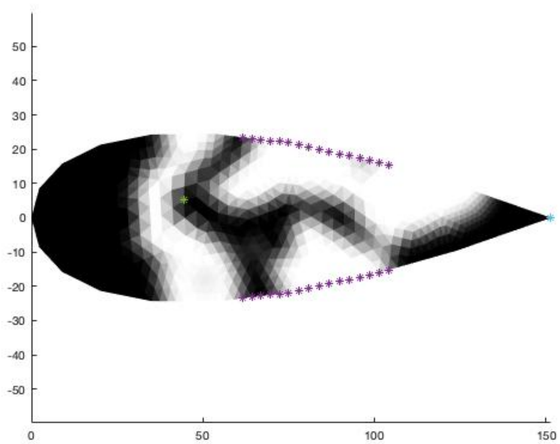

Fig. 12. Topology optimization for boundary condition ranging from 40 to 70% of the chord.

In the figure above as well, the topology optimization wasn't accurate. With all the values of boundary condition range, it was decided to go with the range of 30 to 65% in the present research to get accurate results. The displacement achieved are discussed in the result section.

The effects of the positioning of the external force has also been studied in this. The placement of the external force have a huge impact on the displacement constraint and if the point of force acting has been placed behind the left limit of the BC's, the actuators seems to be impossible to place. Similarly, the positioning of the external force point in the y-direction can bring a lot of changes. The application point if moved in the positive y-direction(15mm) in the author's case, the central chain will get directly linked to the upper BC's and the frontal portion will have a total distinct shape. The major disadvantage came when the displacement constraints are not achieved under this position.

IV. Problem Statement

The first step towards the formulation of a non linear structure is introduced. A linearization procedure is required, like Newton-Raphson method. The structural material is elastic and strain energy density W exist, which upon differentiating can lead to obtaining stress.

$$W(E) = \frac{1}{2} E : D : E \tag{13}$$

where the notation ":" is the contraction operator of tensors such that a:b = $a_{ij}b_{ij}$, with summation in repeated indices and D is the fourth-order constitutive tensor for isotropic materials where D is-

$$D_{ijkl} = \lambda \delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{14}$$

where λ and μ are Lame's constants for isotropic materials where

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \mu = \frac{E}{2(1+\nu)} \tag{15}$$

where E is the Young's modulus and ν is Poisson's ratio.

As the problem is set in 2D, the value of Lagrangian strain E is defined as -

$$E = \frac{1}{2}(\nabla_0 u + \nabla_0 u^T + \nabla_0 u \nabla_0 u^T) \tag{16}$$

The value of E in the matrix form is obtained as-

$$E = \begin{bmatrix} \frac{u_x^2}{2} + \frac{u_y^2}{2} + u_x & \frac{u_y+v_x+u_xv_x+u_yv_y}{2} \\ \frac{u_y+v_x+u_xv_x+u_yv_y}{2} & \frac{v_x^2}{2} + \frac{v_y^2}{2} + v_y \end{bmatrix}$$

The principle of minimizing the potential energy of the system depends upon minimization of the displacement field. Moving ahead with the calculation part, the 9 values of D are calculated based upon the values of indices.

Another relation is used for the calculation of second Piola-Kirchhoff stress S. The relation between the stress S and strain E is linear here.

$$S = \frac{\partial W(E)}{\partial E} \quad (17)$$

With the multiple values of S, the final calculation takes place for W. With this, the shape functions and their derivatives are calculated. All these values are added to the tool under different functions, potential and hessian in our case.

In the development of the tool, a beam structure is taken into consideration. The tool used here provide the user with various nodes and the connectivity matrix "t" to connect the various nodes to form the triangular mesh. We load the .mat file wherein the "p" is the matrix containing all the coordinates of the nodes.

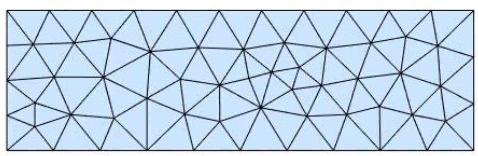

Fig. 13. The initial plot of the rectangular structure obtained by the tool

Then the coordinates of the three nodes of each element are extracted and we also define the area of each element as well. Furthermore, the various shape functions are calculated in both the directions. Upon considering the displacement in x and y direction as u and v respectively and q as the nodal displacement vector of a single element, we will be calculating the shape functions. Therefore,

$$Nux = \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \end{bmatrix}$$

$$Nvx = \begin{bmatrix} 0 & b_1 & 0 & b_2 & 0 & b_3 \end{bmatrix}$$

Similarly, the shape function for y direction is also considered as-

$$Nuy = \begin{bmatrix} c_1 & 0 & c_2 & 0 & c_3 & 0 \end{bmatrix}$$

$$Nvy = \begin{bmatrix} 0 & c_1 & 0 & c_2 & 0 & c_3 \end{bmatrix}$$

where in b and c are the expressions used for the computation of stiffness matrix for example-

$$b_1 = \frac{(y_2 - y_3)}{2Area} \quad (18)$$

$$c_1 = \frac{(x_3 - x_2)}{2Area} \quad (19)$$

Upon calculating the degree of freedoms, the boundary conditions are provided to the structure. There are two types of nodes, forced nodes and imposed nodes on the structure.

Upon giving the boundary conditions, the plot is being generated. Now moving towards the final analysis of the structure, the functions are being called to minimize the potential energy and displacement in order to obtain analytical results. There are two additional functions used for the purpose. The analysis runs with fmincon and fminunc functions respectively. Also, a finite difference check has also been generated in order to substantiate the gradient calculated.

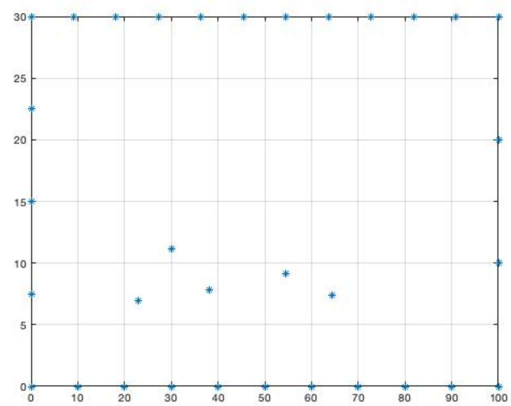

Fig. 14. The cross section with the forced and imposed nodes.

The fmincon is used to find the minimum of constrained nonlinear multi variable function. The syntax used in this coded is

x = fmincon (fun,x0,A,b,Aeq,beq,lb,ub,nonlcon,options)

which minimizes based upon the optimization options specified under **options**.

Similarly, fminunc is used to find minimum of unconstrained multi variable function. The synatx used is

x = fminunc(fun,x0,options)

The two functions "potential" and "hessian" are been entered to evaluate the minimum. Under the potential the major objective will be to calculate the energy of the structure. The potential function will be focusing majorly upon the calculation of the gradient along with the computation of energy with the help of internal and external forces applied on the structure.

$$E_T = \int \phi d\Omega - Fq \quad (20)$$

The main objective of the hessian function is used to get the stiffness matrix. The hessian of the total energy will give us the value of the tangent stiffness matrix as -

$$K_T = \frac{d^2E}{dqdq^T} \tag{21}$$

Once the analysis is performed with the help of the rectangular beam section, the morphing airfoil is taken into consideration. In order to design a morphing airfoil, the selection of NACA 0012 has been based upon its suitability. This is a very basic airfoil and because it has no camber, the lift to drag ratio will certainly be better for certain flight profiles.

Fig. 15. The initial structure input for the morphing analysis.

The structure has been highly meshed. The next step is to provide the boundary conditions that were suggested from the previous works on the morphing wings that were discussed above as 30 to 65% of the chord length. The boundary conditions are provided to the morphing structure and the figure is observed as -

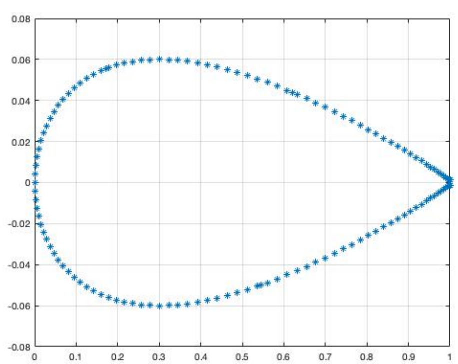

Fig. 16. The boundary conditions applied to the morphing structure.

Once the boundary conditions are set, the analysis are ready to be performed.

V. Results and Analysis

The results from the topology optimization[13] has also been studied in order to learn about the affects of boundary on the airfoil. The displacement achieved by changing the range to 35 to 65% of the chord has been observed as-

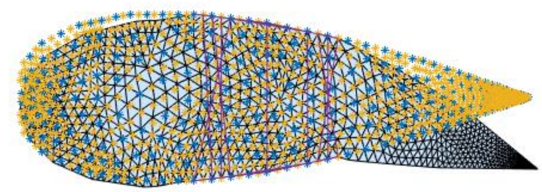

Fig. 17. The final displacement achieved for boundary condition ranging from 35 to 65% of the chord.

The first results are derived from the cantilever beam analysis. The value of external force is taken as -10000N. The functions used for the first case was **fmincon** and the results derived are :

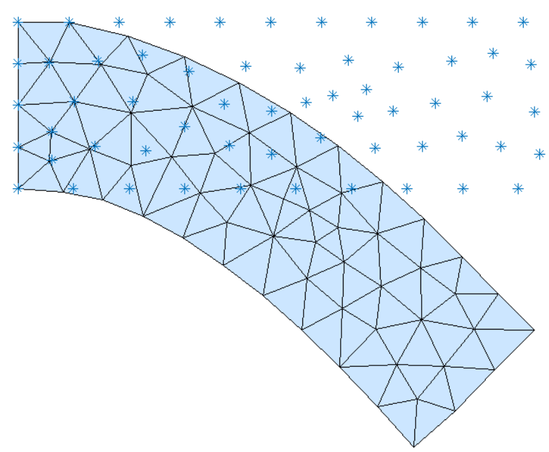

Fig. 18. The displaced beam after the external force has been applied.

The fmincon function provided a large displacement to the beam as shown in the figure and the error has also been cross checked via finite difference method. The error has been calculated for the gradient in order to validate the results obtained by the potential function. The error in this case is generated with respect to 10^{-4} which is less. The error is very low which depicts that the value of the gradient analysed by the tool is close to the analytical value.

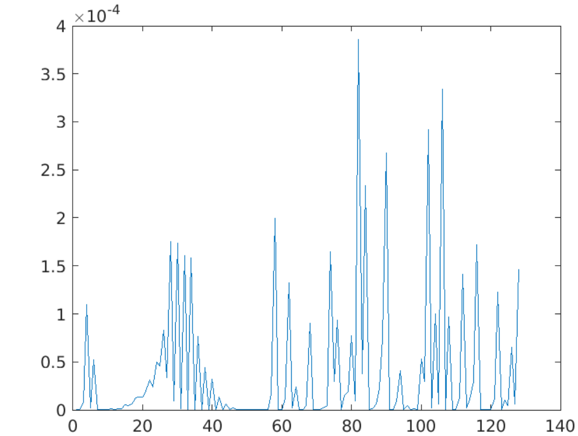

Fig. 19. The error in the gradient calculated for the beam by using finite difference.

The effect of the external force can be observed in the beam by mapping the convergence rate and observe the value of potential energy being minimised by the tool.

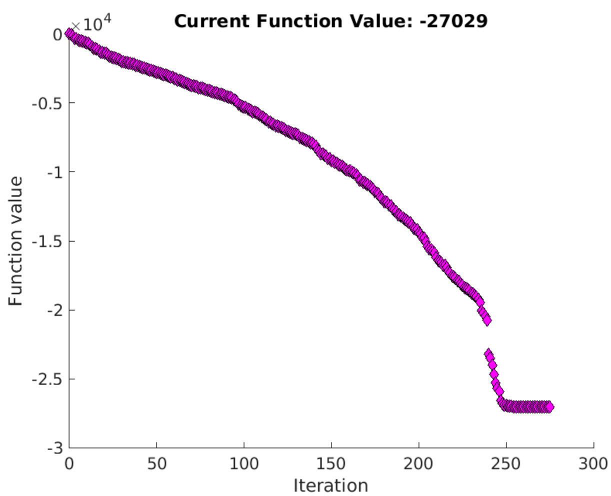

Fig. 20. The final value of the potential function as derived from the tool after convergence.

The final value of minimal potential is -27029. Upon this, the analysis has been run for the usage of **fminunc** function in order to compare the results from both the function.

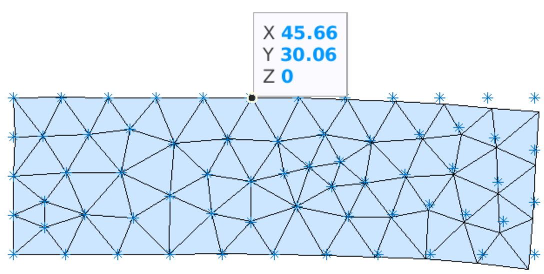

Fig. 21. The displaced beam after the external force has been applied.

Although in the case of fminunc, the error obtained in the gradient is very small than compared to the fmincon function but this cant further validate as which of the two shows more accurate results. In this case, the error has been in the terms of 10^{-6}.

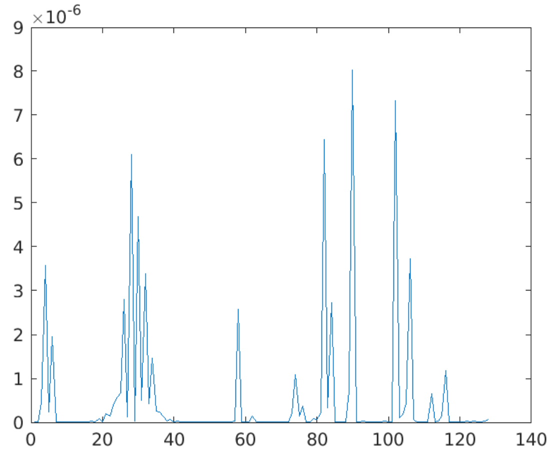

Fig. 22. The error in the gradient calculated for the beam by using finite difference.

The value achieved here are on the contrary value of external force as -1000N applied at the upper right edge of the beam. Hence the value of the function minimised came out to be -2608.05.

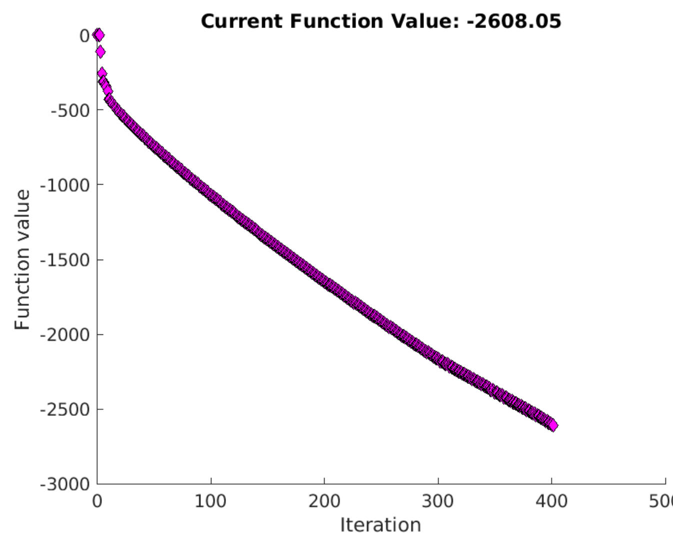

Fig. 23. The final value of the potential function as derived from the tool after convergence.

Once the tool is observed to be working for the analysis and iterations are been run accurately for the convergence, the morphing wing is now used for the analysis in the tool. The fmincon function is used. During this case, the value of external force has been kept high as to ensure proper displacement in the morphing structure. The boundary conditions has been provided as Drichlet to the upper and lower layer as mentioned before.

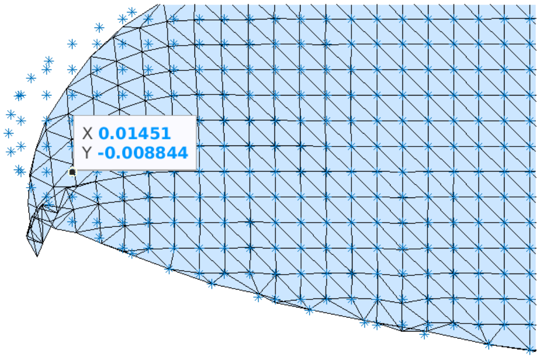

Fig. 24. The displaced airfoil after the external force has been applied.

Also, the potential of the system converges after sometime. It has been observed that though the number of iterations has been less than 200 but the computational time has been very large.

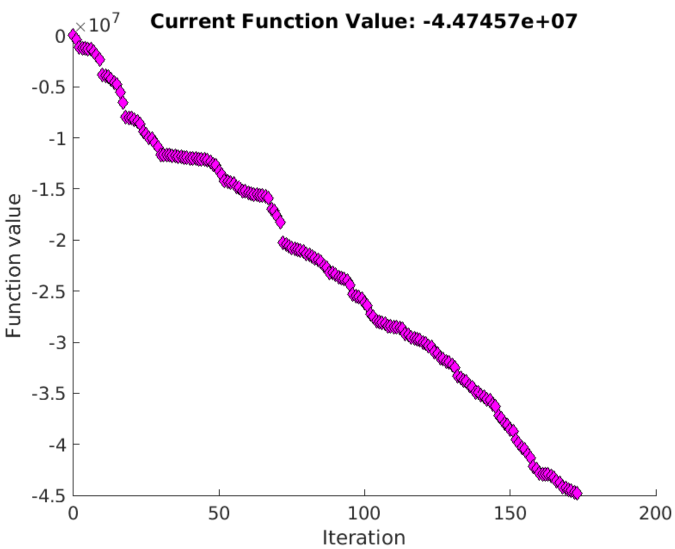

Fig. 25. The final value of the potential function as derived from the tool after convergence.

The minimum potential derived from this is -4.47457e+7.

VI. Conclusion

The tool has been designed in order to solve non-linear elastic problems. During the development of the tool, many aspects of a non linear functions came into light. The use of functions like fmincon and fminunc were well versed and provide a deep insight in understanding the method. The development of this tool also allowed me to understand the numerical analysis using MATLAB and cleared many doubts in the same. Though the results obtained are not validated but the finite difference method implied the errors to be very small which suggests that the potential function has been generating near to accurate results.

The morphing wing has been well studied during this entire period. Though the results for the morphing wings were not as expected but the analysis proved to be favouring fmincon function for the calculation. Fmincon had provided results that can be validated. The major issue had been with the potential function as there are many variables in it and the magnitude of the variables seems to be very high that not just take a lot of computational time but also increases the complexity of the problem. The morphing wing had been analysed multiple times for this and the value of external force played a major role in carrying out the analysis.

The force nodes applied on the morphing wing provide different results. The point of force selected for the wing was the last point at the trailing edge. Even the boundary conditions played an important role in analysing the structure. Furthermore the research project can be carried out for different aspect of numerical analysis. The future work can be included as by changing the point force and the position of the force anywhere on the morphing structure. Also, the topology optimization of the morphing can be carried out in continuation to this code. I propose to change the NACA airfoil in order to attain better results as the used profile may or may not be well suited for topology optimization.

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