SeaThru-NeRF: Neural Radiance Fields in Scattering Media Appendix

Anonymous CVPR submission

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Derivation of Eqs. (10-11)

We suggested the formulation

$$C(\mathbf{r}) = \int_{t_n}^{t_f} T(t)\Big(\sigma^{\text{obj}}(t)\mathbf{c}^{\text{obj}}(t) + \sigma^{\text{med}}(t)\mathbf{c}^{\text{med}}(t)\Big)dt$$

with transmittance:

$$T(t) = \exp\left(-\int_{t_n}^{t}\Big(\sigma^{\text{obj}}(s) + \sigma^{\text{med}}(s)\Big)ds\right) \quad ,$$

using separate color and density parameters for the *object* and *medium*.

In the discretized version, given a sampling $\{s_i\}_{i=0}^{N}$ ($s_0 = t_n$, $s_N = t_f$) with intervals of length $\delta_i = s_{i+1} - s_i$, and assuming the parameters $\sigma^{\text{obj}}, \mathbf{c}^{\text{obj}}, \sigma^{\text{med}}, \mathbf{c}^{\text{med}}$ to be constant per interval, we can write the transmittance at a sampling point s_i, with as a piece-wise constant sum instead of the integral:

$$\begin{aligned} T(s_i) &= \exp\left(-\int_{t_n}^{s_i}\Big(\sigma^{\text{obj}}(s) + \sigma^{\text{med}}(s)\Big)ds\right) \\ &= \exp\left(-\sum_{j=0}^{i-1}(\sigma_j^{\text{obj}} + \sigma_j^{\text{med}})\delta_j\right) \quad . \end{aligned}$$

In the discretized rendering equation

$$\hat{C}(\mathbf{r}) = \sum_i C_i(\mathbf{r})$$

the contribution $C_i(\mathbf{r})$ of the ith interval is given by:

$$\begin{aligned} C_i(\mathbf{r}) &= \int_{s_i}^{s_{i+1}} T(t)(\sigma_i^{\text{obj}}\mathbf{c}_i^{\text{obj}} + \sigma_i^{\text{med}}\mathbf{c}_i^{\text{med}})dt \\ &= (\sigma_i^{\text{obj}}\mathbf{c}_i^{\text{obj}} + \sigma_i^{\text{med}}\mathbf{c}_i^{\text{med}}) \cdot \int_{s_i}^{s_{i+1}} T(t) \end{aligned}$$

where we can compute:

$$\begin{aligned} &\int_{s_i}^{s_{i+1}} T(t) \\ &= \int_{s_i}^{s_{i+1}} \exp\left(-\int_{t_n}^{t}(\sigma^{\text{obj}}(s) + \sigma^{\text{med}}(s))ds\right)dt \\ &= \int_{s_i}^{s_{i+1}} \exp\left(-\int_{t_n}^{s_i}(\sigma^{\text{obj}}(s) + \sigma^{\text{med}}(s))ds\right) \cdot \\ &\qquad \exp\left(-\int_{s_i}^{t}(\sigma_i^{\text{obj}} + \sigma_i^{\text{med}})ds\right)dt \\ &= \int_{s_i}^{s_{i+1}} T(s_i) \cdot \exp\left(-(t - s_i)\cdot(\sigma_i^{\text{obj}} + \sigma_i^{\text{med}})\right)dt \\ &= T(s_i) \cdot \left[-\frac{\exp\left(-(t - s_i)\cdot(\sigma_i^{\text{obj}} + \sigma_i^{\text{med}})\right)}{\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}}\right]_{s_i}^{s_{i+1}} \\ &= T(s_i) \cdot \frac{1 - e^{-(\sigma_i^{\text{obj}} + \sigma_i^{\text{med}})\delta_i}}{\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}} \end{aligned}$$

which finally results in

$$C_i(\mathbf{r}) = T(s_i)\left(1 - e^{-(\sigma_i^{\text{obj}} + \sigma_i^{\text{med}})\delta_i}\right)\frac{\sigma_i^{\text{obj}}\mathbf{c}_i^{\text{obj}} + \sigma_i^{\text{med}}\mathbf{c}_i^{\text{med}}}{\sigma_i^{\text{obj}} + \sigma_i^{\text{med}}}$$

as required.