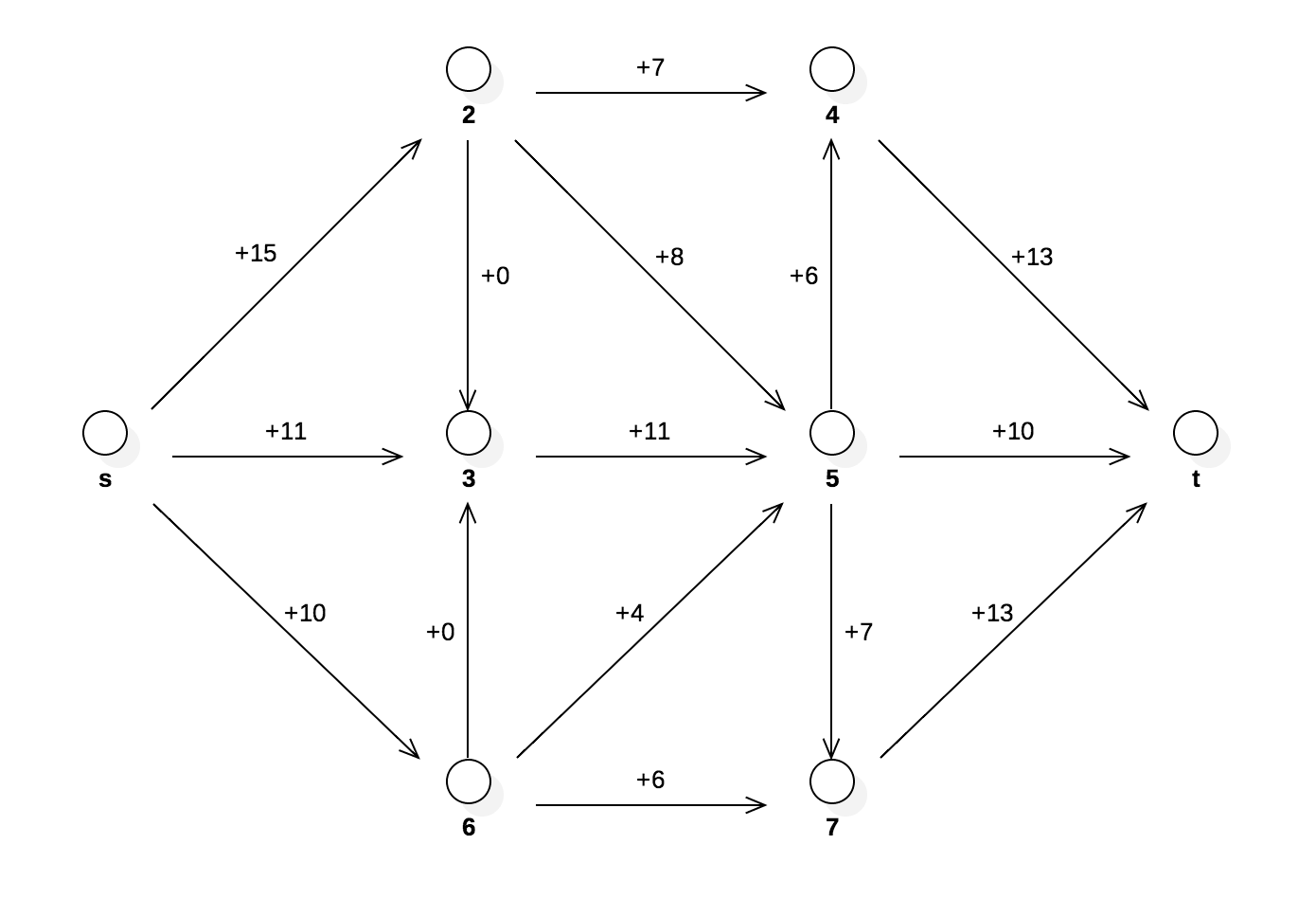
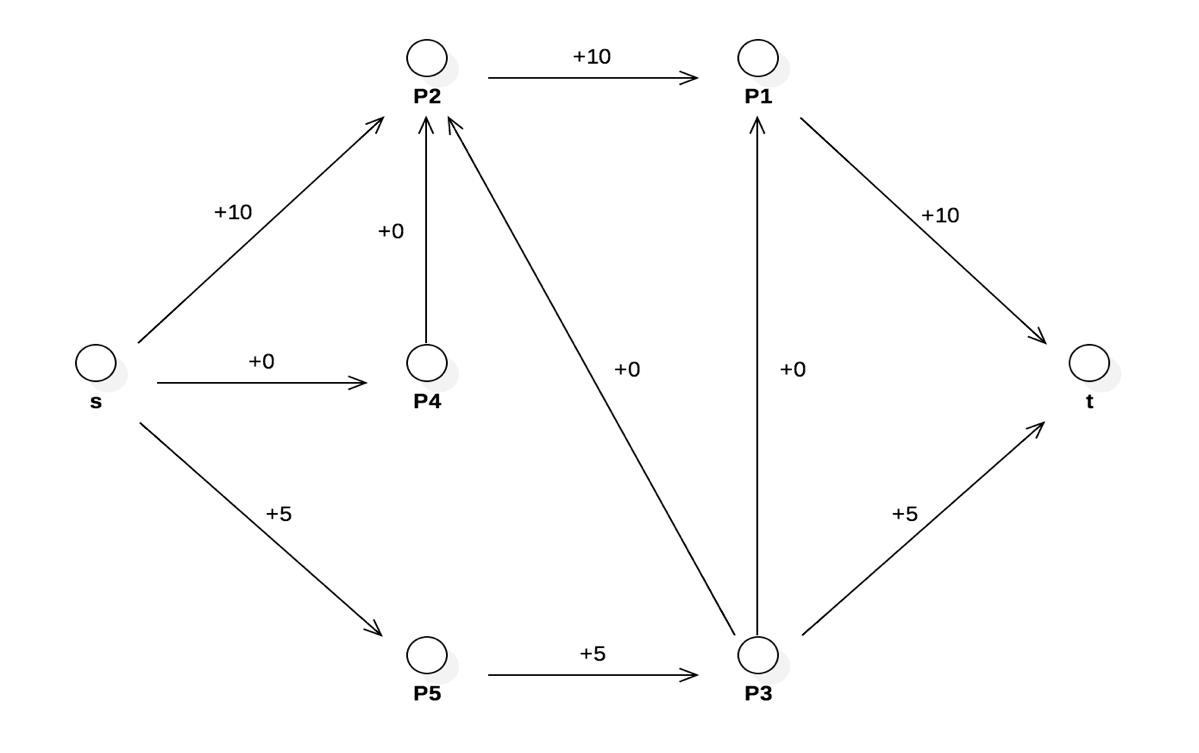
Student Name: Duc Tu Luong  
Last 3-digit ID: 122

# Homework #5

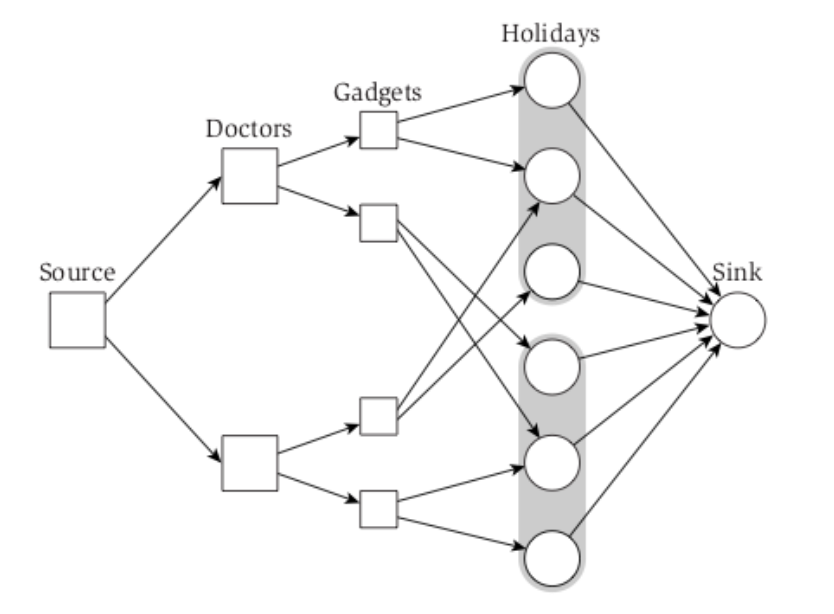
1. **Ford-Fulkerson Algorithm***Credit:* <http://www.geeksforgeeks.org/ford-fulkerson-algorithm-for-maximum-flow-problem/>  
     
   a) Pseudo code:

b) Let be the max flow, is the total number of nodes in graph. When we iterate while there is still augmenting path, the worst case is that we only add 1 unit in every iteration, as a result, the time complexity is   
c) The maximum possible flow is 36, the flow graph is as following:  


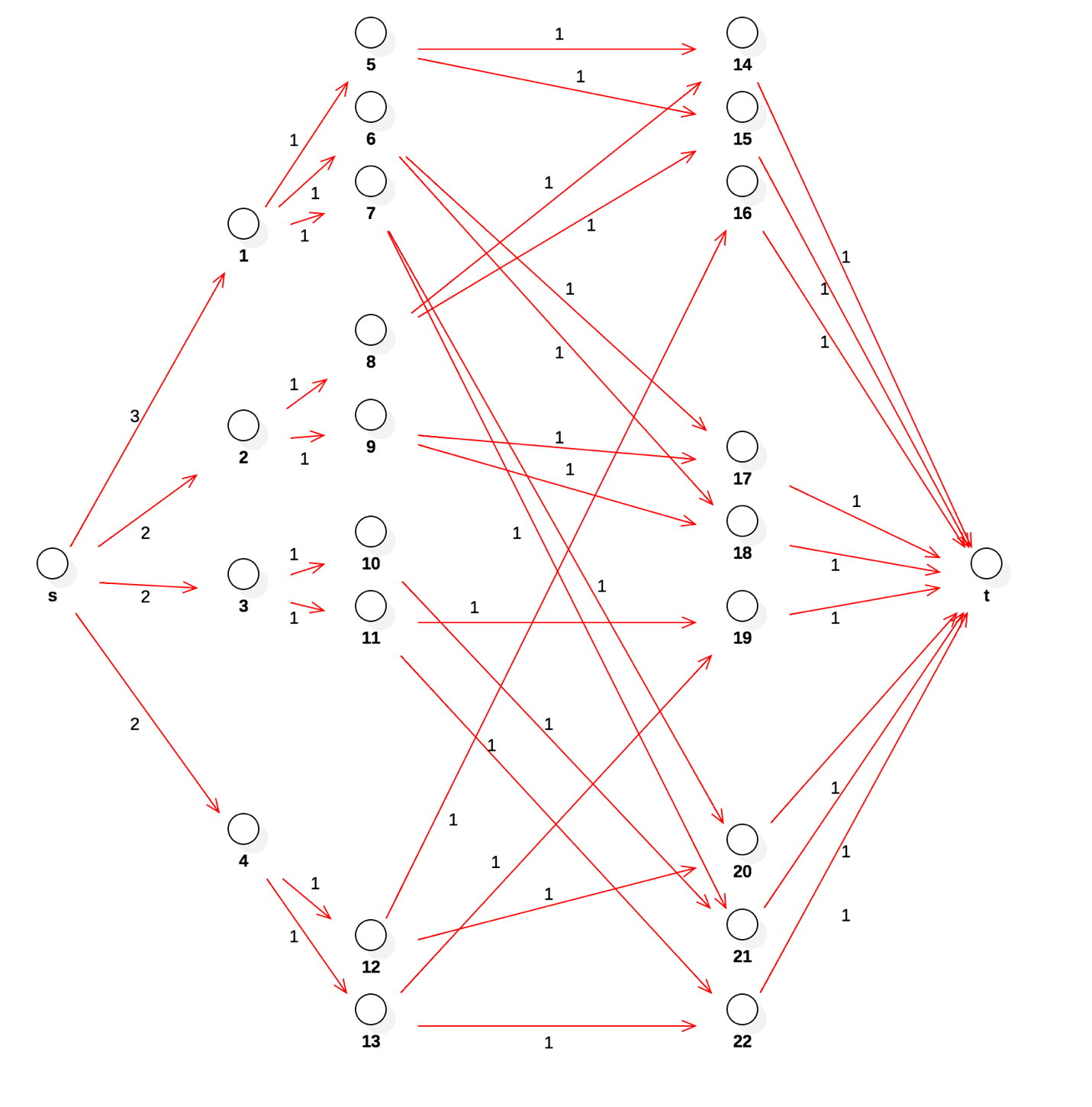
1. We model the problem using network flow diagram. The network flow of profit is as following:  
     
     
   The maximum profit we can make is 15, we will take P1, P2, P3, P5 to generate this profit.
2. a)   
   *Problem*:  
   -: number of doctors  
   - : set of vacation days;   
   - : set of vacation-days doctor can work  
   - : vacation periods  
   - days in period   
   Give a polynomial algorithm which returns assignment of doctors to vacation days, or reports if no such assignment exists; and satisfies following constraints:  
   - For a given parameter , each doctor should be assigned to work at most vacation days total, and only days when he or she is available.  
   - For each vacation period , each doctor should be assigned to work at most one of the days in the set .

*Solution*:

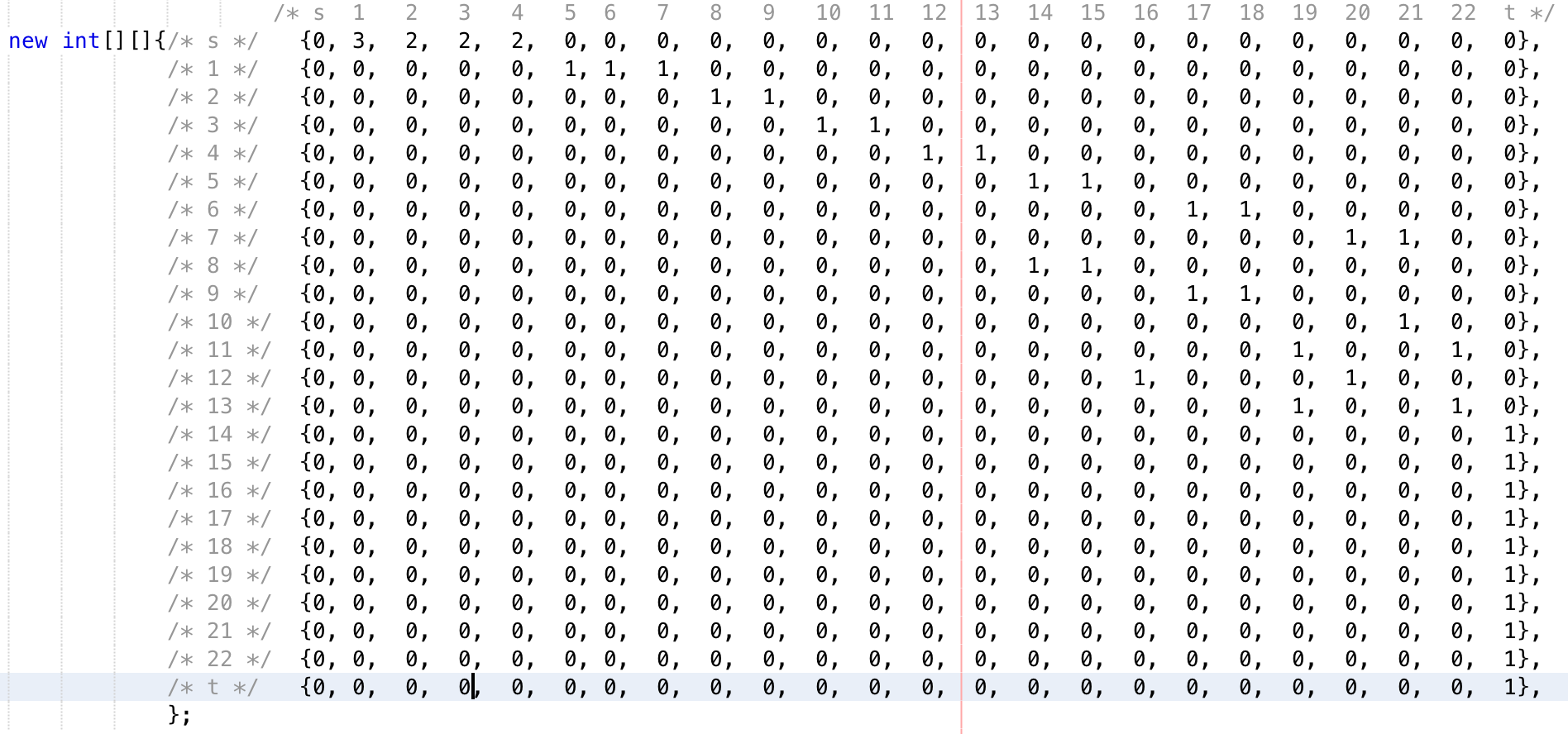
We use network flow model to solve this problem. We have a node representing each doctor attached to a node representing each day when he or she can work; each edge has capacity of 1.   
We attach a super-source to each doctor node by an edge of capacity , and we attach each day node to a super- sink by an edge with upper and lower bounds of 1. Suppose there are vacation days total; we put a demand of on the sink and on the source.

We include a new node with an incoming edge of capacity 1 from the doctor node , and with outgoing edges of capacity 1 to each day in vacation period when doctor is available to work. The network flow model is as following  
  


b) Problem instance with 4 doctors and 3 vacation periods. The network flow diagram is as following

  
  
- Vertices from 1 to 4 represent 4 doctors.  
- Vertices from 5 to 13 represent “gadgets” for 4 doctors, these gadgets guarantee each doctor only cover 1 day in each vacation period.

- Vertices from 14 to 22 represent vacation days, each 3-day block represents each vacation period.

c) We encode the network flow diagram into a matrix with values of each edge as following  


When applying to question 1 implementation, the result is as following:  
Maximum flow is 9, which is all vacation days are covered, the details are as following:  
- Day 1 of vacation period 1 is covered doctor 1

- Day 2 of vacation period 1 is covered doctor 2

- Day 3 of vacation period 1 is covered doctor 4

- Day 1 of vacation period 2 is covered doctor 1

- Day 2 of vacation period 2 is covered doctor 2

- Day 3 of vacation period 2 is covered doctor 3

- Day 1 of vacation period 3 is covered doctor 1

- Day 2 of vacation period 3 is covered doctor 3

- Day 3 of vacation period 3 is covered doctor 4

1. a)   
   *Problem:*  
    users of the website

groups from of the demographic groups, which users belong to  
 advertisers  
Give an efficient algorithm to decide if there is a way to show a single ad to each user so that the site’s contracts with each of the advertisers is satisfied for this minute? (That is, for each , can at least of the users, each belonging to at least one demographic group in , be shown an ad provided by advertiser ?), and if so, to choose an ad to show each user.

*Solution:*  
We design a flow network as following:  
- There is a source   
- Vertices for all users  
- Vertices for all demographic groups

- Vertices for all advertisers

- There is a source   
- There is an edge of capacity 1 from to each group for which user belongs to a demographic group.

- There is an edge of capacity ∞ from each group to at least one advertiser .

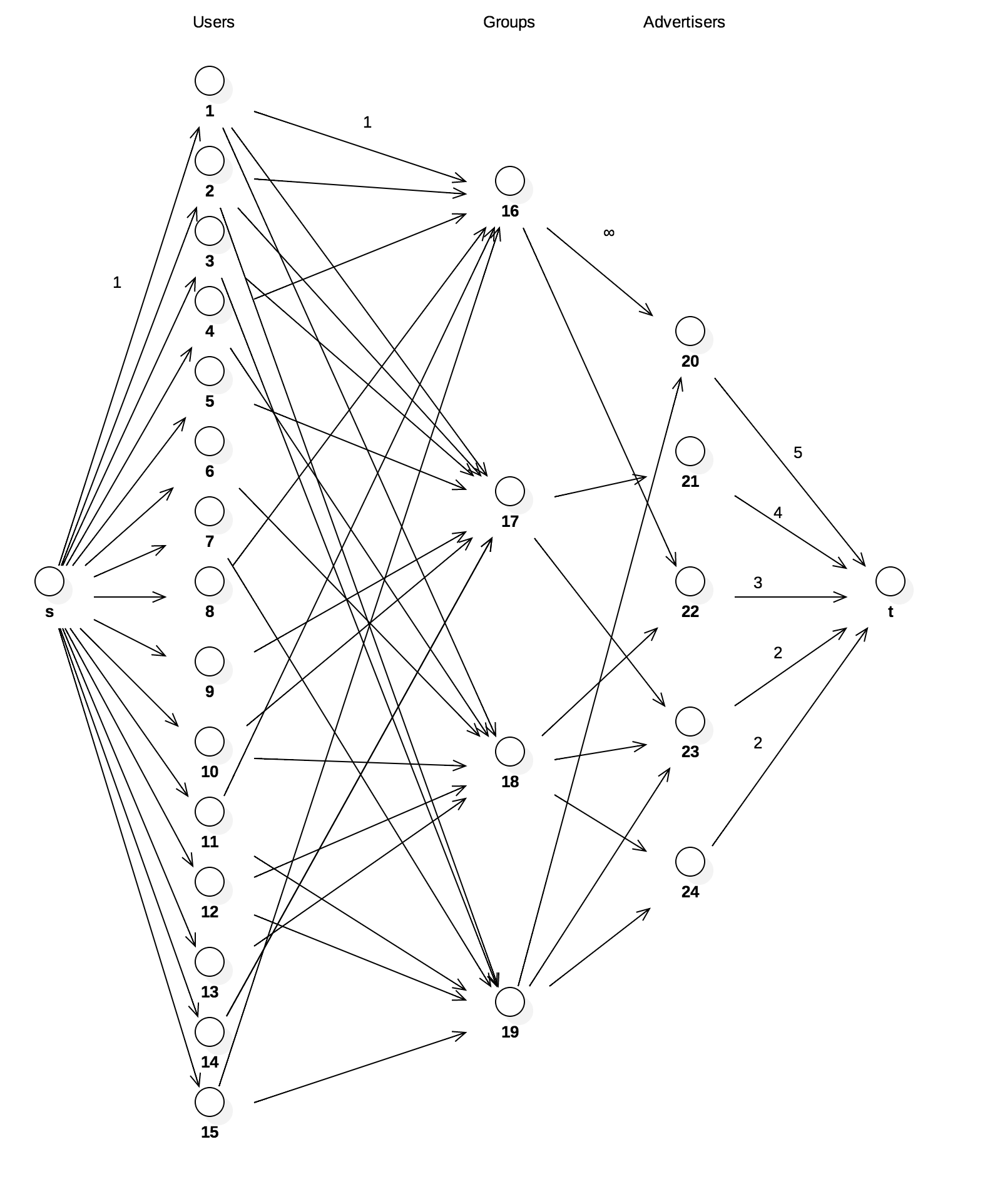
- There is an edge with a capacity of 1 from to each for each .

- There is an edge with a capacity of the site contracts with each of advertisers which is of the users from to for each .

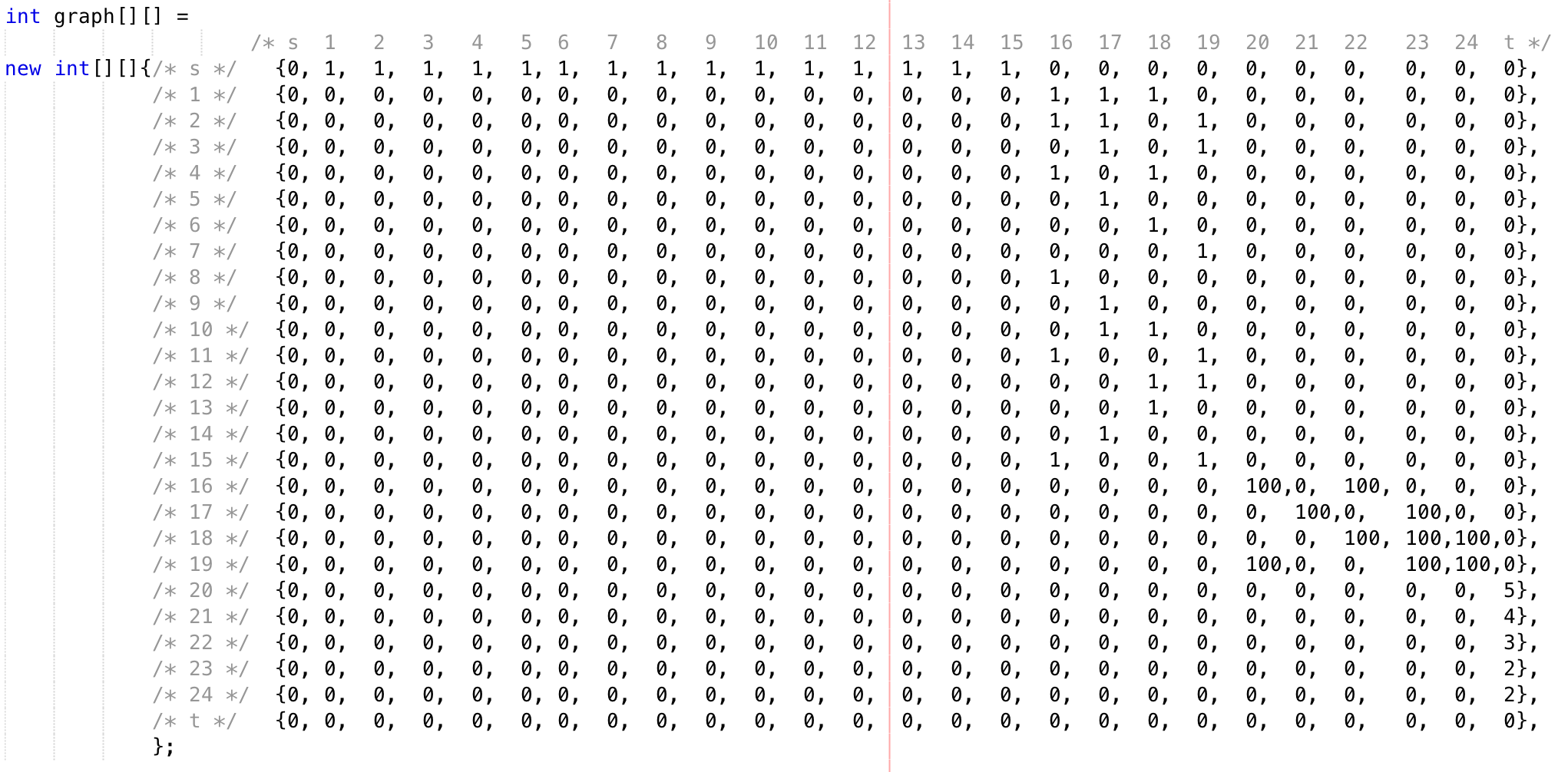
- The source has supply of

- The sink has demand of   
  
If there is a valid circulation in this graph, meaning that advertiser shows ads to a subset of vertices of , in which a subset of users belong to, which mean that advertiser shows ads to appropriate people. We will find out the maximum flow in this graph, if this maximum flow is equal to , it means advertisers show their ads to all site’s users with appropriate content.  
  
b) Problem instance with k=4, n=15, m=5 is represented as following network flow diagram

* Each edge from s to vertices 1 to 15 has capacity of 1 (source to each user vertex)
* Each edge from vertices 1 to 15 to vertices 16 to 19 has capacity of 1 (user vertices to groups)
* Each edge from vertices 16 to 19 to vertices 20 to 24 has capacity of ∞ (groups to advertisers)
* Each edge from vertices 20 to 24 to sink t has capacity of site’s contract to that advertiser (advertisers to sink t).



c) We encode the network flow diagram into a matrix with values of each edge as following, we use value 100 to encode ∞



When applying to question 1 implementation, the result is as following:

The maximum flow is 15, which means all users are shown appropriate ads.

- Node 20 to sink t has flow of 5/5, which means the site’s contract between advertiser 20 has been met.

- Node 21 to sink t has flow of 4/4, which means the site’s contract between advertiser 21 has been met.

- Node 22 to sink t has flow of 3/3, which means the site’s contract between advertiser 22 has been met.

- Node 23 to sink t has flow of 2/2, which means the site’s contract between advertiser 23 has been met.

- Node 24 to sink t has flow of 1/2, which means the site’s contract between advertiser 24 has NOT been met.