Gradient

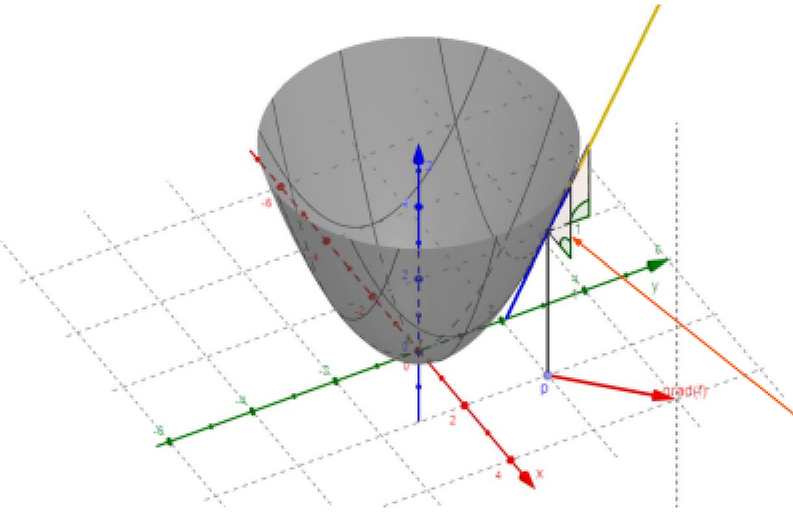
Gradient

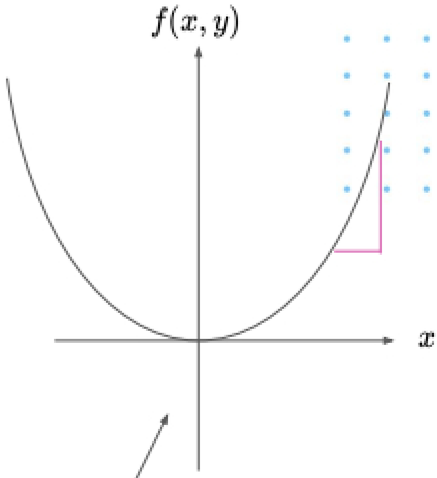
PROTON X

PROTONX

Đạo hàm riêng

Partial Derivative





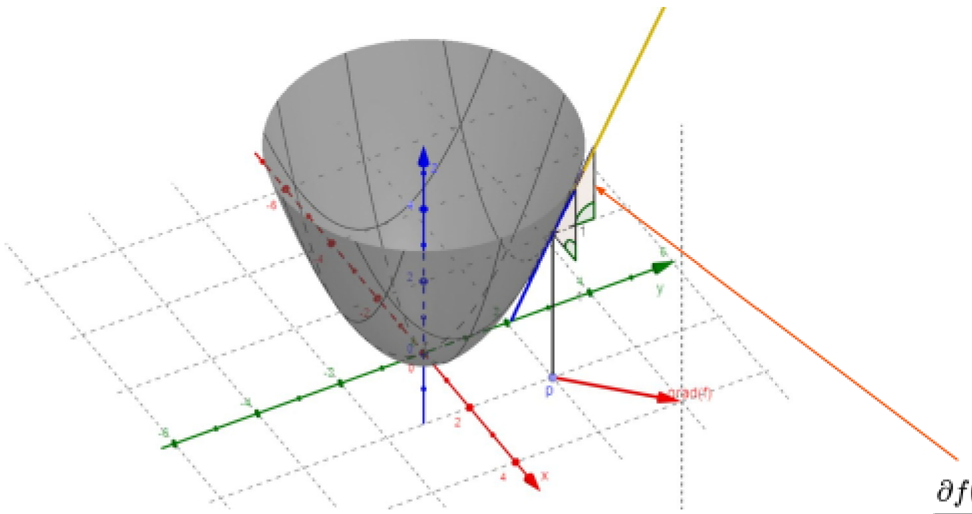
$$\frac{\partial f(x,y)}{\partial x} = x$$

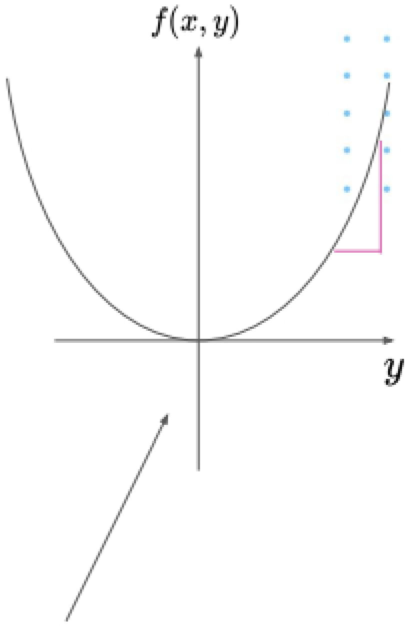
$$f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2$$

Sự **ảnh hưởng tức thì** của x đến f khi **y giữ nguyên**

Đạo hàm riêng

Partial Derivative





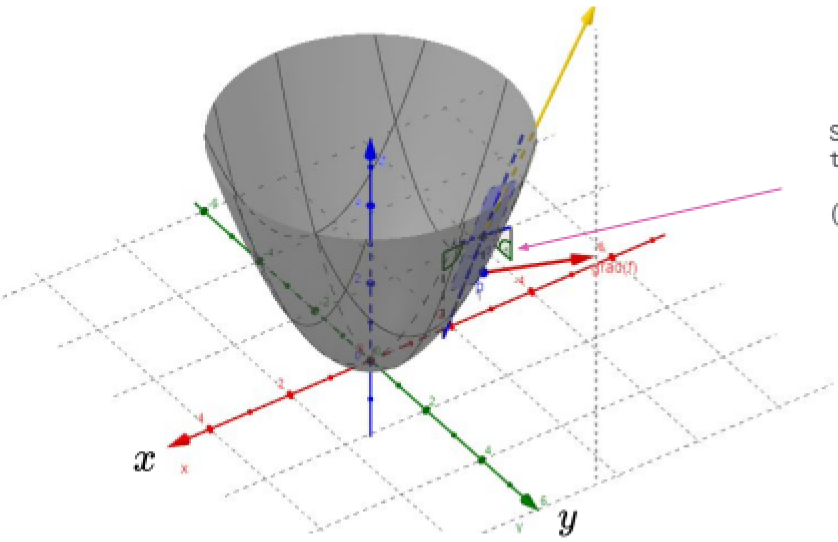
$$\frac{\partial f(x,y)}{\partial y} = y$$

Sự **ảnh hưởng tức thì** của x đến f khi **y giữ nguyên**

$$f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2$$

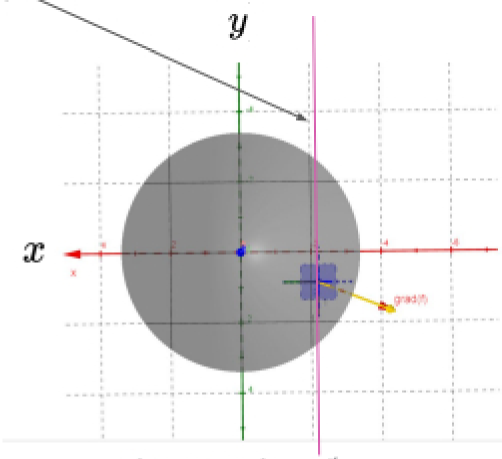
Đạo hàm riêng

Partial Derivative



Sự thay đổi của f khi y thay đổi tức thì.

(**Giữ x không đổi**)

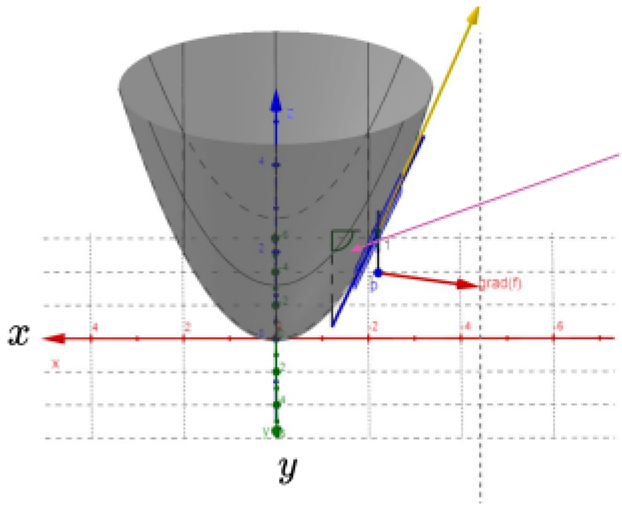


Nhìn từ trên xuống

$$f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2$$

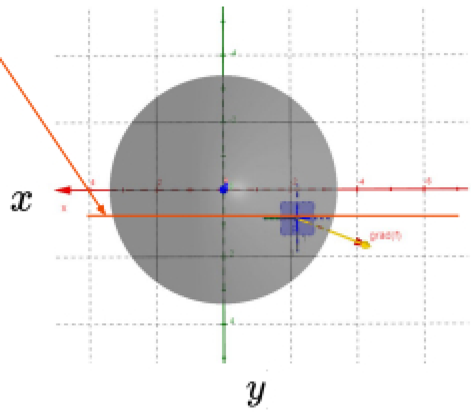
https://www.geogebra.org/m/vubtk9v8

Đạo hàm riêng
Partial Derivative



Sự thay đổi của f khi x thay đổi tức thì.

(**Giữ y không đổi**)

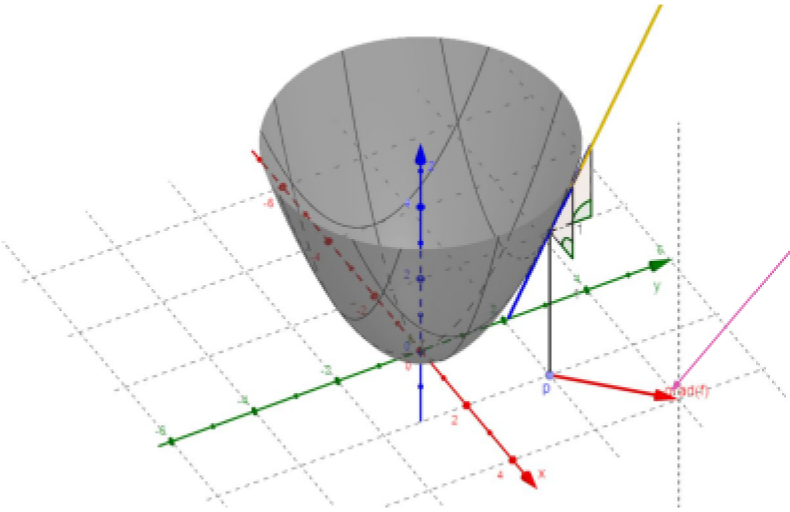


$$f(x,y)=\frac{1}{2}x^2+\frac{1}{2}y^2$$

https://www.geogebra.org/m/vubtk9v8

Gradient

Gradient



Gradient trường hợp này tạo thành **vector**

$$\nabla f = \left[\frac{\partial f(x,y)}{\partial x} , \frac{\partial f(x,y)}{\partial y} \right]$$

$$\left[x \quad , \quad y \right]$$

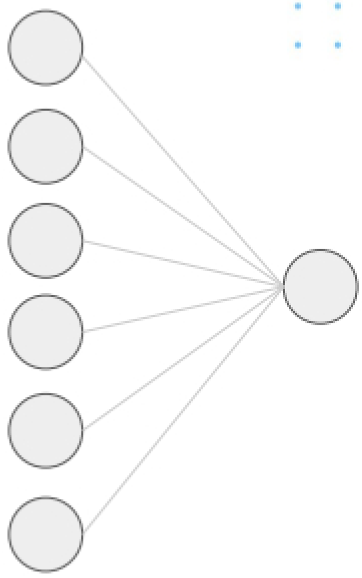
Ý nghĩa:

Thể hiện **độ dốc của mặt phẳng** tại tọa độ (x, y bất kỳ)

$$f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2$$

Đạo hàm riêng Vector - Scalar

Vector - Scalar Partial Derivative



$$f(\mathbf{x}) = f(x_1, x_2, \dots x_n)$$

$$\mathbf{x} \mapsto f(\mathbf{x})$$

$$f : \mathbb{R}^n \to \mathbb{R}$$

Đạo hàm riêng

$$\frac{\partial f}{\partial x_1} = \lim_{h \to 0} \frac{f(x_1+h, x_2, \dots, x_n) - f(x_1, x_2, \dots, x_n)}{h}$$

$$\dots$$

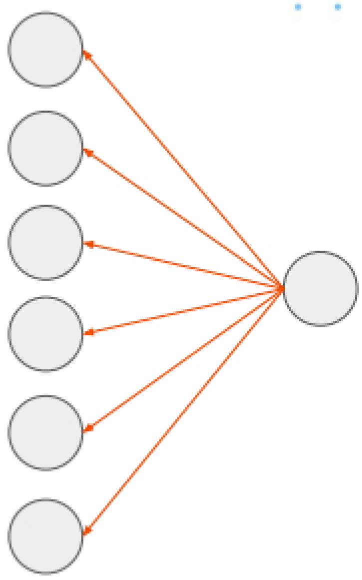
$$\frac{\partial f}{\partial x_n} = \lim_{h \to 0} \frac{f(x_1, x_2, \dots, x_n+h) - f(x_1, x_2, \dots, x_n)}{h}$$

Gradient Vector - Scalar

Gradient

$$\nabla_{\mathbf{x}} f = \frac{df}{d\mathbf{x}} = [\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}] \in \mathbb{R}^{1\times n}$$

Gradient of **f** / Jacobian



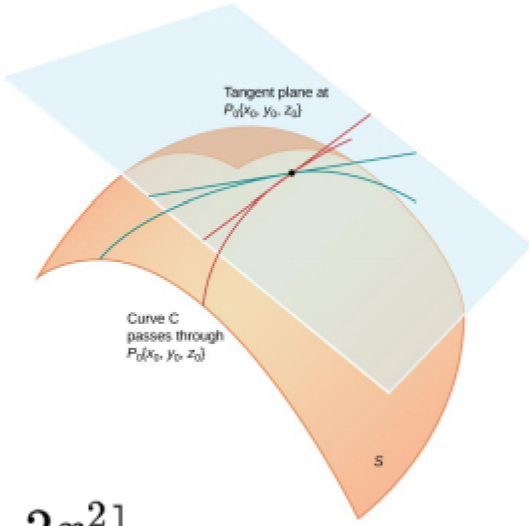
Ví dụ: Tính Gradient của f(x)

$$f(x_1, x_2) = x_1^3 + 2x_1^2 x_2$$

Gradient Vector - Scalar

Gradient of Vector - Scalar Function

Ví dụ: Tính Gradient của f(x)



$$f(x_1, x_2) = x_1^3 + 2x_1^2x_2$$

$$\frac{\partial f}{\partial x_1} = 3x_1^2 + 4x_1x_2$$

$$\frac{\partial f}{\partial x_2} = 2x_1^2$$

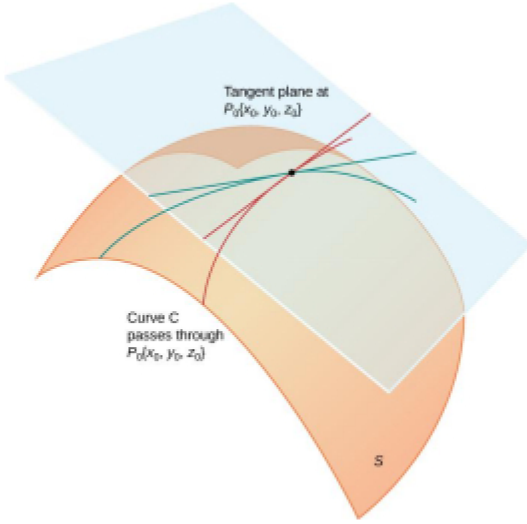
$$\nabla_{\mathbf{x}} f = [\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}] = [3x_1^2 + 4x_1x_2, 2x_1^2]$$

Gradient Vector - Scalar

Gradient of Vector - Scalar Function

$$\nabla_{\mathbf{x}} f = \frac{df}{d\mathbf{x}} = [\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}] \in \mathbb{R}^{1\times n}$$

Gradient of **f** / Jacobian



Bài tập: Tính Gradient của f(x)

$$f(x_1, x_2) = 3x_1^3 x_2^3 - x_1^{\frac{1}{2}} x_2^2$$

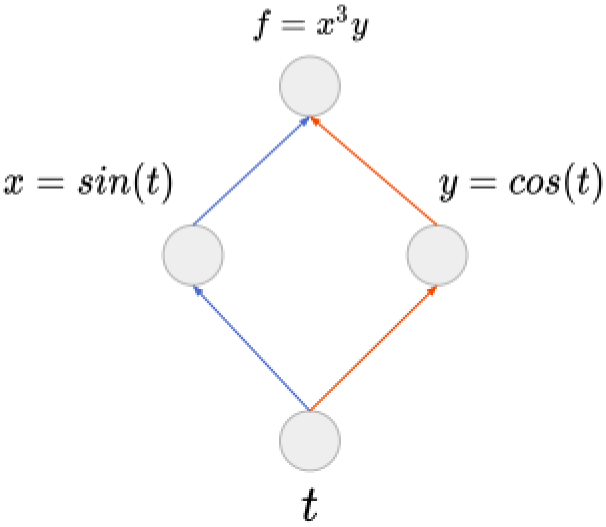
Tính chất đạo hàm riêng

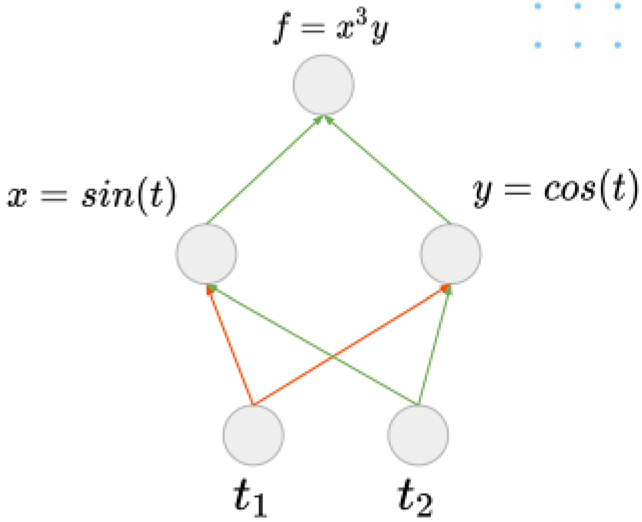
Partial Derivative Properties

	Tổng quát	Đạo hàm riêng
Quy tắc nhân	$(fg)' = f'g + fg'$	$\frac{\partial}{\partial \mathbf{x}}(f(\mathbf{x})g(\mathbf{x})) = \frac{\partial f}{\partial \mathbf{x}} g(\mathbf{x}) + \frac{\partial g}{\partial \mathbf{x}} f(\mathbf{x})$
Quy tắc tổng	$(f+g)' = f' + g'$	$\frac{\partial}{\partial \mathbf{x}}(f(\mathbf{x}) + g(\mathbf{x})) = \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} + \frac{\partial g(\mathbf{x})}{\partial \mathbf{x}}$
Quy tắc chuỗi	$f(g)' = f(g)'g'$	$\frac{\partial}{\partial \mathbf{x}}(f \circ g)(\mathbf{x}) = \frac{\partial}{\partial \mathbf{x}}(f(g(\mathbf{x}))) = \frac{\partial f}{\partial g} \frac{\partial g}{\partial \mathbf{x}}$

Quy tắc chuỗi nhiều biến

Multivariable Chain Rules





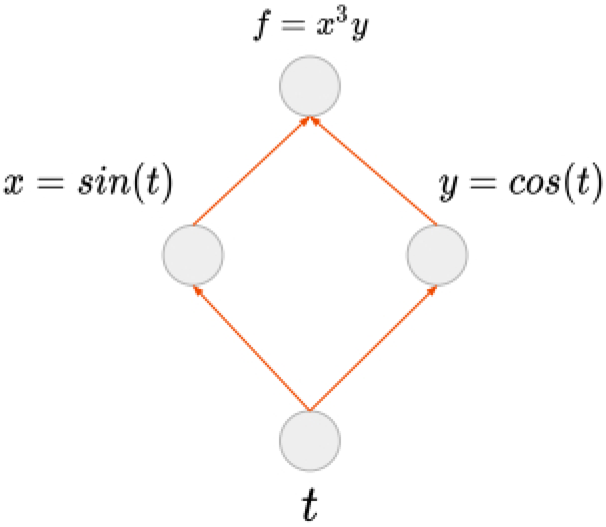
$$\frac{\partial f}{\partial t_1} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t_1} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t_1}$$

$$\frac{\partial f}{\partial t_2} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t_2} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t_2}$$

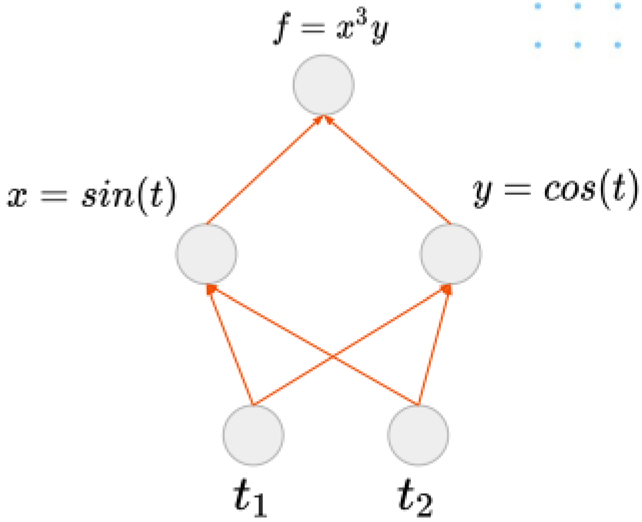
$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

[Quiz] Quy tắc chuỗi nhiều biến

[Quiz] Multivariable Chain Rules



$$\frac{\partial f}{\partial t} = ???$$

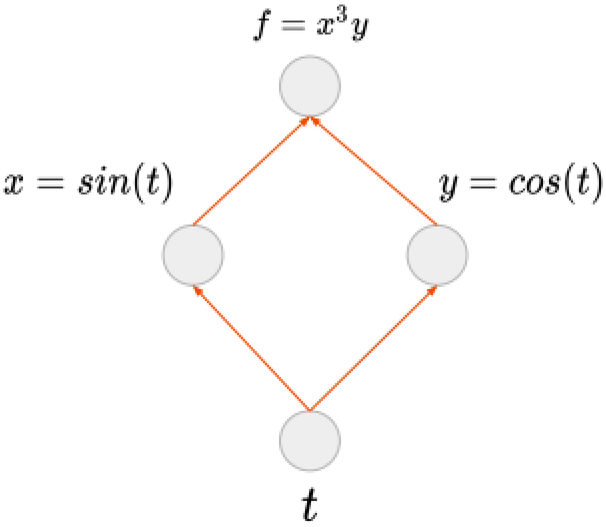


$$\frac{\partial f}{\partial t_1} = ???$$

$$\frac{\partial f}{\partial t_2} = ???$$

[Solution] Quy tắc chuỗi nhiều biến

[Solution] Multivariable Chain Rules

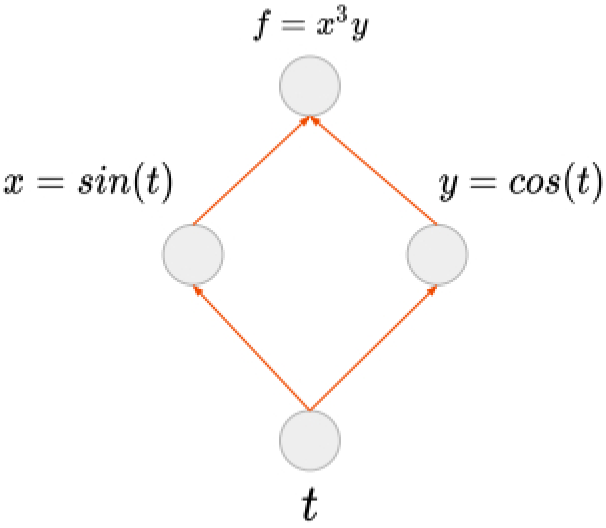


$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

$$= 3x^2ycos(t) - x^3sin(t)$$

[Quiz] Quy tắc chuỗi nhiều biến

[Quiz] Multivariable Chain Rules



$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

$$= 3x^2 y cos(t) - x^3 sin(t)$$

$$t = 4 \rightarrow \frac{\partial f}{\partial t} = \ ???$$

[Solution] Quy tắc chuỗi nhiều biến

[Solution] Multivariable Chain Rules

1 Tính từ công thức đạo hàm

$$3x^2ycos(t) - x^3sin(t)$$

```
def cal(t):
  x = tf.math.sin(t)
  y = tf.math.cos(t)
  return 3 * x ** 2 * y * tf.math.cos(t) - x ** 3 * tf.math.sin(t)
cal(4.0)
```

$$t = 4 \rightarrow \frac{\partial f}{\partial t} =$$

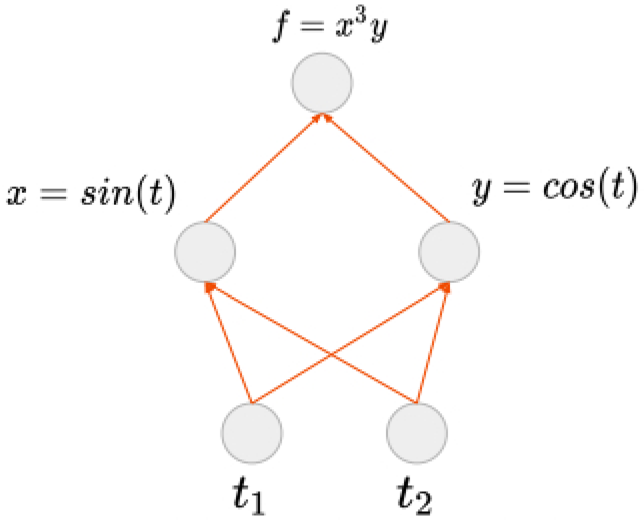
```
0.4060797
```

2 Sử dụng Gradient Tape

```
t = tf.constant(4.0)
with tf.GradientTape() as g:
  g.watch(t)
  x = tf.math.sin(t)
  y = tf.math.cos(t)
  f = x**3*y
dy_dt = g.gradient(f, t)
```

Quy tắc chuỗi nhiều biến

Multivariable Chain Rules



$$\frac{\partial f}{\partial t_1} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t_1} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t_1}$$

$$\frac{\partial f}{\partial t_2} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t_2} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t_2}$$

$$\frac{df}{d(t_1,t_2)} = \frac{\partial f}{\partial(x,y)}\frac{\partial(x,y)}{\partial(t_1,t_2)} = \left[\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y}\right]\begin{bmatrix} \frac{\partial x}{\partial t_1} & \frac{\partial x}{\partial t_2} \\ \frac{\partial y}{\partial t_1} & \frac{\partial y}{\partial t_2} \end{bmatrix}$$

Chuyển quy tắc chuỗi nhiều biến về phép nhân ma trận

Đọc thêm

Extra Materials

The Matrix Calculus You Need For Deep Learning

https://explained.ai/matrix-calculus/

Taylor Series

https://youtu.be/3d6DsjIBzJ4