

1. Multiple choices

(1) D The sample sizes need to be equal

(2) B 0.10

(3) A True

2.

$$\bar{X} \pm z_{\frac{\alpha}{2}} \left( \frac{s}{\sqrt{n}} \right)$$

$$\bar{X} = \text{Mean} = 245.8$$

$$\alpha/2 = (1 - 0.95)/2 = 0.025$$

$$z_{\frac{\alpha}{2}} = 1.96$$

$$\bar{X} \pm z_{\frac{\alpha}{2}} \left( \frac{s}{\sqrt{n}} \right) = 245.8 \pm 1.96 * \frac{23.64}{\sqrt{15}} = 245.8 \pm 11.96$$

$$\Rightarrow \mathbf{[233.84, 257.76]}$$

3.

$$(a) \hat{p} = \frac{864}{1234} = 0.70$$

$$(b) \hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\alpha/2 = (1-0.95)/2 = 0.025$$

$$z_{\frac{\alpha}{2}} = 1.96$$

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.70 \pm 1.96 \sqrt{\frac{0.70(1-0.70)}{1234}}$$

$$[0.674, 0.725]$$

4.

$$L(\theta_1, \theta_2) = (2\pi\theta_2)^{-\frac{n}{2}} \exp \left( -\frac{1}{2\theta_2} \sum_{i=1}^n (x_i - \theta_1)^2 \right)$$

$$\ln L(\theta_1, \theta_2) = -\frac{n}{2} \ln 2\pi\theta_2 + \frac{1}{2\theta_2} \sum_{i=1}^n (x_i - \theta_1)^2$$

- Take derivative of  $\theta_1$

$$\frac{d \ln L(\theta_1, \theta_2)}{d \theta_1} = \sum_{i=1}^n x_i - \sum_{i=1}^n \theta_1 = n\bar{X} - n\theta_1 = 0$$

$$\theta_1 = \bar{X}$$

- Take derivative of  $\theta_2$

$$\frac{d \ln L(\theta_1, \theta_2)}{d \theta_2} = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^n (x_i - \bar{X})^2 = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^n (x_i - \bar{X})^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{X})^2 = v$$

5.

$$\text{pdf } g_4(y) = \frac{\frac{5!}{3!} \left[ \frac{y^2}{4} \right]^3 \left[ 1 - \frac{y^2}{4} \right] y}{2} = 10 \left( \frac{y^7}{64} - \frac{y^9}{256} \right)$$

$$\text{mean} = \int_0^2 10y \left( \frac{y^7}{64} - \frac{y^9}{256} \right) dy = \frac{160}{99} = 1.616$$