Q1

(1) 10^{th} percentile : $\frac{10(21+1)}{100} = 2.2^{th}$

$\Rightarrow 7.4 \times 0.8 + 8.3 \times 0.2$

$= 7.58$

50^{th} percentile : $\frac{50(21+1)}{100} = 11^{th}$

$\Rightarrow 11.6$

(2) Five number summary.

$Q_1 = 0.25(21+1) = 5.5^{th}$

$= 8.7 \times 0.55 + 9.3 \times 0.45$

$= 8.97$

$Q_2 = 0.5(21+1) = 11^{th}$

$= 11.6$

$Q_3 = 0.75(21+1) = 16.5^{th}$

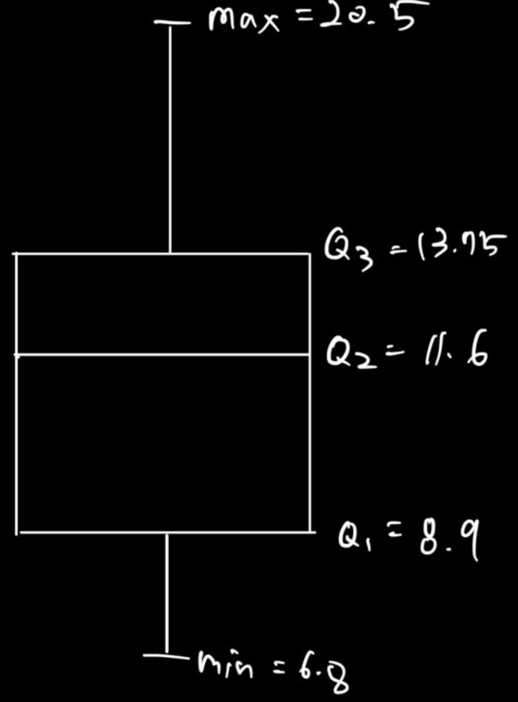
$= 13.6 \times 0.5 + 13.9 \times 0.5$

$= 13.75$

$\therefore$ 5-number summary : 6.8, 8.9, 11.6, 13.75, 20.5

Box plot

Median = 11.6 = Q_2



(3)

$IQR = Q_3 - Q_1 = 13.75 - 8.97 = 4.78$

$Range = Max - Min = 20.5 - 6.8$

$= 13.7$

(4)

$LIF = Q_1 - 1.5 \times IQR$

$= 8.97 - 1.5 \times 4.78$

$= 1.8$

$UIF = Q_3 + 1.5 \times IQR$

$= 13.75 + 1.5 \times 4.78$

$= 20.92$

→ The wind speed on 22nd day is 30 mph which is bigger than UIF. So it is an outlier

→ The wind speed on 23rd day is 20.2 mph which is smaller than UIF. So it is not an outlier

Q_2

(1) mean : 0, variance : $\frac{5\times1^2}{5} = 1$

(2) mean : $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

⇒ in this case $n=5$.

Variance

$var = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

⇒ in this case, $n=5$

(3) $x_1 = 0.4,\ x_2 = -0.3,\ x_3 = 1.3,\ x_4 = -1.7,\ x_5 = 0.1$

$$mean = \frac{0.4 + (-0.3) + 1.3 + (-1.7) + 0.1}{5}$$

$$= \frac{-0.2}{5}$$

$$= -0.04$$

$$var = \frac{1}{4}\left\{(0.44)^2 + (-0.26)^2 + (1.34)^2 + (-1.66)^2 + (0.14)^2\right\}$$

$$= 1.208$$

⇒ The true mean 0 and our sample mean −0.04 are almost same. Also, the true variance 1 and our sample variance 1.208 are close

(4)

$\bar{X}$ is a stat of the samples.

All linear combination of normal random variables is normal with their mean and variance

$\Rightarrow E(\bar{X}) = 0, \quad var(\bar{x}) = \frac{1}{n}$

$\Rightarrow \bar{X} \sim N(0, \frac{1}{5})$

$\Rightarrow$ $\bar{x}$ will have normal distribution with 0 and variance $\frac{1}{5}$

Q3

(1) CDF of Y_i, $F_r(x)$

$= P(Y_i \le x)$

$$F_r(x) = \sum_{i=r}^{n} \binom{n}{i} [F(x)]^{i} [1-F(x)]^{n-i}$$

$\Rightarrow$ put $r=2$

$$F_2(y) = \sum_{i=2}^{n} \binom{n}{i} [F(x)]^{i} [1-F(x)]^{n-i}$$

(2) PDF of rth order stat $\Rightarrow f_r(y)$

$$f_r(y) = F_r'(y) = \sum_{k=r}^{n-1} \binom{n}{k}(k)[F(y)]^{k-1} \cdot f(y)[1-F(y)]^{n-k}$$

$$+ \sum_{k=r}^{n-1} \binom{n}{k}[F(y)]^{k}(n-k)[1-F(y)]^{n-k-1}[-f(y)]$$

$$+ n[F(y)]^{n-1} f(y)$$

$\Rightarrow \binom{n}{k}k = n\binom{n-1}{k-1}$, $\binom{n}{k}(n-k) = n\binom{n-1}{k}$,

it follows that the pdf of $Y_{(r)}$ is

$$\Rightarrow f_r(y) = \frac{n!}{(r-1)!(n-r)!}[F(y)]^{r-1}[1-F(y)]^{n-r} f(y) \quad \text{for } a<y<b$$

(3) $n=5$

$f(x) = \frac{1}{3}x, \quad 0 \le x \le 2$

$F(y) = \frac{1}{6}x^2, \quad 0 \le x \le 2$

$\Rightarrow$ CDF

$\Rightarrow \sum_{i=2}^{5} \binom{5}{2} \left[\frac{1}{6}x^2\right]^{i} \left[1-\frac{1}{6}x^2\right]^{5-i}$

$\Rightarrow 10\left(\frac{1}{6}x^2\right)^2 \left(1-\frac{1}{6}x^2\right)^3 + 10\left(\frac{1}{6}x^2\right)^3\left(1-\frac{1}{6}x^2\right)^2 + 10\left(\frac{1}{6}x^2\right)^4\left(1-\frac{1}{6}x^2\right) + 10\left(\frac{1}{6}x^2\right)^5$

(4) CDF of $Y_{(4)}$

$\Rightarrow \sum_{i=4}^{5} \binom{5}{4} \left[\frac{1}{6}x^2\right]^{i} \left[1-\frac{1}{6}x^2\right]^{5-i}$

$\Rightarrow 5\left(\frac{1}{6}x^2\right)^4\left(1-\frac{1}{6}x^2\right) + 5\left(\frac{1}{6}x^2\right)^5$

PDF of $Y_{(4)}$

$\Rightarrow \frac{5!}{3!}\left(\frac{1}{6}x^2\right)^3\left(1-\frac{1}{6}x^2\right)\cdot\frac{1}{3}x$