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# Homework #1

**Problem 1. Matching Residents to Hospitals**

1. Model of the problem:   
     
   Let S be the set of n medical students, let H be the set of m hospitals. Because we assume that there were more students graduating than there were slots available in the m hospitals, there will be a set S’ of medical students who are not assigned to any hospitals and S’⊆S.

Each hospital h ranks all students s in order of preference for which s ∉ S’, and each student s ranks all hospitals in order of preference.  
  
We say that an assignment of students to hospitals is stable if it does not have neither of below instability:  
(i) There are students s and s’ and a hospital h, so that

- s is assigned to h, and

- s’ is assigned to no hospital, and

- h prefers s’ to s

(ii) There are students s and s’, and hospitals h and h’, so that

- s is assigned to h, and

- s’ is assigned to h’, and

- h prefers s’ to s, and

- s’ prefers h to h’.

1. Overall idea of the algorithm: we loop through all the hospitals and find the students on their preference list from the highest to lowest, offer each student in this list if that student is still free. On the student’s perspective, each student will be offered by a hospital, the student may change their mind and accept a better offer based on their preference list. Once all the hospitals’ slots are filled, there will be a surplus of remaining students who do not receive any offer.
2. Pseudo code:  
   *Initially all s S and h H are free  
   While there is a hospital which is free and has not offered to every student  
    Choose such a hospital h  
    Let s be the highest-ranked medical student in h’s preference list to whom h has not yet offered  
    If s is free then  
    (s, h) become matched  
    Else s is currently matched with h’  
    If s prefers h’ to h then  
    h remain free  
    Else s prefers h to h’  
    (s, h) become matched  
    h’ become free  
    Endif  
    Endif  
   Endwhile  
   Return the set S of matched pairs*
3. To show that the algorithm works, we show that it resolves 2 problems of instability (i) and (ii)  
     
   First, for the problem instance (i), suppose we have students s and s’, and a hospital h. If h prefers s’ to s, h would offer to s’ before s because s’ is free (in order of h’s preference list). As a result, it is impossible for s’ to be free at the end, if s’ is preferred by h to s.  
     
   Second, for the problem (ii), suppose we have students s and s’, and hospitals h and h’ so that s is assigned to h and s’ is assigned h’. This problem is similar to the (i) problem, if h prefers s’, then it would offer s’ before s, and if s’ prefers h, it would accept h’s offer. This problem will create a contradiction to the algorithm.
4. Suppose we have n hospitals, the while loop of this algorithm will terminate after at most n2 iterations, so the time complexity is O(n2).

**Problem 2. Implementation of Propose-and-Reject Algorithms**

1. Please see source code file
2. From Victor to Zeus: {1=1, 2=3, 3=2, 4=5, 5=4}  
   Victor - Amy  
   Wyatt - Clare  
   Xavier - Bertha  
   Yancey - Erika  
   Zeus - Diane
3. From Zeus to Victor: {1=1, 2=3, 3=2, 4=5, 5=4}  
   Zeus - Diane  
   Yancey - Erika  
   Xavier - Bertha  
   Wyatt - Clare  
   Victor - Amy
4. From Amy to Erika: {1=4, 2=3, 3=2, 4=5, 5=1}  
   Amy - Yancey  
   Bertha - Xavier  
   Clare - Wyatt  
   Diane - Zeus  
   Erika - Victor