Kane-Mele model

In 2005, Kane and Mele [PRL 95, 226801 (2005)] generalized the Haldane model to the graphene lattice model with the time reversal invariant spin-orbit coupling, which is known as the Kane-Mele model.

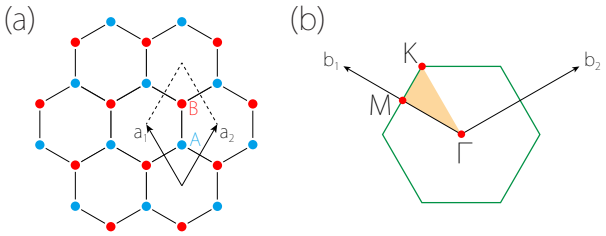

Figure 1. (a) Honeycomb structure. (b) Hexagonal Brillouin zone.

The graphene lattice has the following translation vectors:

$$\boldsymbol{a}_1 = a(-\frac{\sqrt{3}}{2},\ \frac{3}{2}), \qquad \boldsymbol{a}_2 = a(\frac{\sqrt{3}}{2},\ \frac{3}{2})$$

where a is the bond length. The fractional coordinates of A and B sites are written as

$$\boldsymbol{r}_A = (\frac{1}{3},\ \frac{1}{3}), \qquad \boldsymbol{r}_B = (\frac{2}{3},\ \frac{2}{3})$$

The Kane-Mele model is expressed as,

$$\mathcal{H} = t\sum_{\langle i,j\rangle} c_i^\dagger c_j + i\lambda_{SO}\sum_{\langle\langle i,j\rangle\rangle} v_{ij}c_i^\dagger s_z c_j + i\lambda_R\sum_{\langle i,j\rangle} c_i^\dagger(\boldsymbol{s}\times\boldsymbol{d}_{ij})_z c_j + \lambda_v\sum_i \xi_i c_i^\dagger c_i,$$

where $c_i^\dagger = (c_{i,\uparrow}^\dagger, c_{i,\downarrow}^\dagger)$ and the Pauli matrices s_i describe the electron spin. The first term is the nearest neighbor hopping term on a graphene lattice. The second term is a mirror symmetric spin-orbit interaction which connects second neighbors with a spin dependent amplitude. Here, $v_{ij} = \frac{2}{\sqrt{3}}(\boldsymbol{d}_i \times \boldsymbol{d}_j)_z = \pm 1$, where $\boldsymbol{d}_i$ and $\boldsymbol{d}_j$ are two unit vectors along the two bonds the electron traverses going from site j to i. The third term is the nearest neighbor Rashba term, which violates the M_z mirror symmetry, breaks the conservation of s_z and couples the electrons with spin-up and spin-down. The last term is a staggered sublattice potential with $\xi_i = \pm 1$.

A magnetic field $\boldsymbol{B}$ breaks time-reversal symmetry, which allows the terms in the Hamiltonian as

$$H_B = \sum_i c_i^\dagger(\boldsymbol{B}\cdot\boldsymbol{s})c_i$$