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Research Track

On MRI Denoising through Enhanced Complex Diffusion

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Abstract

A magnetic resonance image denoising technique is proposed in this article. The technique is based on an enhanced complex diffusion process and the application of the two-dimensional Gabor filter. The enhancement is made through the introduction of an enhancement operator. Best possible denoising is achieved through the selection of the best diffusion coefficient. Results from an experiment support the validity and applicability of the proposed technique.

Keywords *MRI; Image denoising; Rician noise*

1. Introduction

Magnetic Resonance Imaging (MRI) is a medical technique that uses a magnetic field and computer-generated radio waves to create detailed images of organs and tissues in the body. It is a non-invasive way used by the medical community to produce images of internal organs of the body without the need for surgical procedures. These images are used to display pathological and physiological variations experienced by living tissues. It is a very desirable imaging technique which is considered harmless and produces accurate results. The produced images are used for medical analysis and diagnostics; thus, it is very important that such images be noise-free [1].

Quadrature displaced coils are part of the structure of an MRI system. They form a quadrature detector subsystem used to produce images. It has already been demonstrated that such coil physical arrangement can improve the image's signal-to-noise ratio (SNR) by a factor of $\sqrt{2}$ with respect to a non-quadrature coil structure of similar dimensions. The signals that are measured with the quadrature detector are essentially two or three-dimensional arrays of complex numbers. Consequently, a pair of images are produced representing real and imaginary parts of a complex image [2, 3]. The magnitude of the complex image is used for clinical analysis; however, the image is inevitably corrupted by Rician noise [4]. It is vital to remove, or at least as much as possible reduce this noise from MRI images. Many processes have been used to remove this noise [5-9].

For better diagnoses and medical analyses, the structural details of MRI images must be preserved while the images are being denoised. Edges' shapes and locations in these images must be conserved, and no spurious details should be generated when denoising is performed. In this article, a denoising technique based on these criteria is proposed. The technique is founded on an enhanced complex diffusion process and the application of the two-dimensional Gabor filter. The enhancement is made through the introduction of an enhancement operator. The coefficient of the diffusion process is a function of an angle, θ, and a threshold, κ. It is a monotonically decreasing positive weight function. Best possible denoising is achieved through the selection of best θ value, θ_{Best}, at a given κ. This value is produced through the application of the two-dimensional Gabor filter and image entropy evaluation.

The performance of the proposed technique is evaluated through an experiment where three metrics are used to support its validity. The metrics are the structural similarity index measure (SSIM), the peak signal-to-noise ratio (PSNR), and the normalized absolute error (NAE). The proposed technique is given in section 2, and its performance evaluation is presented in section 3.

2. MRI Denoising Technique

As shown in Figure 1, the proposed technique is based on the use of a complex diffusion process within which an enhancement operator is embedded, the application of the two-dimensional Gabor filter, and image entropy evaluation. Description of these parts of the technique are presented in the following subsections.

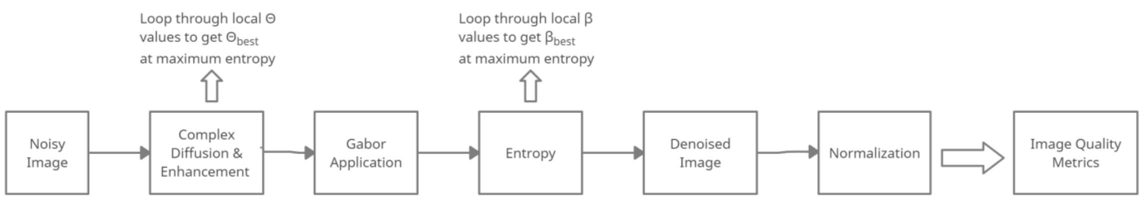


Figure 1: *The Proposed MRI Denoising Technique Block Diagram*

2.1. Modified Complex Diffusion

The complex diffusion process is presented mathematically as:

$$I = \nabla \cdot \left[\, c\big(Im(I)\big)\nabla I \,\right] \tag{1}$$

Where (I) *is* the treated image, ($\nabla\cdot$) represents divergence, (∇) represents gradient, and $Im(I)$ is the imaginary part of I. The complex diffusion coefficient, c, is given as:

$$c\big(Im(I)\big) = \frac{e^{j\theta}}{1 + \left(\frac{\tau\{Im(I)\}}{k\theta}\right)^2} \tag{2}$$

k is the threshold, $\theta$is an angular value, and τ is the enhancement operator. This diffusion process is a modification of that given in [10]

2.2. Enhancement Operator

The contrast of an image is improved using a two-dimensional nonlinear filter developed in [11]. The formulation of this filter is based on the generalization of an algorithm developed in [12] which is used to calculate the energy of signals employing Teager's algorithm presented in [13]. The filter proposed in [11] works like a local-mean weighted band pass filter; it enhances the contrast of an image by combining the image with its filtered version.

Application of Newton's law of motion equation to the motion of a mass is described with a second-order ordinary differential equation, with the discrete solution for an oscillatory motion given as: $x(n) = A\cos[\Omega n + \psi]$. In this solution, A, is the amplitude, Ω, is the discrete sampling frequency, and ψ is an arbitrary initial phase. It is shown in [12] and as seen from equation (3) through equation (5) that the energy, E, associated with this oscillation is proportional to A and Ω as: $E \propto A^2\Omega^2$.

$$E = \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 \tag{3}$$

Noting that $\Omega^2 = k/m$ and substituting for $x(n)$ in equation (3), we obtain

$$E = \frac{1}{2}m\Omega^2 A^2 \tag{4}$$

or

$$E \propto A^2\Omega^2 \tag{5}$$

To characterize, $A, \Omega, and\ \psi$, three equations are used:

$$x(n-1) = A\,cos[\Omega(n-1) + \varphi] \tag{6}$$

$$x(n) = A\,cos[\Omega(n) + \varphi] \tag{7}$$

$$x(n+1) = A\,cos[\Omega(n+1) + \varphi] \tag{8}$$

These equations can be manipulated to produce,

$$A^2 sin^2(\Omega) = x(n)^2 - x(n+1)\cdot x(n-1) \tag{9}$$

Small values of Ω result in a formula for the corresponding oscillation as

$$E = A^2\Omega^2 \approx x(n)^2 - x(n+1)\cdot x(n-1) \tag{10}$$

Subsequently, a nonlinear filter is obtained by applying the filtering of equation (10) along both the vertical and horizontal directions resulting in a 2-D version given by:

$$y[p,q] = 2x^2[p,q] - x[p-1,q]\cdot x[p+1,q] - x[p,q-1]\cdot x[p,q+1] \tag{11}$$

The produced enhancement operator in equation (11), τ, is adopted here where it is applied in equation (2).

2.3. Gabor Filter

The Gabor filter is used in image processing applications for edge detection, texture analysis, and feature extraction. The filters have been shown to possess optimal localization properties in both the spatial and the frequency domain. The two-dimensional Gabor filter can be viewed as a sinusoidal signal of frequency and orientation, modulated by a Gaussian wave, and it is given as follows [1]:

$$G_{f,\sigma1,\sigma2}(x,y,\beta,\psi) = \exp\left[-0.5\left(\frac{u^2}{\sigma1^2} + \frac{v^2}{\sigma2^2}\right)\right] cos(2\pi f u + \psi) \tag{12}$$

$$u = x\,sin(\beta) + y\,cos(\beta)$$
$$v = x\,cos(\beta) + y\,sin(\beta)$$

This filter is used here to produce the best θ value, θ_{Best}. To produce this value, the following values are substituted in equation (12): $\psi = \pi/2$, $\beta \in [0, 2\pi]$, $f = 1/4$, and σ1 = σ1 = σ = 4

2.4. Entropy Calculation

Entropy is a statistical measure of randomness that is used to characterize the texture of an image. The highest entropy value is used here to identify θ_{Best}, and β_{Best} values. Given that P contains the image's normalized histogram counts; image entropy, $H(X)$, where X is a discrete random variable is calculated here using Shannon's formula [1]:

$$H(X) = -\sum_{i=1}^{n}\left(P(x_i)log_2P(x_i)\right) \tag{13}$$

2.5. Denoising

The noise reduction starts with the application of the complex diffusion process given by equations (1) and (2) to a given Rician noise corrupted MRI image. The application is iterated for a selected number of iterations at a given threshold value, κ. The result from this application is a complex image. The real or active component and the imaginary or reactive component of the produced complex are squared and added, the result is square rooted, and a corresponding magnitude image (MI) is produced. This denoising procedure is made best possible by using θ_{Best} and β_{Best} values. The method through which these values are generated is given in the section below.

2.6 θ_{Best} and β_{Best} Values Generation Method

The values of θ_{Best} and β_{Best} are generated through the application of Gabor filter and image entropy computation. The value of θ in equation (2) is incremented one degree at a time, and the corresponding diffused image is generated at every increment. For every value of θ, Gabor filter is applied to the corresponding diffused image, and the value of β in equation (12) is incremented one degree at a time for a total of 360 degrees. The value of the image entropy at every θ and β increments is calculated. Once the sweep through all θ and β values is complete, the global maximum entropy value is identified. The values of θ and β at this entropy level are recognized as θ_{Best} and β_{Best}.

2.7 Used Metrics [14]

Selecting the noise-free image as being the reference image, SSIM, PSNR, and NAE metrics are applied here to validate the proposed technique. Brief descriptions of these metrics are given below.

- **Structural Similarity (SSIM)**: This is an index that is used to measure the similarity between two images, x and y. It is given as:

$$SSIM = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \tag{14}$$

  where c_1 and c_2 are constants, μ_x is the average of x, μ_y is the average of y, ${\sigma_x}^2$ is the variance of x, ${\sigma_y}^2$ is the variance of y, and σ_{xy} is the covariance of x and y,
- **Normalized Absolute Error (NAE)**: The value of the normalized absolute error between two images $f(x,y)$ and $f'(x,y)$ is:

$$NAE = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}[f(i,j) - f'(i,j)]}{\sum_{i=1}^{M}\sum_{j=1}^{N}[f(i,j)]^2} \tag{15}$$

  where M and N are the number of rows and columns in the input images.
- **Peak Signal to Noise Ratio (PSNR)**: The PSNR is a function of the mean squared error, MSE, it is calculated as:

$$PSNR = 10\log\left(\frac{(2^n - 1)^2}{MSE}\right) \tag{16}$$

3. Performance Evaluation

Due to quadrature coils displacement in the MRI system, two images are generated; one is identified as the real part and the other as the imaginary part. A corresponding magnitude image, MI, is obtained by individually squaring the real and imaginary parts, adding them and then, taking the square root of the result. See Table 1.

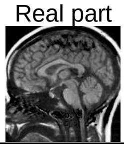
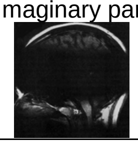
Real part	Imaginary part	Magnitude image
		

Table 1: MRI Real, Imaginary and Magnitude Images

To obtain a Rician noise corrupted MRI image, additive white Gaussian noise is added to the real and imaginary components at the desired signal-to-noise, SNR, level before finding the corresponding Rician noise corrupted MI which is found by adding the squares of these images and square rooting the result. An experiment was conducted to evaluate the performance of the proposed technique. In this experiment, three SNR levels were investigated, namely $\mathrm{SNR} = 7, 10, \text{and } 15$. At each level, the proposed technique was applied with a complex diffusion of five iterations. Results from the experiment are shown in Table 2.

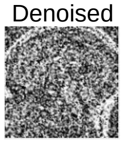
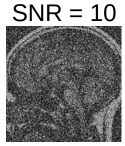
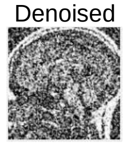
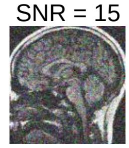
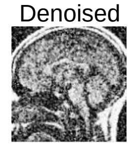
SNR = 7	Denoised	SNR = 10	Denoised	SNR = 15	Denoised
					
SSIM= 0.101	SSIM= 0.975	SSIM= 0.18	SSIM= 0.97	SSIM=0.37	SSIM= 0.98
PSNR= 7.91	PSNR= 51.92	PSNR= 10.4	PSNR= 52.4	PSNR=15.23	PSNR= 53.1
NAE = 0.93	NAE = 2.04	NAE= 0.68	NAE = 1.83	NAE=0.38	NAE = 1.58
θ°_{Best} N/A	θ°_{Best}= 345	θ°_{Best} N/A	θ°_{Best}= 109	θ°_{Best} N/A	θ°_{Best}= 323

Table 2: Experiment Results

4. Conclusion

A technique based on complex diffusion and nonlinear enhancement is proposed in this article for denoising MRI images. Results from a performed experiment demonstrate the applicability of the technique and support its validity. The technique is effective and machine adaptable.

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