

Homework 4

All homework must be turned in on PDF format. This can be scanned or typed in any paper size, but the format must be PDF and the file must be readable. This document can be modified for your homework submission. An additional homework template is available on Canvas to assist you in creating your answers, and content from lecture notes can be used.

- All final answers must be circled or in green.
- All homework must have a name on the top of **every** page.
- Submission errors (not in PDF, illegible, etc.) will not be re-graded.

Problem 1

Simplify the following equations into their minimal sum-of-products form using Boolean algebra. Every step annotated with a postulate or theorem.

$$\overline{\bar{a}b} \cdot \overline{\bar{a}\bar{b}} + c \cdot (\bar{c} + \bar{c}d) :: \text{given}$$

The image shows a handwritten derivation of a Boolean algebra simplification. The steps are as follows:

- $\overline{\bar{a}b} \cdot \overline{\bar{a}\bar{b}} + c \cdot (\bar{c} + \bar{c}d)$ (given)
- $= \overline{\bar{a}b} \cdot \overline{\bar{a}\bar{b}} + c \cdot (\bar{c} + \bar{c}d)$ (De Morgan's theorem ($\overline{A+B} = \overline{A} \cdot \overline{B}$))
- $= (\overline{\bar{a}b} + \overline{\bar{a}\bar{b}}) (\bar{c} + \overline{\bar{c}d})$ (De Morgan's theorem ($\overline{A+B+C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$))
- $= (\bar{a}b + \bar{a}\bar{b}) (\bar{c} + \overline{\bar{c}(1+d)})$
- $= (\bar{a}b + \bar{a}\bar{b}) (\bar{c} + \bar{c})$ (Postulate $\bar{c} + c = 1$)
- $= (\bar{a}b + \bar{a}\bar{b}) \cdot 1$
- $= \bar{a}b + \bar{a}\bar{b}$
- $= \bar{a}(\bar{b} + b)$ (Postulate $\bar{b} + b = 1$)
- $= \bar{a}$ (Postulate $\bar{a} + a = 1$)

A vertical bracket on the right side of the derivation is labeled "Mengyan Yang".

Problem 2

Minimize the Boolean equations represented by the following equations switching equations. Postulates and theorems of intermediate steps are required.

1. $f(a, b, c) = \sum m(0, 1, 5, 7)$ (into minimum SOP form)
2. $f(a, b, c, d) = \sum m(0, 1, 10, 11, 15)$ (into minimum SOP form)
3. $f(a, b, c, d) = \prod M(2, 3, 6, 7, 10, 11, 14, 15)$ (into minimum POS form)
4. $f(a, b, c, d) = \sum m(2, 3, 6, 7, 8, 9, 12, 13)$ (into minimum SOP form)

2.

- ① $f_{ab}(c) = \sum_m (a, b, c, m)$

$$\begin{aligned}
 &= ab(\bar{c} + \bar{a}\bar{b}) + \bar{a}b(c + \bar{a}b) \\
 &= \bar{a}b(\bar{c} + c) + (\bar{a} - a)b(c + \bar{a}b) \\
 &\quad \text{d}(\bar{a}, b, c) = \bar{a}\bar{b} + b\bar{c}
 \end{aligned}$$

- ② $f_{abc}(d) = \sum_m (a, b, c, d, m)$

$$\begin{aligned}
 &= abc\bar{d} + \bar{a}bc\bar{d} + a\bar{b}cd + ab\bar{c}d \\
 &= \bar{a}\bar{b}\bar{c}(\bar{d} + d) + \bar{a}\bar{b}c(d + \bar{d}) + a\bar{b}cd \\
 &\quad (\bar{a} + a = 1) \\
 &\quad \text{d} = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + a\bar{b}cd \\
 &A - BC = (A + B)(C + D) \\
 &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}c\bar{b} + a\bar{b}cd \\
 &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}c\bar{b} + a\bar{b}cd \\
 &\quad \text{d} = \bar{b}c(a + \bar{a}) + a\bar{b}cd \\
 &\quad \text{d} = \bar{b}c(a + \bar{a}) + a\bar{b}cd \\
 &\quad \text{d} = \bar{b}c(a + \bar{a}) + a\bar{b}cd
 \end{aligned}$$

(3) $\bar{f}(a, b, c, d) = \bar{f}_M(2, 3, 6, 4, 1, 4, 15)$

$$\begin{aligned}
 &= \Sigma_m(0, 1, 4, 5, 8, 9, 12, 13) \\
 &= \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d}) + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} \\
 &\quad + \bar{a}\bar{b}\bar{c}\bar{d} \\
 &= \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d}) + \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}) + \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}) \\
 &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} \\
 &= \bar{a}\bar{c}(\bar{b} + \bar{a}) + \bar{a}\bar{c}(\bar{b} + \bar{a}) \\
 &= \bar{a}\bar{c} + \bar{a}\bar{c} \\
 &= \bar{c}(\bar{a}, \bar{b}, \bar{c}, \bar{d}) \quad (\text{circled})
 \end{aligned}$$

(4) $f(a, b, c, d) = \Sigma_m(1, 2, 3, 6, 7, 8, 9, 12, 13)$

$$\begin{aligned}
 &= \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} \\
 &= \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d}) + \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}) + \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}) \\
 &= \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d}) + \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}) + \bar{a}\bar{b}\bar{c}(\bar{d} + \bar{a}) \\
 &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} \\
 &= \bar{a}\bar{c}(\bar{b} + \bar{a}) + \bar{a}\bar{c}(\bar{b} + \bar{a}) \\
 &= \bar{a}\bar{c} + \bar{a}\bar{c} \\
 &= \bar{c}(\bar{a}, \bar{b}, \bar{c}, \bar{d}) \quad (\text{circled})
 \end{aligned}$$

Mengary f_M

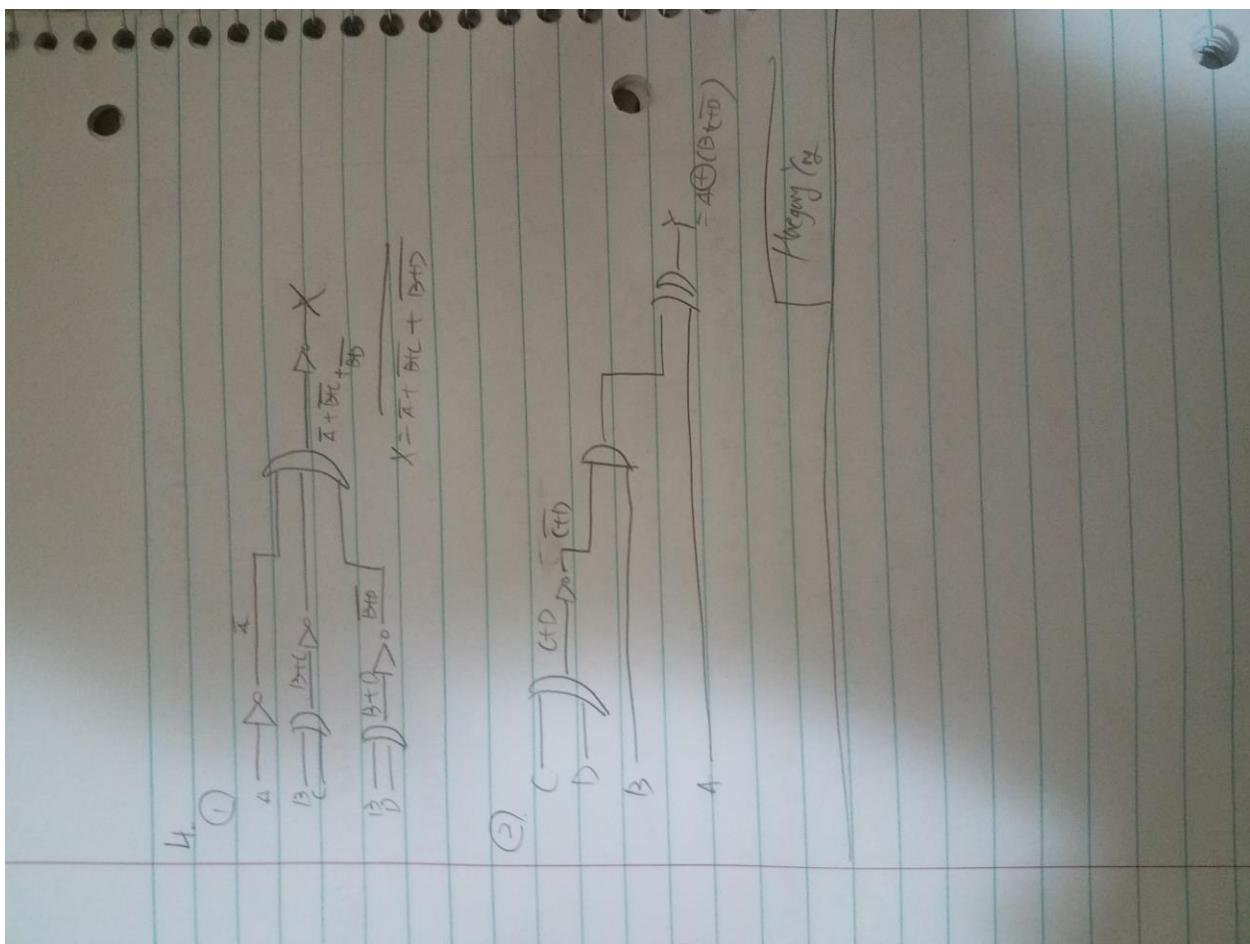
Problem 3

For the four equations of the previous problem, fill in the truth table below (no intermediate work is necessary). (The table is split into two to save space.)

Problem 4

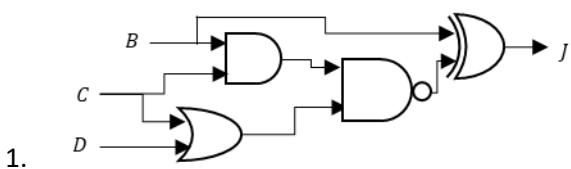
Convert the following Boolean equations into equivalent logic/switching/Boolean circuits (no intermediate work is necessary). Do **not** perform any optimization of these equations.

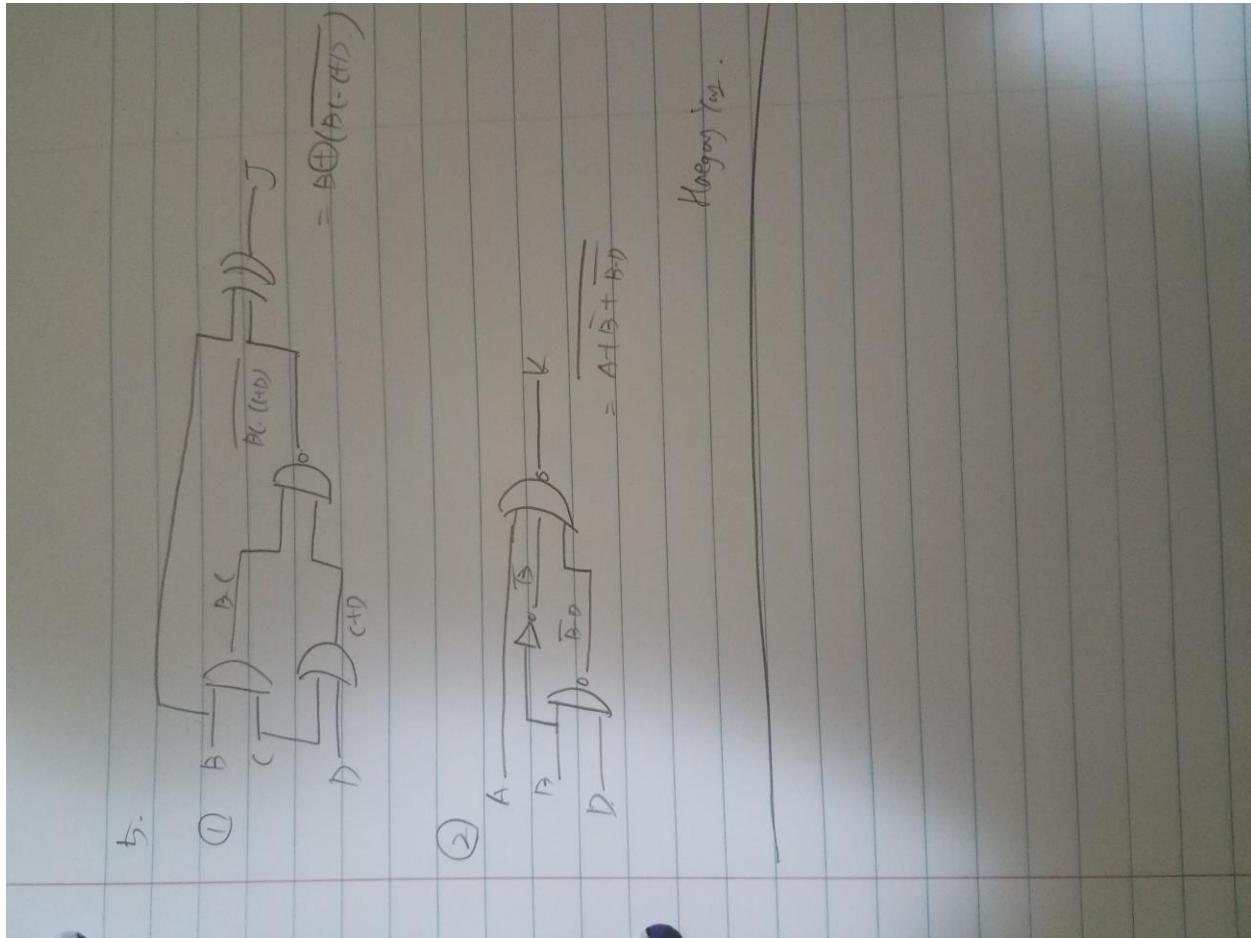
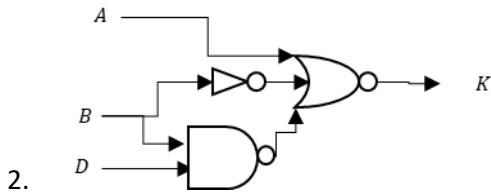
- $X = \overline{\overline{A} + \overline{B} + \overline{C} + \overline{B} + D}$
 - $Y = A \oplus (B \cdot \overline{\overline{C} + D})$



Problem 5

Write the equation represented by this switching circuits. Do **not** perform any optimization of the equation. No intermediate work is required.





	Expression	Dual
P2	$a + 0 = a$	$a \cdot 1 = a$
P3	$a + b = b + a$	$ab = ba$
P4	$a + (b + c) = (a + b) + c$	$a(bc) = (ab)c$
P5	$a + bc = (a + b)(a + c)$	$a(b + c) = ab + ac$
P6	$a + \bar{a} = 1$	$a \cdot \bar{a} = 0$
T1	$a + a = a$	$a \cdot a = a$
T2	$a + 1 = 1$	$a \cdot 0 = 0$
T3	$\bar{\bar{a}} = a$	
T4	$a + ab = a$	$a(a + b) = a$
T5	$a + \bar{a}b = a + b$	$a(\bar{a} + b) = ab$

T6	$ab + a\bar{b} = a$	$(a + b)(a + \bar{b}) = a$
T7	$ab + a\bar{b}c = ab + ac$	$(a + b)(a + \bar{b} + c) = (a + b)(a + c)$
T8	$\overline{a + b} = \bar{a}\bar{b}$	$\overline{ab} = \bar{a} + \bar{b}$
T9	$ab + \bar{a}c + bc = ab + \bar{a}c$	$(a + b)(\bar{a} + c)(b + c) = (a + b)(\bar{a} + c)$
T10(a)	$f(x_1, x_2, \dots, x_n) = x_1(f(1, x_2, \dots, x_n) + \bar{x}_1 f(0, x_2, \dots, x_n))$	
T10(b)	$f(x_1, x_2, \dots, x_n) = [x_1 + f(0, x_2, \dots, x_n)][\bar{x}_1 + f(1, x_2, \dots, x_n)]$	