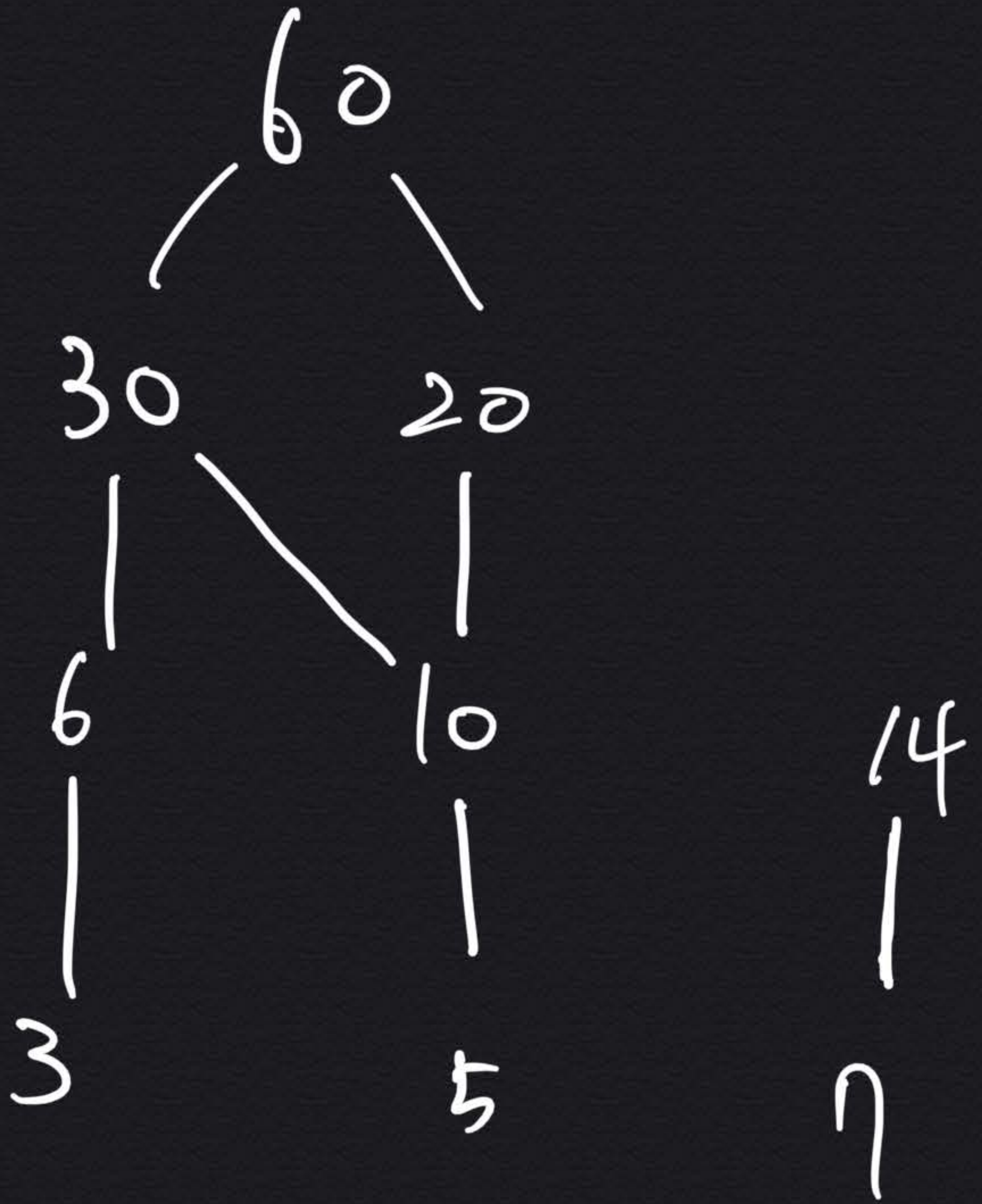
1.

1- a.



1-b

- The domain is the set of all integers. xRy if $x+y$ is odd.

  An integer z is odd if $z=2k+1$ for some integer k.

  =) Not an equivalence relation.

  X reflexive for all integer

- The domain is the set of all integers. aRb if and only if $a^2+b^2=0$

  =)  It is equivalent relation. Because there is only $0R0$.

  reflexive, symmetric, transitive.

1-C

- The domain is the set of all positive integers. x is related to y if $y = 3nx$, for some positive integer n.

=) This is a strict order, not total order.

- n is a positive integer =) X reflexive
- $y = 3nx$, $z = 3ny$, $z = (3n)^2 x$ =) transitive.
- 1 and 2 is not comparable =) X total order.

2-a

```
min1 := a1     }
min2 := a2     }  Because n≥2
For i=1 to n
    if (min1 < ai)
        min1 := ai
    End-if
End-for
For i=1 to n
    if (min2 < ai)
        if (min2 > min1)
            min2 := ai
        End-if
    End-if
End-for
Return(min2)
```

2-b

(a) $a = 2 * (b-1)-1 \ \{a>0\}$

$\Rightarrow 2*(b-1)-1 > 0$

$\Rightarrow 2*b - 2 - 1 > 0$

$\Rightarrow 2*b - 3 > 0$

$\Rightarrow 2*b > 3$

$\Rightarrow b > \frac{3}{2}$

(b) $b = (c+10)/3 \ \{b>6\}$

$\Rightarrow (c+10)/3 > 6$

$\Rightarrow c+10 > 18$

$\Rightarrow c > 8$

(c) $a = a+2*b-1 \ \{a>1\}$

$\Rightarrow a+2*b-1 > 1$

$\Rightarrow a+2*b > 2$

$\Rightarrow 2*b > 2-a$

$\Rightarrow b > \frac{2-a}{2}$

$\Rightarrow b > 1 - \frac{a}{2}$

(d) $a=2*b+1, b=a-3 \ \{b<0\}$

$\Rightarrow a-3<0$

$\Rightarrow a<3$

$\therefore a=2*b+1 \ \{a<3\}$

$\Rightarrow 2*b+1<3$

$\Rightarrow 2*b<2$

$\Rightarrow b<1$

(e) $a=3*(2*b+a); \ b=2*a-1 \ \{b>5\}$

(e) $a = 3*(2*b+a)$; $b = 2*a-1$ {$b>5$}

$\Rightarrow 2*a-1>5$

$\Rightarrow 2*a>6$

$\Rightarrow a>3$

$\therefore 3*(2*b+a)>3$

$\Rightarrow 2*b+a>1$

$\Rightarrow 2*b>1-a$

$\Rightarrow b>\frac{1-a}{2}$

2-C

- $f(n) = n \log_{41} n$ $\quad g = n \log n$

- $f(n) = n(\log \log n) + 3(\log n) + 12n$ $\quad g = \log \log n$

- $f(n) = n^{21} + (1.1)^n$ $\quad g = 1.1^n$

2-d

$f(n)=\frac{1}{2}n^5-100n^3+3n-1$, $g(n)=n^5$

By the definition

$f(n)=\Theta(n^5)$ if $f(n)=O(n^5)$ and $f(n)=\Omega(n^5)$

① proof $f(n)=O(n^5)$

$c=\frac{1}{2}$, $n_0=0$

So obviously for any $n\geq 0$ $\frac{1}{2}n^5\underbrace{-100n^3+3n-1}_{\text{always } <0 \text{ when } n\geq 0} \leq \frac{1}{2}n^5$

$\therefore f(n)=O(n^5)$

② proof $f(n)=\Omega(n^5)$

same way.

② proof $f(n) = \Omega(n^5)$

same way.

$c = 1, \quad n_0 = 1$

$\frac{1}{2}n^5 - 100n^3 + 3n - 1 \le n^5$

$n^5 \ge \frac{1}{2}n^5$, $\quad -100n^3 + 3n - 1 \le 0$ when $n \ge 1$

$\therefore f(n) = \Omega(n^5)$

Therefore, $f(n) = \Theta(n^5)$

3-a

- 2 6 18 54 162 486

- 2 5 8 11 14 17

- 27 9 3 1 $\frac{1}{3}$ $\frac{1}{9}$

3-b

$a_0 = 30.000\$$

$a_n = (1.005)a_{n-1} - 600$