Reactor Design Project

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1 Introduction

The intended product nuclei of this reactor design is Europium-151. Discovered in 1901 by Eugne-Anatole Demaray, it is part of the Lathanides group on the periodic table, and appears as a soft, silvery metal that blemishes easily and reacts with water at room temperature. With data from the British Geological Survey, the Royal Society of Chemistry has rated the "relative supply risk" of Europium as a 9.5 out of 10, based on factors like reserve distribution, production concentration, and political stability scores[1]. Like other rare-earth elements, Europium may soon be in critical supply in the future, and while creating it via nuclear fission may not be remotely efficient, it will be the focus of this reactor design. Commercially, Europium has been used for fluorescent lighting, preventing forgery of banknotes, developing control rods for nuclear reactors (due to its large absorption cross section), alloy production, and as a dopant for laser materials. Its phosphorescent quality is what is exploited for most applications in industry. Because of its rarity, 1 gram of 99.999% pure Europium can cost about $120 USD[2]. More important than its rarity or its price however, is the fact that it was named after Europe, and I couldn't find an element that was more relevant to studying abroad in Rome.

2 Core Composition

A **homogeneous 3000 MW(t) sodium-cooled fast reactor** was considered for the production of Europium by the fission of ^{238}U from PuO_2. At the high speeds the neutrons would be generated, it was assumed that the sodium coolant and structure would be invisible to neutrons, and the scalar flux ϕ would only be a function of energy. The multiplication factor k_∞ can then be written as

$$k_\infty = \frac{\int_0^\infty \nu\Sigma_f(E)\phi(E)dE}{\int_0^\infty \Sigma_a(E)\phi(E)dE},$$

[1] From the same source, Russia and China are both the world's largest producers and reserve holders of the element

[2] Price estimates were obtained from online distributors, namely Alibaba and Sigma-Aldrich

where ν is the neutron generation per fission and Σ is the macroscopic cross-sections of absorption and fission reactions. By defining the energy averaged cross-sections of ϕ and Σ, as well as the fission in the fertile material of the reactor, k_∞ can be rewritten as

$$k_\infty = \frac{\widetilde{e}\nu\bar{\sigma}_f^{fi} + (1-\widetilde{e})\nu\bar{\sigma}_f^{fe}}{\widetilde{e}\bar{\sigma}_f^{fi} + (1-\widetilde{e})\bar{\sigma}_f^{fe} + \frac{V_c N_c}{V_f N_f}\bar{\sigma}_a^c + \frac{V_{st} N_{st}}{V_f N_f}\bar{\sigma}_a^{st}},$$

where $\widetilde{e}$ is the enrichment of the core, the bars represent the energy averaged cross sections of the respective quantities, V represents the volume, N represents the number density, fe denotes the fertile material, fi denotes the fissile material, c denotes the coolant, and st denotes the structural material.

A reactor with a volumetric composition of 40% fuel, 50% coolant, and 10% iron structural material was considered. The fuel was taken to be UO_2, 20% enriched with PuO_2. Values for the relevant materials are given below in Table 1[3].

Table 1: Microscopic cross sections and densities

	σ_f b	σ_a b	σ_t b	ρ g/cm^3
PuO_2	1.95	2.40	8.6	11.0
UO_2	0.05	0.404	8.2	11.0
Na	-	0.0018	3.7	0.97
Fe	-	0.0087	3.6	7.87

The values for ν in the fast spectrum were assumed to be 2.98 and 2.47 for plutonium and uranium respectively[4]. With these values, and the molecular weights of the four molecules, the equation above was evaluated to obtain a value of 1.5136 for k_∞.

Now considering a critical reactor with $k = 1$, the probability of non-leakage P_{NL} can be found, as

$$k = k_\infty P_{NL},$$

which, in this case, is equal to 0.661.

3 Core Dimensions

To close the neutron balance equation, Fick's Law was used to approximate neutron flow as a fluid, and is said to be valid when the material scatters more neutrons than it absorbs, the regions are large relative to the average distance

[3]Taken from *Fundamentals of Nuclear Reactor Physics*, by Elmer E. Lewis
[4]Also from Lewis, as well as with data from the World Nuclear Association

neutrons travel between adsorptions, and the reactions are far from strong localized sources. For the uranium target nuclei, its absorption cross-section is relatively small at high energies, it is assumed that the reactor will be relatively large compared to the scale of neutron absorption distances, and no localized sources are present in the reactor. Vacuum boundaries are also assumed for the reactor core for simplicity's sake.

With this approximation, P_{NL} can also be expressed as

$$P_{NL} = \frac{1}{1 + L^2 B_g^2}, \quad L = \sqrt{\frac{D}{\Sigma_a}}, \quad D = \frac{1}{3\Sigma_t},$$

where L is the diffusion length, D is the diffusion coefficient, Σ_t is the total cross-section, and B_g is the geometric buckling value of the reactor. The total macroscopic cross-section and total macroscopic absorption cross-section can be found by summing up the individual number densities multiplied by the microscopic cross-sections found in Table 1. This gives that $\Sigma_t = 0.8068$ cm^{-1} and $\Sigma_a = 0.06826$ cm^{-1}. From this, L and D can be evaluated, and used in the equation for P_{NL} to solve for the value of $B_g^2 = 0.08473$.

Assuming we have a finite cylindrical reactor of height 5 m, the geometric buckling is given as

$$B_g^2 = (\frac{2.405}{R_{ex})})^2 + (\frac{\pi}{H_{ex}})^2,$$

where $R_{ex} = R + 2D$, and $H_{ex} = H + 4D$. With a calculated B_g and a set height, the equation was solved for R, and the critical radius was found to be 7.44 cm.

4 Reactor Power

With the dimensions of the reactor determined, the scalar flux profile of a cylindrical reactor can be given as

$$\phi(r) = CJ_0(\frac{2.405r}{R_{ex}}) \cos(\frac{\pi z}{H_{ex}}),$$

where C is a coefficient based on the power of the reactor, r is the radial distance from the center of the reactor, and z is the vertical position of the reactor. As before, Σ_f can be found by multiplying the number density of the fuel components with their respective microscopic cross-sections for fission, and $\bar{\Sigma_f}$ was found to equal 0.048 cm^{-1}. Taking γ, the energy released per fission to be $3.1 * 10^{-11}$ J/fission, and the power P as 3000 MW, the constant C can be found as

$$C = \frac{3.63P}{\gamma \bar{\Sigma_f} V}$$

for a cylindrical reactor, where V is the volume of the reactor, given as $V = \pi R_{ex}^2 H_{ex}$. Using the obtained values, C was found to be $4.316 * 10^{14}$. Now with the fully defined flux equation, the average flux over the core can be found as

$$\bar{\phi} = \frac{1}{V}\int_{-H/2}^{H/2} dz \int_0^R dr(2C\pi r J_0(\frac{2.405r}{R_{ex}})\cos(\frac{\pi z}{H_{ex}})).$$

This can easily be worked out on the back of a napkin[5], to find that $\bar{\phi} = 2.9253 * 10^{13}$ neutrons cm^{-2} s^{-1}.

The maximum fission power production rate is found at the maximum value of ϕ, where both the Bessel function and cosine function are equal to unity. From this, it can be determined that the **maximum power density**, $P''' = C\gamma\Sigma_f = 642.22$ MW/m^3.

5 Europium Production

With $\bar{\phi}$ determined, the production rate of europium nuclides from fission can be found. The differential equation can be given as

$$\frac{dE}{dt} = \gamma_{fp}\bar{\Sigma_f}\bar{\phi} - \sigma_a\bar{\phi}E(t),$$

where γ_{fp} is the fraction of fissions that result in the production of europium, $\bar{\Sigma_f} = 0.00991$ is the averaged macroscopic fission cross-section for the target nuclei, ^{238}U, and $E(t)$ is the number of europium molecules per unit volume at time t. At high energies (>10 MeV), the average microscopic absorption cross-section appears to be around 0.0025 barns[6]. The fraction of fissions for europium is 0.0081[7]. Europium is produced by fission from the uranium core and lost from absorption at a rate proportional to its accumulation. The full solution can be solved for this first order ODE with the initial condition that $E(t = 0) = 0$, and is given below as

$$E(t) = 3.21084 * 10^{22} - 3.21084 * 10^{22} e^{-7.31325 * 10^{-14} t}.$$

From this equation, the steady state solution can easily be seen to equal $3.21 * 10^{22}$ nuclides/m^3 as t approaches infinity, which is approximately 8.0474 grams of europium produced. At some point, the absorption from the accumulated europium will match the generation of it from fission, and no more europium will be accumulated. To estimate the time it could take to yield a kilogram of europium, it will be assumed that the absorption cross section of 0.0025 barns

[5] Or with Wolfram Mathematica 11.3.
[6] From the EXFOR library of experimental nuclear reaction data
[7] From the IAEA's "Live Chart of Nuclides"

is negligible, or that any accumulated europium is instantly removed from the system. In either case, the resulting differential equation becomes

$$\frac{dE}{dt} = \gamma_{fp}\bar{\Sigma_f}\bar{\phi},$$

and the solution is simply linear with respect to time, given as

$$E(t) = 2.34817 * 10^9 t.$$

With this assumption, it can be seen that the production rate of europium is around 2.35*10^9 molecules/m^3 every second, which, after converting to mass and multiplying by the volume of the reactor, is **roughly** $5.89 * 10^{-13}$ **grams of europium produced/s**. With this in mind, it would take **roughly 54,000 years to produce 1 kilogram of europium.**

6 Economics of Alchemy

As mentioned before, europium is quite valuable due to its rarity. Extrapolating from how much a gram was worth as mentioned earlier, a kilogram of europium could theoretically be **worth around $120,000 USD**. This would be, of course, after 54,000 years of non-stop production. Since $^{238]}$U is the target nuclei, it can be estimated that $1/\gamma_{fp}$ atoms of Uranium would be needed to generate one atom of europium every second. Assuming that the target nuclei is only lost through fission, the number of uranium molecules needed every second would simply be the production rate of europium multiplied by $1/\gamma_{fp}$, which is 2.90*10^{11} atoms/m^3 per second, or, after converting, 1.30*10^{-10} grams per second. Uranium oxide is estimated to cost around $1.4 USD[8] per gram. Multiplying this cost by the needed mass of uranium per second, as well as the number of seconds to produce a kilogram of europium, yields the total cost to be **$310 million dollars in USD**. Of course, this assumes that the sole cost of running the reactor is the supply of uranium, and that the only revenue comes from europium generation, neither of which is true.

7 Conclusions

Clearly, this reactor design is far from an efficient way of generating either europium or revenue[9]. If one was truly set on creating europium however, there are several avenues to explore. A source generating an element that decays into europium could be added to the core, the power, reactivity, and flux of the reactor could be increased, or the enrichment of the reactor could also be adjusted to obtain higher levels of europium production. Because of europium's minuscule fission fraction from uranium however, it is unlikely that this venture would ever turn a profit. While a potentially useful by-product of a nuclear reactor, trying to create europium through fission seems inadvisable.

[8] Data from the World Nuclear Association

[9] That is, assuming all the calculations thus far have been correct