Huffman's result.

1. $s_1 = 1111$ $p_1 = .40$ — s_1 1
$s_2 = 1110$ $p_2 = .15$ — s_2 010
$s_3 = 10$ $p_3 = .10$ — s_3 011
$s_4 = 01$ $p_4 = .09$ — s_4 0000
$s_5 = 1101$ $p_5 = .09$ — s_5 0001
$s_6 = 1100$ $p_6 = .04$ — s_6 0010
$s_7 = 00$ $p_7 = .08$ — s_7 0011

(a) shannon

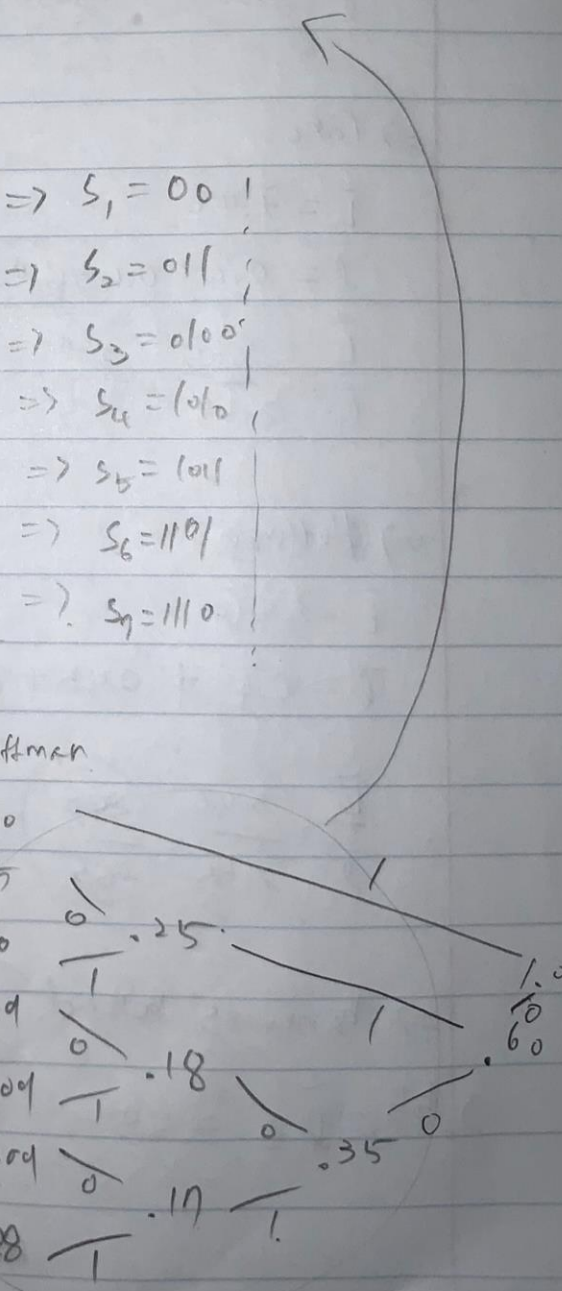
$(\frac{1}{2})^{le} \le .40$ $le = 2$ $F_1 = 0$ $(.00\ldots)_2$ $\Rightarrow s_1 = 00$
$(\frac{1}{2})^{le} \le .15$ $= 3$ $F_2 = .4$ $(.011\ldots)_2$ $\Rightarrow s_2 = 011$
$(\frac{1}{2})^{le} \le .10$ $= 4$ $F_3 = .55$ $(.0\,1000\ldots)_2$ $\Rightarrow s_3 = 0100$
$(\frac{1}{2})^{le} \le .09$ $= 4$ $F_4 = .65$ $(.1010\ldots)_2$ $\Rightarrow s_4 = 1010$
$(\frac{1}{2})^{le} \le .09$ $= 4$ $F_5 = .74$ $(.1011\ldots)_2$ $\Rightarrow s_5 = 1011$
$(\frac{1}{2})^{le} \le .09$ $= 4$ $F_6 = .83$ $(.1101\ldots)_2$ $\Rightarrow s_6 = 1101$
$(\frac{1}{2})^{le} \le .08$ $= 4$ $F_7 = .92$ $(.1110\ldots)_2$ $\Rightarrow s_7 = 1110$

(b) Fano

s_1 $s_1 \to 0$ $\Rightarrow 0$
s_2 $s_2 \to 1\,0\,0$ $\Rightarrow 100$
s_3 $s_3 \to 1\,0\,1$ $\Rightarrow 101$
s_4 $s_4 \to 1\,1\,0\,0$ $\Rightarrow 1100$
s_5 $s_5 \to 1\,1\,0\,1$ $\Rightarrow 1101$
s_6 $s_6 \to 1\,1\,1\,0$ $\Rightarrow 1110$
s_7 $s_7 \to 1\,1\,1\,1$ $\Rightarrow 1111$

(c) Huffman

s_1 .40
s_2 .15
s_3 .10
s_4 .09
s_5 .09
s_6 .04
s_7 .08



(d). Compression ratio.

=> shannon

$\bar{L} = 1.6 + 0.6 + 0.2 + 0.18 + 0.36 + 0.36 + 0.16 = 3.46$

$\bar{l} = 0.8 + 0.45 + 0.4 + 0.36 + 0.36 + 0.36 + 0.32 = 3.05$

$\frac{\bar{L}}{\bar{l}} = \frac{3.46}{3.05} = \frac{346}{305}$

=> Fano

$\bar{L} = 3.46$

$\bar{l} = 0.4 + 0.45 + 0.3 + 0.36 + 0.36 + 0.36 + 0.32 = 2.55$

$\frac{\bar{L}}{\bar{l}} = \frac{3.46}{2.58} = \frac{346}{255}$

=> Huffman

$\bar{L} = 3.46$

$\bar{l} = 0.4 + 0.45 + 0.36 + 0.36 + 0.36 + 0.32 + 0.3 = 2.55$

$\frac{\bar{L}}{\bar{l}} = \frac{3.46}{2.55} = \frac{346}{255}$

=> shannon's bound.

$2.55 \leq \bar{l} \leq 3.55 \quad \Rightarrow \frac{3.46}{3.55} \leq \frac{\bar{L}}{\bar{l}} \leq \frac{3.46}{2.55} = \frac{346}{355} \leq \frac{\bar{L}}{\bar{l}} \leq \frac{346}{255}$

2. $S = \{0,1\}^2 = \{s_1, s_2, s_3, s_4\}$

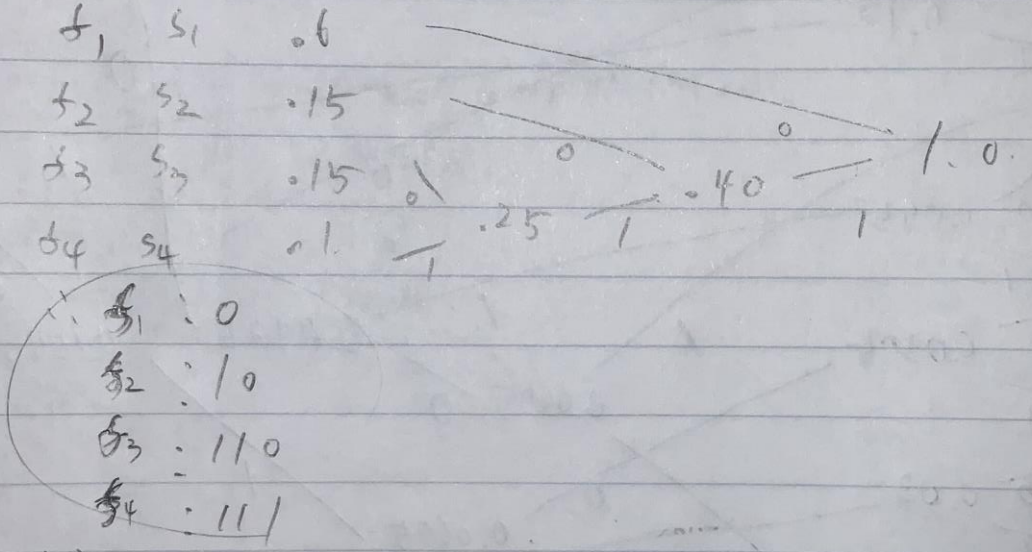
$s_1 = 00$ $\quad t_1 = .6$

$s_2 = 01$ $\quad t_2 = .15$

$s_3 = 10$ $\quad t_3 = .15$

$s_4 = 11$ $\quad t_4 = .1$

(a)



$s_1 : 0$

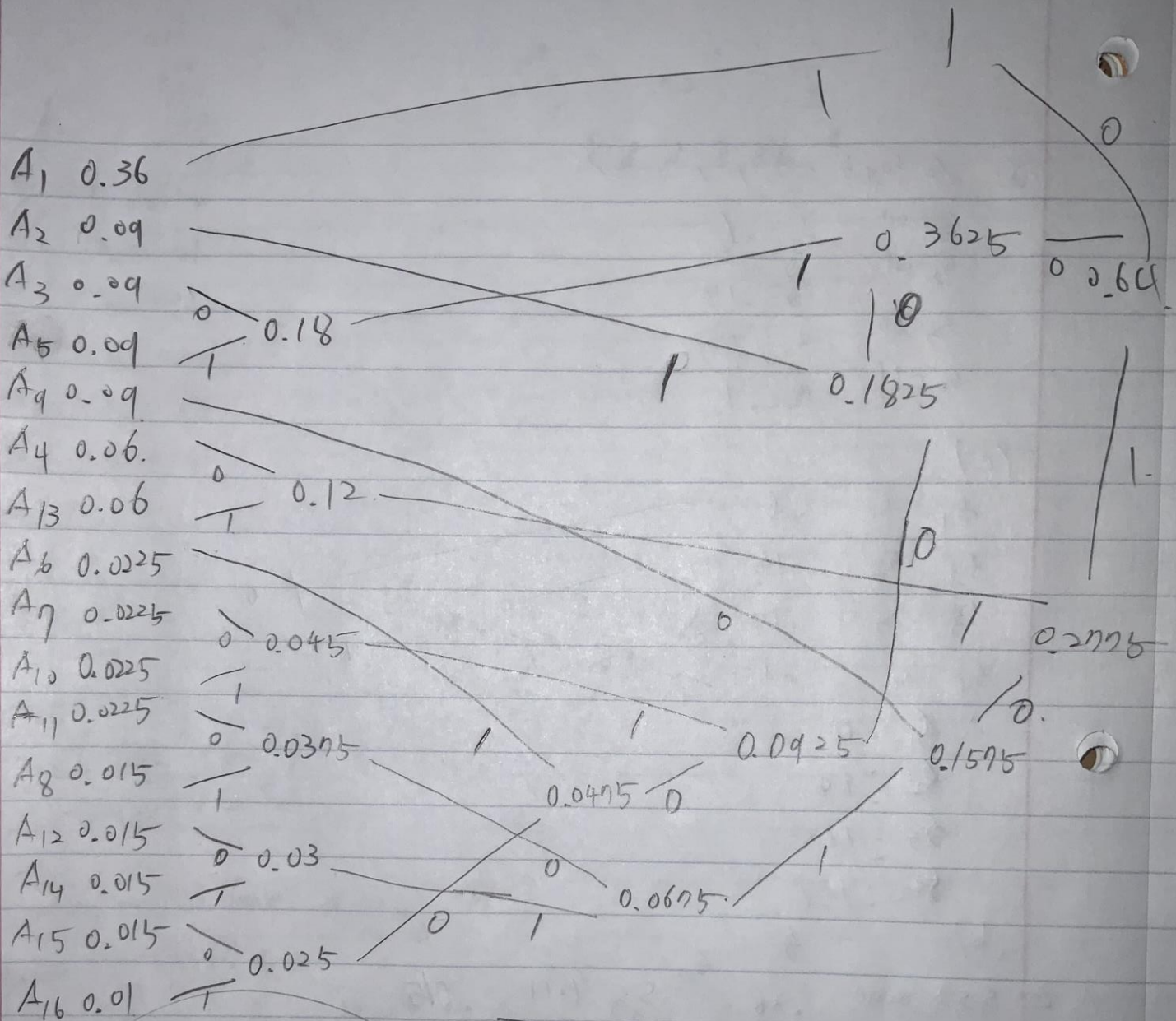
$s_2 : 10$

$s_3 : 110$

$s_4 : 111$

(b)

		t		t
$A_1 \leftarrow s_1 s_1$	0000	.36	$A_{12} \leftarrow s_3 s_4$	1011	.015
$A_2 \leftarrow s_1 s_2$	0001	.09	$A_{13} \leftarrow s_4 s_1$	1100	.06
$A_3 \leftarrow s_1 s_3$	0010	.09	$A_{14} \leftarrow s_4 s_2$	1101	.015
$A_4 \leftarrow s_1 s_4$	0011	.06	$A_{15} \leftarrow s_4 s_3$	1110	.015
$A_5 \leftarrow s_2 s_1$	0100	.09	$A_{16} \leftarrow s_4 s_4$	1111	.01
$A_6 \leftarrow s_2 s_2$	0101	.0225			
$A_7 \leftarrow s_2 s_3$	0110	.0225			
$A_8 \leftarrow s_3 s_4$	0111	.015			
$A_9 \leftarrow s_3 s_1$	1000	.09			
$A_{10} \leftarrow s_3 s_2$	1001	.0225			
$A_{11} \leftarrow s_3 s_3$	1010	.0225			



A_1: 1
A_2: 0001
A_3: 0010
A_4: 0110
A_5: 0011
A_6: 000001
A_7: 000010
A_8: 01101
A_9: 0100
A_{10}: 000011
A_{11}: 01 0100
A_{12}: 01 0110
A_{13}: 0111
A_{14}: 01011
A_{15}: 0000000
A_{16}: 0000001

$\bar{L} = 4(0.36 + 4\times0.09 + 2\times0.06 + 4\times0.0225 + 4\times0.015 + 0.01)$
$= 4$

$\bar{l} = 0.36 + 4\times0.09 + 4\times0.09 + 4\times0.06 + 4\times0.09 + 6\times0.0225 + 6\times0.0225 + 6\times0.015 + 4\times0.09 + 6\times0.0225 + 6\times0.0225 + 6\times0.015 + 4\times0.06 + 6\times0.015 + 4\times0.05 + 4\times0.01 = 3.265$

$\therefore \frac{\bar{L}}{\bar{l}} = \frac{4}{3.265}$ or 2. Compression ratio

(c)

- Source entropy $H(S)$

$$= \sum_{i=1}^{m} p_i l_i$$

$$= \bar{l}$$

$$= 3.265$$

(same in Huffman's method.)

- Shannon's bound of compression ratio

$$3.265 \leq \bar{l} \leq 4.265$$

$$\Rightarrow \frac{4}{4.265} \leq \frac{L}{\bar{l}} \leq \frac{4}{3.265}$$

3. $s_1 = 000$

$s_2 = 001$

$s_3 = 01$

$s_4 = 010$

$s_5 = 110$

$s_6 = 101$

$s_7 = 111$

$s_8 = 100$

$s_9 = ?$

- Find s_9

$1 - (2 \cdot 2^{-4} + 6 \cdot 2^{-3}) = 2^{-3}$.

$\Rightarrow$ (011) $= s_9$

- All possible leaves of binary files

$0 \leq \text{length}(v) \leq 3$.

Length 0 : λ = empty word.

Length 1 : 0, 1

Length 2 : 00, 01, 10, 11

Length 3 : 010.