1. $f_1 = 0.3$, $f_2 = 0.22$, $f_3 = 0.2$, $f_4 = 0.1$, $f_5 = 0.06$, $f_6 = 0.05$, $f_7 = 0.04$, $f_8 = 0.03$

- shannon's

$(\frac{1}{2})^{le} \le 0.3$ $le = 2$ $F_1 = 0$ $(.00\ldots)_2 \Rightarrow S_1 = 00$

$(\frac{1}{2})^{le} \le 0.22$ $= 3$ $F_2 = .3$ $(.010\ldots)_2 \Rightarrow S_2 = 010$

$(\frac{1}{2})^{le} \le 0.2$ $= 3$ $F_3 = .52$ $(.100\ldots)_2 \Rightarrow S_3 = 100$

$(\frac{1}{2})^{le} \le 0.1$ $= 4$ $F_4 = .72$ $(.1011\ldots)_2 \Rightarrow S_4 = 1011$

$(\frac{1}{2})^{le} \le 0.06$ $= 5$ $F_5 = .82$ $(.11010\ldots)_2 \Rightarrow S_5 = 11010$

$(\frac{1}{2})^{le} \le 0.05$ $= 5$ $F_6 = .88$ $(.11100\ldots)_2 \Rightarrow S_6 = 11100$

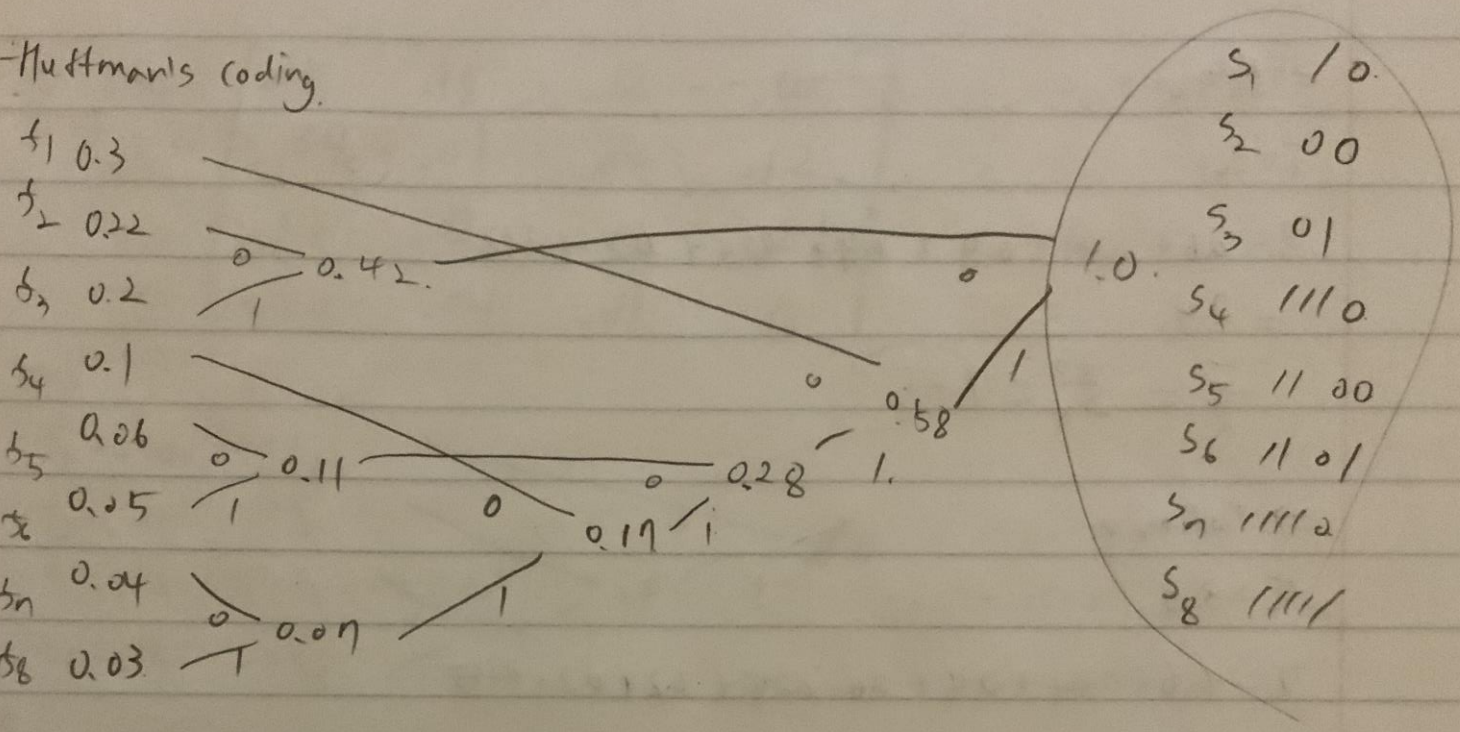
$(\frac{1}{2})^{le} \le 0.04$ $= 5$ $F_7 = .93$ $(.11101\ldots)_2 \Rightarrow S_7 = 11101$

$(\frac{1}{2}) \le 0.03$ $= 6$ $F_8 = .97$ $(.11110\ldots)_2 \Rightarrow S_8 = 11110$

- fano's

f_1	S_1	0	0				$\Rightarrow 00$
f_2	S_2	0	1				$\Rightarrow 01$
f_3	S_3	1	0				$\Rightarrow 10$
f_4	S_4	1	1	0	0		$\Rightarrow 1100$
f_5	S_5	1	1	0	1		$\Rightarrow 1101$
f_6	S_6	1	1	1	0		$\Rightarrow 1110$
f_7	S_7	1	1	1	1	0	$\Rightarrow 11110$
f_8	S_8	1	1	1	1	1	$\Rightarrow 11111$

-Huffmanis coding.



-Compression ratio

=>shannon

$\bar{L} = 0.4 + 0.66 + 0.6 + 0.3 + 0.18 + 0.15 + 0.12 + 0.04$

$= 3.00$

$L = 0.6 + 0.66 + 0.6 + 0.4 + 0.3 + 0.25 + 0.2 + 0.18$

$= 3.19$

$\frac{\bar{L}}{L} = \frac{3.00}{3.19} \simeq 0.9404$

=> Fano

$\bar{L} = 3.0$

$\bar{l} = 0.6 + 0.4 + 0.4 + 0.4 + 0.2 + 0.2 + 0.2$

$= 2.4$

$\frac{\bar{L}}{\bar{l}} = \frac{3.0}{2.4} = 1.25$

=> Huffman

$\bar{L} = 3.0$

$\bar{l} = 0.6 + 0.4 + 0.4 + 0.4 + 0.2 + 0.2 + 0.15$

$= 2.63$

$\frac{\bar{L}}{\bar{l}} = \frac{3.0}{2.63} = 1.1407$

2. $[s_{ij}] = \begin{bmatrix} .08 & .18 & .12 & .02 \\ .08 & .01 & .04 & .12 \\ .14 & .02 & .03 & .01 \\ .10 & .04 & .01 & 0 \end{bmatrix}$

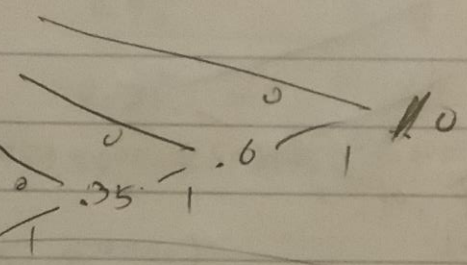
(a)

$f_1 = .40 \rightarrow s_1$

$f_2 = .25 \rightarrow s_2$

$f_3 = .20 \rightarrow s_3$

$f_4 = .15 \rightarrow s_4$



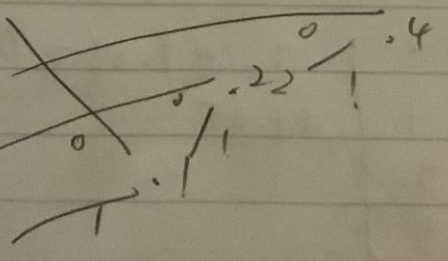
$s_1 = 0, \; s_2 = 10, \; s_3 = 110, \; s_4 = 111$

- $\bar{L} = .40 + .5 + .6 + .45 = 1.95 \rightarrow$ average number of bits

(b)

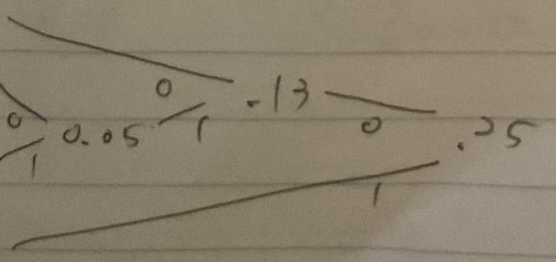
Context s_1

s_1 .08
s_2 .18
s_3 .12
s_4 .02

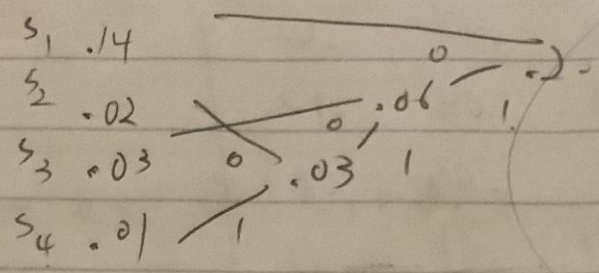


Context s_2

s_1 .08
s_2 .01
s_3 .04
s_4 .12

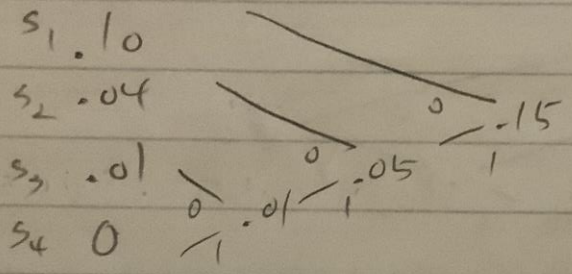


Context S_3



	S_1	S_2	S_3	S_4
S_1	110	00	0	0
S_2	0	010	110	10
S_3	10	011	10	110
S_4	111	1	111	111

Context S_4



S_2	S_1	S_1	S_2	S_4	S_2	S_1
10	00	110	0	1	10	00

(1) $T = \sum_{i=1}^{4}\sum_{j=1}^{4} l_{ij}\, s_{ij} = .24 + .14 + .24 + .06 + .16 + .03 + .12 + .12$

$+ .14 + .06 + .06 + .03 + .1 + .08 + .03$

$= 1.65$

$$[l_{ij}] = \begin{bmatrix} 3 & 1 & 2 & 3 \\ 2 & 3 & 3 & 1 \\ 1 & 3 & 2 & 3 \\ 1 & 2 & 3 & 3 \end{bmatrix}$$

16+8+4+2+1
4 3 2 1 0

249
256

1111011₀

1111100

128

0.25
0.125

129
128

3.

$t_a = .5,\ t_b = .3,\ t_c = .15,\ t_d = .05$

(a)

Number	a	l
	0	1
d	0.95	0.05
a	0.95	0.025
b	0.95+0.025×0.5=0.965	0.025×0.3=0.00175

$[0.965, 0.97500)$

$\frac{1}{2^8} \le 0.00175 \le \frac{1}{2^n}$

$0.9625 \le \frac{x}{256} \le 0.97$

$246.4 \le x \le 248.32$ $x = 248 \therefore \Rightarrow$ $\frac{248}{256} = \frac{124}{128} = \frac{62}{64} = \frac{31}{32}$

1111_2 $1111_2 \Rightarrow \therefore$ (11111)

(b) $r = (.011)_2 = 0.375$

a	l	$\frac{r-\alpha}{l}$	decode
0	1	0.375	a
0	0.5	$\frac{0.375}{0.5} = 0.75$	b
0.5×0.5=0.25	0.15	$\frac{0.125}{0.15} = 0.83$	c

∴ abc

1011 101

125 2.25

0.625

4. $s_a = s_b = s_c = s_d = \frac{1}{4}$

(a)

$H^{(1)}(S) = H(S^1)$

$= -\sum_{w \in S} s(w) \log_2 s(w) = 8$

(b)

$H^{(4)} = 64 - 8 = 56$

$= H(S^2) - H(S^1)$