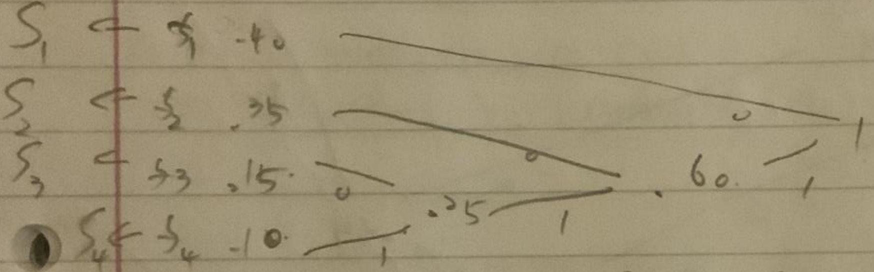
$S = \{s_1, s_2, s_3, s_4\}$ $s_1 = 000,\ s_2 = 001,\ s_3 = 01,\ s_4 = 1.$

$$[S_{ij}] = \begin{bmatrix} .04 & .25 & .04 & .07 \\ .20 & .08 & .06 & .01 \\ .09 & .01 & .04 & .01 \\ .07 & .01 & .01 & .01 \end{bmatrix}$$

7.1. (a) $s_1 = .40$ $s_2 = .35$ $s_3 = .15$ $s_4 = .10$



$S_1 = 01$ $S_2 = 10$ $S_3 = 110$ $S_4 = 111$

$\bar{l} = .8 + .7 + .45 + .3 = 2.25$ compression ratio $= \frac{\bar{L}}{\bar{l}} = \frac{2.65}{2.25}$

$\bar{L} = 1.2 + 1.05 + .3 + .1 = 2.65$

(b)

$6 \times s_1 s_1 = .04$ $5 \times s_3 s_1 = .09$ $\bar{L} = 2 \times 2.65 = 5.3$

$6 \times s_1 s_2 = .25$ $5\ s_3 s_2 = .01$

$5 \times s_1 s_3 = .04$ $4 \times s_3 s_3 = .04$

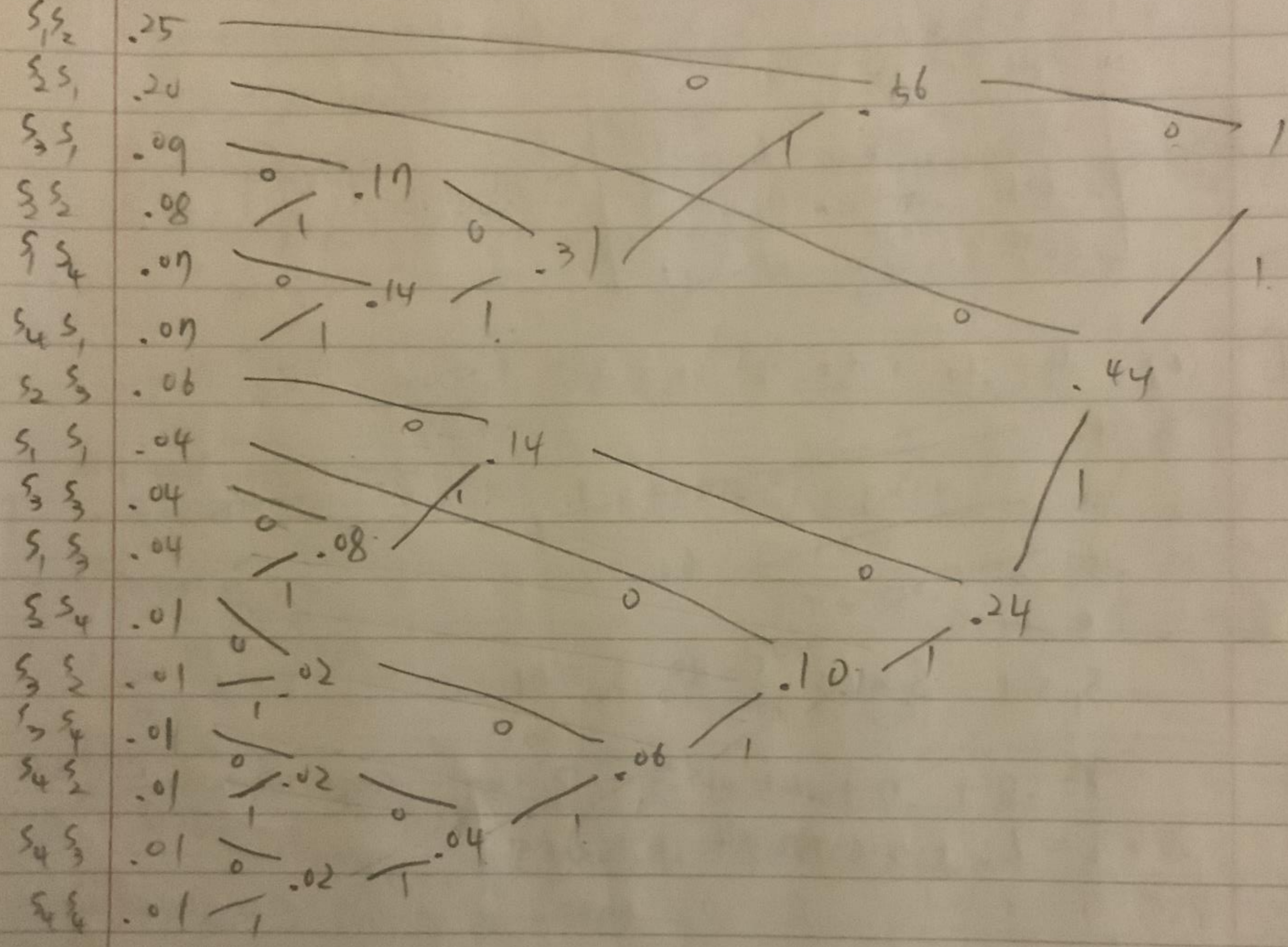
$4 \times s_1 s_4 = .07$ $3 \times s_3 s_4 = .01$

$6 \times s_2 s_1 = .20$ $4 \times s_4 s_1 = .07$

$6 \times s_2 s_2 = .08$ $4\ s_4 s_2 = .01$

$5 \times s_2 s_3 = .06$ $3\ s_4 s_3 = .01$

$4 \times s_2 s_4 = .01$ $2\ s_4 s_4 = .01$



s_1s_2	.25	00
s_2s_1	.20	10
s_3s_1	.09	0100
s_3s_2	.08	0101
s_1s_4	.07	0110
s_4s_1	.07	0111
s_2s_3	.06	1100
s_1s_1	.04	1110
s_3s_3	.04	11010
s_1s_3	.04	11011
s_2s_4	.01	111100
s_3s_2	.01	111101
s_3s_4	.01	1111100
s_4s_2	.01	1111101
s_4s_3	.01	1111110
s_4s_4	.01	1111111

$$\bar{l} = 0.5 + 0.4 + 0.36 + 0.32 + 0.28 + 0.28 + 0.24 + 0.16 + 0.20 + 0.20 + 0.06 + 0.06 + 0.07 + 0.07 + 0.07 + 0.07$$

$$= 3.34$$

$$\frac{\bar{L}}{\bar{l}} = \frac{5.3}{3.34}$$

(c)

	s_1	s_2	s_3	s_4
s_1	100	0	0	0
s_2	0	10	110	10
s_3	101	110	10	110
s_4	11	111	111	111

Context s_1: s_1 0.4, s_2 .25, s_3 .04, s_4 .07; .08, .15, .40

Context s_2: s_1 0.20, s_2 .08, s_3 .06, s_4 .01; .07, .15, .35

Context s_3: s_1 .04, s_2 .01, s_3 .04, s_4 .01; .02, .06, .15

Context s_4: s_1 .07, s_2, s_3 .01, s_4 .01; .02, .03, .10

s_2 s_3 s_1 s_3 s_1 s_1 s_3 s_2 s_3 s_3 s_1 s_4

10 10 0 101 0 100 101 110 110 10 0 11

Context s_2 s_2 s_1 s_3 s_1 s_1 s_3 s_2 s_3 s_3 s_1

(d) already got the possible context schemes from (c)

$$[l_{ij}] = \begin{bmatrix} 3 & 1 & 3 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 3 & 2 & 3 \\ 1 & 2 & 3 & 3 \end{bmatrix}$$

$$\bar{l}^{(1)}(s) = \sum_{i=1}^{4} \sum_{j=1}^{4} l_{ij} f_{ij} = 0.12 + 0.25 + 0.12 + 0.14 + 0.20 + 0.16 + 0.18 + 0.03 + 0.04 + 0.03 + 0.08 + 0.03 + 0.07 + 0.02 + 0.03 + 0.03 = 1.58$$

$$C.r. = \frac{2.65}{1.58}$$

7.1.2.

(a) This person believes they have achieved compression ratio of S^2 whose frequency actually had to be f_{ij}

(b) 27 is the compression ratio achieved by first-order Huffman encoding of the source alphabet

7.1.3.

(a) This person believes they achieved compression ratio of first-order Huffman-encoding

(b) He got the right compression ratio

7.1.4.

$$[f_{ij}] = \begin{bmatrix} .75 & .06 & .04 \\ .05 & .01 & .02 \\ .05 & .01 & .01 \end{bmatrix} \quad S = \{s_1, s_2, s_3\}$$

$f_1 = .85$ → 0

$f_2 = .08$ → 10

$f_3 = .07$ → 11

$.15$

$\bar{l}^{(0)} = 1 \times 0.85 + 2 \times 0.08 + 2 \times 0.07$

$= 0.85 + 0.16 + 0.14$

$= 1.15$

context s_1: s_1 0.75, s_2 0.06, s_3 0.04 — 0.10, 0.15

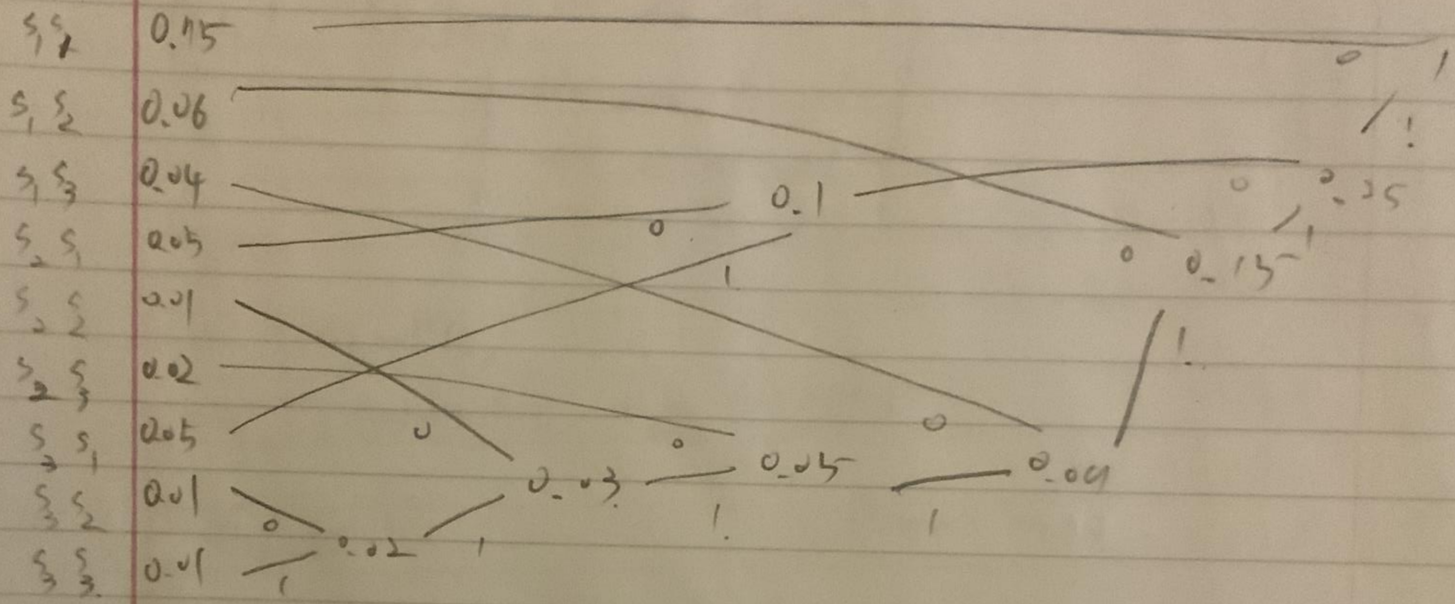
context 2 s_2: s_1 0.05, s_2 0.01, s_3 0.02 — 0.03, 0.08

context 3 s_3: s_1 0.05, s_2 0.01, s_3 0.01 — 0.02, 0.07

	s_1	s_2	s_3
s_1	0	0	0
s_2	10	10	10
s_3	11	11	11

$\bar{l}^{(1)} = 0.75 + 0.12 + 0.08 + 0.05 + 0.02 + 0.04$

$+ 0.05 + 0.02 + 0.02 = 1.15$



$s_1 s_1$: 0 $\quad$ $s_2 s_1$: 100 $\quad$ $s_3 s_1$: 101

$s_1 s_2$: 110 $\quad$ $s_2 s_2$: 111110 $\quad$ $s_3 s_2$: 1111110

$s_1 s_3$: 1110 $\quad$ $s_2 s_3$: 11110 $\quad$ $s_3 s_3$: 1111111

$$\bar{l}\ S^2 = 0.75 + 0.18 + 0.16 + 0.15 + 0.06 + 0.1 + 0.15 + 0.07 + 0.07$$

$$= 1.69$$

$$\frac{\bar{l}(S^2)}{2} = 0.845 < \bar{l}^{(1)} = 1.15$$

$$\therefore \bar{l}^{(1)} > \frac{\bar{l}(S^2)}{2}$$

7.2.1 $[t_{ij}] = \begin{bmatrix} .04 & .23 & .04 & .07 \\ .20 & .08 & .06 & .01 \\ .09 & .01 & .04 & .01 \\ .07 & .01 & .01 & .01 \end{bmatrix}$

$H^{(0)}$

$= H(S^1) - H(S^0)$

$= -\sum_{w \in S^1} f(w) \log_2 f(w) + \sum_{\substack{w \in S^0 \\ w > 0}} f(w) \log_2 f(w)$

$= 1.80$

$\bar{L} = 2.65$

$\Rightarrow$ shannon's bound

$\frac{\bar{L}}{\bar{l}^{(0)}} = \frac{2.65}{2.25} < \frac{\bar{L}}{H^{(0)}(S)} = \frac{2.65}{1.80}$

already got in 7.1.1.(a)

$H^{(1)}$

$= H(S^2) - H(S^1)$

$= H(S^2) - 1.80$

$= 3.30 - 1.80 = 1.5$

$\Rightarrow$ shannon's bound

$\frac{\bar{L}}{\bar{l}^{(1)}} = \frac{2.65}{1.58} < \frac{\bar{L}}{H^{(1)}(S)} = \frac{2.65}{1.5}$

already got in 7.1.1.(d)

7.2.2

$\eta^{(0)}$

$= H(s^{1}) - H(s^{0})$

$= 0.63$

$\eta^{(1)}$

$= H(s^{2}) - H(s^{1})$

$= 1.48 - 0.63$

$= 0.85$