1. $S = \{a, b, c, d\}$

$.4, .35, .15, .1$

bbbb

| Next letter | $\alpha$ | $l$ |
|---|---|---|
| | 0 | 1 |
| b | $0 + (0.4)(1) = 0.4$ | $(1)(0.35) = 0.35$ |
| b | $0.4 + 0.4 \cdot 0.35 = 0.54$ | $(0.35)^2 = 0.1225$ |
| b | $0.54 + 0.4 \cdot 0.1225 = 0.589$ | $(0.1225)(0.35) = 0.042875$ |
| b | $0.589 + 0.4 \cdot 0.042875 = 0.60615$ | $(0.042875)(0.35) = 0.01500625$ |

Final interval

$[.60615 \sim .62115625)$

$0.62115625 - 0.60615 \approx 0.0150$

$\frac{1}{2^6} \leq 0.0150 \leq \frac{1}{2^7}$

$0.60615 \leq \frac{x}{64} \leq 0.62115625$

$\Rightarrow 38.7936 \leq x \leq 39.754$

$x = 39$

$39 = 32 + 4 + 2 + 1$

$\frac{39}{64} = \frac{32}{64} + \frac{4}{64} + \frac{2}{64} + \frac{1}{64}$

$= \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6}$

$= (.100111)_2$

$\frac{39}{64} = (.100111)_2$

Code: 1001

-abcd

| Next letter | $\alpha$ | $l$ |
|---|---|---|
| | 0 | 1 |
| a | 0 | 0.4 |
| b | $0 + 0.4 \cdot 0.4 = 0.016$ | $0.4 \cdot 0.35 = 0.14$ |
| c | $0.016 + 0.14 \times 0.75 = 0.121$ | $0.14 \times 0.15 = 0.021$ |
| d. | $0.121 + 0.021 \times 0.9 = 0.1399$ | $0.021 \times 0.1 = 0.0021$ |

Final interval

$[0.1399, 0.1420)$

dfwld

$\frac{1}{2^9} \leq 0.0021 \leq \frac{1}{2^8}$

$\therefore \quad 0.1399 \leq \frac{x}{512} \leq 0.1420$

$71.688 \leq x \leq 72.704$

$x = 72$

$\frac{72}{512} = (.001001)_2$

Code: 0010

$\frac{72}{64}$

$\frac{64}{8}$

$\frac{64}{512} + \frac{8}{512}$

$= \frac{1}{2^3} + \frac{1}{2^6}$

$= (.001001)_2$

- dcba

| Next letter | $\alpha$ | $l$ |
|---|---|---|
| | 0 | 1 |
| d | $0 + 0.9 \times 1 = 0.9$ | $1 \times 0.1 = 0.1$ |
| c | $0.9 + 0.1 \times 0.75 = 0.975$ | $0.1 \times 0.15 = 0.015$ |
| b | $0.975 + 0.015 \times 0.4 = 0.981$ | $0.015 \times 0.35 = 0.00525$ |
| a | $0.981$ | $0.00525 \times 0.4 = 0.0021$ |

Final interval

$[0.981, 0.9831)$

$\frac{1}{2^9} \leq 0.0021 \leq \frac{1}{2^8}$

$0.981 \leq \frac{x}{512} \leq 0.9831$

$\Rightarrow 502.272 \leq x \leq 503.3472$

$x = 503$

$\frac{503}{512} = (.11111000)_2$

$\Rightarrow$ Code = 1111

$503 = 256 + 128 + 64 + 32 + 16 + 4 + 1$

$\frac{503}{512} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^9}$

$= (0.111110001)_2$

0.2

- badd

| Next letter | $\alpha$ | $l$ |
|---|---|---|
| | 0 | 1 |
| b | 0.4 | 0.35 |
| a | 0.4 | $0.4 \times 0.35 = 0.14$ |
| d | $0.4 + 0.14 \times 0.9 = 0.526$ | $0.14 \times 0.1 = 0.014$ |
| d | $0.526 + 0.014 \times 0.9 = 0.5386$ | $0.014 \times 0.1 = 0.0014$ |

Final interval

$[0.5386, 0.5400)$

$$\frac{1}{2^{10}} \leq 0.0014 \leq \frac{1}{2^9}$$

$$0.5386 \leq \frac{x}{1024} \leq 0.5400$$

$$\Rightarrow 551.5264 \leq x \leq 552.960$$

$x = 552$

$$\frac{552}{1024} \Rightarrow (.1000\,1)_2$$

code: 1000

552
512
40
32
8
8

$\frac{2^5}{2^{10}}$

$$\frac{512}{1024} + \frac{32}{1024} + \frac{8}{1024}$$

$$= \frac{1}{2} + \frac{1}{2^5} + \frac{1}{2^7}$$

$$= (.1000101)_2$$

2. $f_a[0, 0.4)$, $f_b[0.4, 0.75)$, $f_c[0.75, 0.9)$, $f_d[0.9, 1.0)$

$p_a = 0.4$, $p_b = 0.35$, $p_c = 0.15$, $p_d = 0.1$

| $\alpha$ | $l$ | $\frac{r-\alpha}{l}$ | Decode |
|---|---|---|---|
| 0 | 1 | 0.75 | c |
| 0 + 0.75 | $1 \cdot 0.15 = 0.15$ | 0 | a |
| 0.75 | $0.15 \times 0.4 = 0.06$ | $\frac{0.8 - 0.75}{0.06} = 0.8$ | a |
| 0.75 | $0.06 \times 0.4 = 0.024$ | 0 | a |

$0.25 + 0.015625 \simeq 0.266 = r$

| $\alpha$ | $l$ | $\frac{r-\alpha}{l}$ | Decode |
|---|---|---|---|
| 0 | 1 | 0.266 | a |
| 0 | 0.4 | $\frac{0.266}{0.4} \simeq 0.665$ | b |
| $0 + 0.4 \times 0.4 = 0.16$ | $0.4 \times 0.35 = 0.14$ | $\frac{0.266 - 0.16}{0.14} = \frac{0.106}{0.14} \simeq 0.757$ | c |
| $0.16 + 0.14 \times 0.75 = 0.265$ | $0.14 \times 0.15 = 0.021$ | $\frac{0.266 - 0.265}{0.021} = \frac{0.001}{0.021} \simeq 0.048$ | a |

- 1 0 1 ≈ 0.656

| α | l | r = α / l | Decode: |
|---|---|---|---|
| 0 | 1 | 0.656 | b |
| 0 + 0.4 = 0.4 | 0.35 | (0.656 − 0.4) / 0.35 = 0.256 / 0.35 ≈ 0.731 | b |
| 0.4 + 0.35 × 0.4 = 0.54 | 0.35 × 0.35 = 0.1225 | (0.656 − 0.54) / 0.1225 = 0.116 / 0.1225 ≈ 0.947 | d |
| 0.54 + 0.1225 × 0.4 ≈ 0.650 | 0.1225 × 0.35 ≈ 0.043 | (0.656 − 0.650) / 0.043 ≈ 0.488 | b |

- 0 1 0 1 ≈ 0.313

| α | l | r = α / l | Decode: |
|---|---|---|---|
| 0 | 1 | 0.313 | a |
| 0 | 1 × 0.4 = 0.4 | 0.313 / 0.4 ≈ 0.783 | c |
| 0 + 0.4 × 0.75 = 0.3 | 0.4 × 0.15 = 0.06 | 0.013 / 0.06 ≈ 0.217 | a |
| 0.3 | 0.06 × 0.4 = 0.024 | 0.013 / 0.024 ≈ 0.542 | b |

1420

3. $f_a = .4$, $f_b = .35$, $f_c = .15$, $f_d = .1$

(a) – bbbb

| Next letter | $\alpha$ | $\ell$ | cfwld |
|---|---|---|---|
| | 0 | 1 | 0 |
| b | 0.4 | 0.35 | $\frac{1}{2}$ |
| b | 0.54 | 0.1225 | $\frac{5}{8}$ |
| b | 0.589 | 0.0429 | $\frac{5}{8}$ |
| b | 0.6062 | 0.0150 | $\frac{39}{64}$ |

=> no new representative.

– abcd

| Next letter | $\alpha$ | $\ell$ | cfwld |
|---|---|---|---|
| | 0 | 1 | 0 |
| a | 0 | 0.4 | $\frac{1}{4}$ |
| b | 0.016 | 0.14 | $\frac{1}{8}$ |
| c | 0.121 | 0.021 | $\frac{9}{64}$ |
| d | 0.1399 | 0.0021 | $\frac{72}{512}$ |

=> no new representative

- dcba

| Next letter | $\alpha$ | $\ell$ | dtwld |
|---|---|---|---|
| | 0 | 1 | 0 |
| d | 0.9 | 0.1 | $\frac{15}{16}$ |
| c | 0.975 | 0.015 | $\frac{125}{128}$ |
| b | 0.981 | 0.0053 | $\frac{252}{256}$ |
| a | 0.981 | 0.0021 | $\frac{503}{512}$ |

=> no new representative

- badd

| Next letter | $\alpha$ | $\ell$ | dtwld |
|---|---|---|---|
| | 0 | 1 | 0 |
| b | 0.4 | 0.35 | $\frac{1}{2}$ |
| a | 0.4 | 0.14 | $\frac{1}{2}$ |
| d | 0.526 | 0.014 | $\frac{68}{128}$ |
| d | 0.5386 | 0.0014 | $\frac{552}{1024}$ |

=> no new representative.

* There is no source words that have different code representatives.

0.36 0.3744

0.0144 0.24

0.04×0.4

0.25 0.3125 0.3125

16

(b) $f_a[0, .4)$ $f_b[.4, .75)$ $f_c[.75, .9)$ $f_d[.9, 1.0)$

011000 $r = (011000)_2 = 0.375$

| A | l | dfwld | $\frac{r-q}{l}$ | Decode |
|---|---|---|---|---|
| 0 | 1 | 0. | 0.375 | a. |
| 0.0 | 0.4 | $\frac{1}{4} = 0.25$ | $\frac{0.375}{0.4} = 0.9375$ | d |
| 0.4×0.4=0.36 | 0.04 | $\frac{3}{8} = 0.375 = r$ | $\frac{0.015}{0.04} = 0.375$ | a. |
| 0.36 | 0.016 | $\frac{3}{8}$ | $\frac{0.015}{0.016} = 0.9375$ | d |
| 0.36+0.016×0.0, 0.3744 | 0.016×0.1=0.0016 | $\frac{3}{8}$ | $\frac{0.0006}{0.0016} = 0.375$ | a. |
| 0.3744 | 0.0016×0.4=0.00064 | $\frac{3}{8}$ | $\frac{0.0006}{0.00064} = 0.9375$ | d. |

=> number of zeros = 3 = k-1

=> k=4 ∴ The final dfwld which is 0.375 is repeated 4 times.

=> The length is 6

=> adadad