Set 1

(a)

- $A = \{x \in \mathbb{N} : x < 0\}$

→ $A = \{\} = \emptyset$

- A consists of the first five prime numbers

→ $A = \{2, 3, 5, 7, 11\}$

- $A = \{x \in \mathbb{Z} : 2x^2 + x - 1 = 0\}$

→ $2x^2 + x - 1 = 0$

→ $(2x - 1)(x + 1) = 0$

→ $x = \frac{1}{2}, -1$

→ $A = \{-1\}$

b.

- $A = \{1, 2, 3, 4, 5\}$

→ $A = \{x \in N : x < 6\}$

- $A = \{4, 6, 8, 9, 10\}$

→ $A = \{x \in N : 1 < x < 11$ and x is not prime number $\}$

c

→ $\{a, b, c, d\}$ and $\{d, b, a, c\}$ are equal

→ $\{d, a, c\}$ and $\{a, a, d, e, c, e\}$ are equal

d

(d) {a, a, d, e, c, e} and {a, a, d, e, c, e} are equal

$A = \{c, d, f, g\}$

$B = \{f, j\}$

$C = \{d, g\}$

- is $B \subseteq A$?

$B \not\subseteq A$ $\Rightarrow$ $j \notin A$ False

- is $C \subseteq A$?

$C \subseteq A$ $\Rightarrow$ $d \in A$, $g \in A$ True

- is $C \subseteq C$

$\Rightarrow C = C$

$C \subseteq C$ True

e

- $A \cup B$

→ $A \cup B = \{ a, b, c, d, f, g \}$

- $A \cap B$

→ $A \cap B = \{ b, c \}$

- $A - B$

→ $A - B = \{ d, f, g \}$

5.

- $\mathbb{Z}^+ \subset \mathbb{Q}$

True

- $\mathbb{R}^- \subset \mathbb{Q}$

False $-\sqrt{2}$ is not rational

- $\mathbb{Q} \subset \mathbb{Z}$

False $\frac{1}{4}$ is not integer

9.

- $A \times B$

$A \times B = \{(x,a), (x,b), (y,a), (y,b), (z,a), (z,b), (w,a), (w,b)\}$

- $B \times A$

$B \times A = \{(a,x), (a,y), (a,z), (a,w), (b,x), (b,y), (b,z), (b,w)\}$

- $A \times A$

$A \times A = \{(x,x), (x,y), (x,z), (x,w), (y,x), (y,y), (y,z), (y,w), (z,x), (z,y), (z,z), (z,w), (w,x), (w,y), (w,z), (w,w)\}$

h.

$A \times (B \cup C) = (A \times B) \cup (A \times C)$ ↙ Definition of Cartesian product

$\Rightarrow [x \in A) \cap (y \in (B \cup C))$

$\Rightarrow (x \in A) \cap ((y \in B) \cup (y \in C))$ ↙ Def. of union

$\Rightarrow ((x \in A) \cap (y \in B)) \cup ((x \in A) \cap (y \in C))$ ↙ Distribution law

$\Rightarrow ((x,y) \in A \times B) \cup ((x,y) \in A \times C)$ ↙ Def of Cartesian product

$\Rightarrow (A \times B) \cup (A \times C)$

i

Product

i

- $\bar{A} \cap (A \cup B) = \bar{A} \cap B$

$\Rightarrow (\bar{A} \cap A) \cup (\bar{A} \cap B)$ ← Distribution rule

$\Rightarrow \emptyset \cup (\bar{A} \cap B)$ ← Complement rule

$\Rightarrow \bar{A} \cap B$ ← identity law

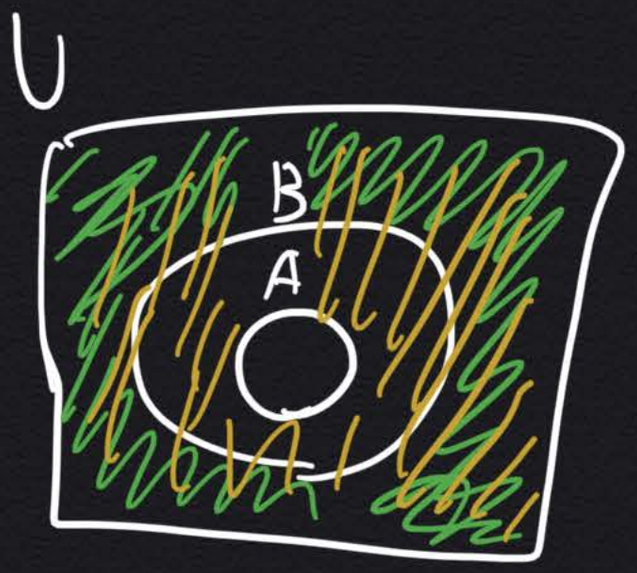
- $\bar{A} \cup (A \cap B) = \bar{A} \cup B$

$\Rightarrow (\bar{A} \cup A) \cap (\bar{A} \cup B)$ ← Distribution rule

$\Rightarrow U \cap (\bar{A} \cup B)$ ← Complement rule

$\Rightarrow \bar{A} \cup B$ ← Domination law

- if $A \subseteq B$ then $\bar{B} \subseteq \bar{A}$



$\bar{A}$

$\bar{B}$

$\Rightarrow \bar{B} \subseteq \bar{A}$

- $A - (B - A) = A$

$\Rightarrow A \cap (B \cap \bar{A}) = A$

$\Rightarrow A \cap (\bar{B} \cup A) = A$ (De Morgan's law)

$\Rightarrow A \cap (A \cup \bar{B}) = A$ (commutative law)

$\Rightarrow A$ (Absorption law)